

Desk Copy

6th Grade

See page 151 - 154

New work - page 214

Page 226

Home work p. 219

6

Trade

WENTWORTH-SMITH MATHEMATICAL SERIES

NEW YORK STATE ARITHMETIC

YEARS FIVE, SIX, SEVEN, AND EIGHT

BY

GEORGE WENTWORTH

AND

DAVID EUGENE SMITH



GINN AND COMPANY

BOSTON • NEW YORK • CHICAGO • LONDON
ATLANTA • DALLAS • COLUMBUS • SAN FRANCISCO

COPYRIGHT, 1909, 1911, 1912, BY GEORGE WENTWORTH AND

DAVID EUGENE SMITH

ENTERED AT STATIONERS' HALL

ALL RIGHTS RESERVED

318.6

The Athenæum Press

GINN AND COMPANY • PROPRIETORS • BOSTON • U.S.A.

PREFACE

In deciding upon the plan for a textbook to meet the needs of the schools of New York state, the authors have been guided by the Syllabus in Arithmetic issued by the State Education Department. With the Syllabus constantly before them they have prepared what they believe to be a thoroughly usable book. They believe that this book, with its clear-cut topical arrangement, will appeal to teachers and pupils alike, and that it will meet the needs of the schools in every respect.

The rule that has guided the authors in their other works is followed in this one: Present the reason briefly but clearly, and then furnish such an amount of practice that the pupil cannot forget the principle. Theory is reduced to a minimum, and practice is abundant, with both abstract and concrete numbers, since the experience of a century proves beyond question that each of these two lines must be treated fully to prepare the pupils for their future work. This is the spirit of the Syllabus, and it is a thoroughly sound one, representing as it does the sanest modern educational thought.

The authors have always insisted that all applied problems should appeal as far as possible to the pupil's interests. In conformity with the spirit of this principle the concrete problems are modern in the best sense. Those that seem to be real are real; modern business customs are followed, and the needs of the future citizen are always kept in mind. But in this book is found no trace of the unfortunate tendency of some of our recent writers to arrange their work without system, to withhold from the pupil the feeling of mastery which is his due, and to consider a problem concrete when

it treats of topics or contains technicalities that no pupil understands and few teachers are expected to know.

The book covers exactly the topics, grade by grade, laid down in the Syllabus, and also provides for a review of the fundamental operations with integers (pages 1-56), to be used or not as occasion demands. The topics of Longitude and Time, and the Metric System, commonly taught but not included in the Syllabus, are placed in the Appendix so as to be available if desired. Under each topic will be found an unusually large number of well-graded examples, and at frequent intervals collections of problems without numbers have been inserted as exercises in terse mathematical statement. The arithmetical topics properly close with sets of exercises that relate to the vocational interests of our people, to the end that pupils may come to know of the dignity of labor, and may leave school with the real applications of arithmetic to common life clear in their minds. Following the wise suggestion of the Syllabus, a chapter on Algebraic Notation is inserted, in which the most common uses of algebra in the life of the artisan are set forth.

Any corrections or suggestions relating to the work will be thankfully received.

The authors hope that teachers who recognize the strong features of the topical arrangement, who wish for an abundance of well-graded problems requiring thought in their solution, who are sympathetic with the movement to replace what is obsolete by the genuine applications of arithmetic to the American life of to-day, who are opposed to certain of the extreme attempts of the present time that are sure to result disastrously to scholarship—that such teachers will find in this work a sane, modern, and helpful treatment of the subject.

GEORGE WENTWORTH
DAVID EUGENE SMITH

CONTENTS

CHAPTER	PAGE
I. WRITING AND READING NUMBERS	1
II. ADDITION OF INTEGERS	9
III. SUBTRACTION OF INTEGERS	17
IV. MULTIPLICATION OF INTEGERS	23
V. DIVISION OF INTEGERS	35
VI. DECIMAL FRACTIONS	57
VII. DENOMINATE NUMBERS	99
VIII. BILLS AND ACCOUNTS	171
IX. FACTORS, MEASURES, MULTIPLES	179
X. COMMON FRACTIONS	191
XI. PERCENTAGE	231
XII. PROFIT AND LOSS	243
XIII. THE EQUATION	249
XIV. TRADE DISCOUNT	255
XV. COMMISSION	269
XVI. INTEREST	277
XVII. RATIO AND PROPORTION	327
XVIII. POWERS AND ROOTS	343
XIX. PRACTICAL MEASUREMENTS	357
XX. RAPID CALCULATION AND BUSINESS FORMS	401
XXI. STOCKS AND BONDS	429
XXII. INSURANCE	443
XXIII. TAXATION	447
XXIV. MISCELLANEOUS PROBLEMS	453
XXV. VOCATIONAL PROBLEMS	461
XXVI. ALGEBRAIC NOTATION	479
APPENDIX	487
NEW YORK STATE EXAMINATIONS	510
DEFINITIONS OF COMMON TERMS	513
INDEX	519

THE NEW YORK STATE SYLLABUS

The following synopsis shows how the work is arranged to meet the requirements of the Syllabus. Schools not feeling the necessity for reviewing the work in the fundamental operations with integers should begin with page 57. Schools wishing to complete arithmetic at the end of the seventh year should, according to the suggestion of the Syllabus, after completing the work to page 401, select such topics from the eighth year's work as are deemed necessary to round out the subject; those wishing to complete arithmetic at the middle of the eighth year may omit part or all of Chapters XXIV and XXV.

	PAGES
Review of the Fundamental Operations	1-56
Fifth Year	57-178
Sixth Year	179-292
Seventh Year	293-400
Eighth Year	401-512

NEW YORK STATE ARITHMETIC

YEARS FIVE TO EIGHT

CHAPTER I

WRITING AND READING NUMBERS

1. Units. The standards by which we count or measure are called *units*.

Thus in speaking of \$4 the unit is \$1; in speaking of 5 eggs the unit is 1 egg, but in speaking of 10 dozen eggs the unit is 1 dozen eggs; in speaking of 40 thousand bricks the unit is a thousand bricks; in counting 1, 2, 3, 4, and so on, the unit is 1; in measuring long distances the unit may be 1 mile.

2. Numbers. That which shows how many times a unit is taken, or what part of a unit is taken, is called a *number*.

Thus 3 indicates that 1 has been taken 3 times; \$6 indicates that \$1 has been taken 6 times; $\frac{4}{5}$ indicates that four fifths of 1 is taken; hence 3, \$6, and $\frac{4}{5}$ are all numbers.

3. Abstract and Concrete Numbers. A number used without reference to any particular unit is called an *abstract number*; if the unit is named, the whole expression is called a *concrete number*.

For example, 5 is an abstract number, but \$5 and 5 bushels are concrete numbers. It must be understood, however, that the true number part is always abstract. That is, in \$5 the true number part is 5. It is convenient, however, in explaining the solution of problems to make the distinction between abstract numbers and concrete numbers.

4. Integers. Numbers applied to whole units are called *whole numbers*, *integral numbers*, or *integers*.

Fractional numbers, already familiar to the class from work in the earlier grades, and mentioned in § 2, are treated later.

5. Notation and Numeration. The writing of numbers is called *notation*; the reading of numbers is called *numeration*.

6. Kinds of Notation. The class is already familiar with the common or Arabic notation (so called because it came to Europe through the Arabs), and the Roman notation.

The Arabic characters, sometimes called *figures* or *ciphers*, originated in India. They were employed without the zero more than 2000 years ago, and the zero or naught has been in use over 1200 years. These figures have been known in Europe more than 900 years, but did not become generally used until about 400 years ago, when they finally displaced the clumsy notation of the ancient Romans.

7. Numerals. The characters used in any system of notation are called *numerals*.

Thus we speak of Arabic numerals and of Roman numerals. The Arabic numerals are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The Roman numerals are I, V, X, L, C, D, M. The Arabic characters, excluding 0, are sometimes called *digits*.

8. Place Value. The value of a figure in a number written in Arabic numerals depends upon its position in the number and is called the *place value* of the figure.

Thus, in the number 53, the 3 occupies units' place and means 3 units; the 5 occupies tens' place and means 5 tens. In the Roman system VIII, although made up of V (five) and III (three), like 53, means only eight, because the numerals have no such place value.

9. Orders of Units. Each successive place in a number that may be occupied by a figure is called an *order of units*.

These orders increase from right to left in a tenfold ratio. Hence the Arabic system is called a *decimal system* (from the Latin *decem*, ten). The ones (or units) are called *units of the first order*; the tens, *units of the second order*; and so on.

10. Names of the Orders. As already learned in school, the number names of the first few orders are as follows :

Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units
1	4	3	2	6	5	7	1

11. Periods. When the figures of a number are five or more, we separate them into groups of three figures each by commas or by spaces, beginning at the right. These groups are called *periods*.

Thus we write thirty-two thousand, seven hundred fifty-six, 32,756 ; it is often printed with a space instead of the comma, thus, 32 756.

12. Separatrix. The comma separating two periods of a number is called the *separatrix*.

13. Names of the Periods. Beginning to name the periods at the right, the first period is called the *units' period* ; the second, the *thousands' period* ; the third, the *millions' period* ; and the fourth, the *billions' period*.

We rarely use number names above millions. The names of the periods above billions are trillions, quadrillions, quintillions, sextillions, septillions, and so on ; but these need not be learned.

14. Reading Numbers. In reading numbers we read each period by itself and add the name of the period. Avoid all useless words.

Thus we read 2,341,406 "two million, three hundred forty-one thousand, four hundred six." It is customary to omit the name of the units' period.

EXERCISE 1

Read aloud the following numbers :

1. 46.	11. 2052.	21. 1,250,000.
2. 85.	12. 3030.	22. 2,500,000.
3. 70.	13. 52,176.	23. 3,333,333.
4. 24.	14. 31,427.	24. 10,010,010.
5. 132.	15. 50,050.	25. 12,000,112.
6. 201.	16. 22,222.	26. 25,150,150.
7. 357.	17. 215,215.	27. \$25,000,000.
8. 777.	18. 600,600.	28. \$32,125,000.
9. 1234.	19. 303,303.	29. \$57,000,250.
10. 1001.	20. 123,456.	30. \$92,350,206.

Read aloud the following sentences :

31. The area of Alabama is 52,250 square miles.
32. The area of Arkansas is 53,850 square miles.
33. The area of Florida is 58,680 square miles.
34. The area of Georgia is 59,475 square miles.
35. The area of Indiana is 36,350 square miles.
36. The area of Iowa is 56,025 square miles.
37. The area of Kansas is 82,080 square miles.
38. The area of Kentucky is 40,400 square miles.
39. The area of Minnesota is 83,365 square miles.
40. The area of Nevada is 110,700 square miles.
41. The area of New York State is 49,170 square miles.
42. The area of Texas is 265,780 square miles.
43. The area of the United States is 3,616,484 square miles.

15. Writing Numbers in Arabic Figures. The class has already learned to write numbers. Beginning at the left, we write each period by itself, followed by a comma, putting a zero in every vacant place.

Thus, to write thirty million, forty thousand, seven hundred six, we first write the order of millions, 30, followed by a comma; then 40 thousand, placing a zero in the place of hundred-thousands; then, after the comma, 706. The number then stands as follows:

30,040,706.

EXERCISE 2

Write in figures, arranged in periods, the following:

1. Five thousand, eight.
2. Nine thousand, fifty-seven.
3. Seven thousand, two hundred.
4. Eight thousand, one hundred one.
5. Twelve thousand, four hundred forty.
6. Fifteen thousand, six hundred fifty-six.
7. Eighteen thousand, seven hundred forty-nine.
8. Twenty-five thousand, eight hundred eleven.
9. One hundred fifty-two thousand, eighty-eight.
10. Two hundred six thousand, four hundred one.
11. Four hundred forty thousand, one hundred six.
12. Eight hundred seventy-five thousand, twenty.
13. Nine hundred ninety-nine thousand, eighty-four.
14. One million, one thousand, one hundred one.
15. Three million, three hundred fifty-five thousand, six.
16. Twenty-three million, two hundred forty-eight thousand, six hundred forty-four.

17. Eighty-four million, seven hundred six.
18. Forty-two million, one thousand, one hundred.
19. Seventy-eight million, two hundred forty thousand.
20. Sixty-eight million, eight hundred seventy-three thousand.
21. Ninety-one million, two hundred thousand, one hundred fifty-one.
22. One hundred forty million, seven hundred forty-six.
23. Two hundred thirty-three million, one hundred seventeen thousand, eight.
24. Two hundred twenty-two million, two thousand, two hundred two.
25. Three hundred twenty-four million, five hundred sixty-seven thousand, eight hundred ninety.
26. Three hundred ninety-two million, six hundred forty thousand, five hundred seventy-nine.
27. Four hundred forty-four million, two hundred sixty-seven thousand, three hundred eighty-seven.
28. Five hundred twenty-eight million, six hundred twenty-six thousand, eight hundred forty-seven.
29. Nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one.
30. Seven hundred seventy-two million, four hundred seventy-three thousand, five hundred eighty-nine.
31. Eight hundred eighty-eight million, eight hundred eighty-eight thousand, eight hundred eighty-eight.
32. One hundred ninety-eight billion, seven hundred sixty-five million, four hundred thirty-two thousand, one hundred fifty-seven.

16. Writing United States Money. In writing United States money the *dollar sign* (\$) is written before the number. A period, called a *decimal point*, is placed between the dollars and the dimes.

For example, 125 dollars is written \$125; 10 dollars and 48 cents (that is, 10 dollars, 4 dimes, 8 cents) is written \$10.48; 5 dollars and 8 cents is written \$5.08.

EXERCISE 3

Read aloud the following:

- | | | | |
|------------|-------------|----------------|----------------|
| 1. \$15. | 9. \$2.25. | 17. \$21.00. | 25. \$4825.25. |
| 2. \$24. | 10. \$3.75. | 18. \$35.25. | 26. \$7275.05. |
| 3. \$40. | 11. \$6.42. | 19. \$42.75. | 27. \$427.82. |
| 4. \$90. | 12. \$7.28. | 20. \$33.03. | 28. \$5207.60. |
| 5. \$2.70. | 13. \$5.05. | 21. \$125.50. | 29. \$4270.07. |
| 6. \$1.30. | 14. \$7.09. | 22. \$325.75. | 30. \$500.05. |
| 7. \$4.60. | 15. \$3.04. | 23. \$2450.25. | 31. \$5000.05. |
| 8. \$5.20. | 16. \$9.08. | 24. \$3575.50. | 32. \$9999.99. |

33. In 1800 the public debt of our country was \$15.63 for each inhabitant; in 1900 it was \$14.52.

34. The average amount for each depositor in the savings banks of the United States recently was \$429.64.

35. Recently the public debt of the United States was \$2,492,231,518.54.

36. The average pay of locomotive engineers on the railroads in our country is about \$4.35 a day; conductors average about \$3.75 a day.

37. In a certain year the United States produced iron and steel to the value of \$804,034,918.75, the value of that produced by the Middle Atlantic States being \$478,687,519.80.

17. The Roman Numerals. The Roman notation is used chiefly for numbering chapters and for dates. It employs seven capital letters, as follows :

<i>Letters,</i>	I	V	X	L	C	D	M
<i>Values,</i>	1	5	10	50	100	500	1000

The first nine numbers are written thus :

I, II, III, IV or IIII, V, VI, VII, VIII, IX.

The tens are written thus :

X, XX, XXX, XL, L, LX, LXX, LXXX, XC.

The hundreds are written thus :

C, CC, CCC, CD, D, DC, DCC, DCCC, DCCCC or CM.

The numbers from eleven to nineteen are written thus :

XI, XII, XIII, XIV, XV, XVI, XVII, XVIII, XIX.

The following are examples of other numbers :

XXII = 22	XCVIII = 98	CLXV = 165
XXXIV = 34	LXXV = 75	CCLIX = 259
XLIX = 49	LXXXI = 81	MDCCXC = 1790
MDCCCCXI or MCMXI = 1911		

EXERCISE 4

Write in Arabic figures :

1. XIV.	4. LIX.	7. LXXX.	10. MDCC.
2. XXIX.	5. LXVI.	8. XCVI.	11. MDCXX.
3. XXXI.	6. LXXV.	9. DCCXI.	12. MDCCCCX.

Write in Roman numerals :

13. 26.	16. 69.	19. 135.	22. 525.
14. 32.	17. 81.	20. 241.	23. 1892.
15. 50.	18. 99.	21. 363.	24. 1928.

CHAPTER II

ADDITION OF INTEGERS

18. Addition. You have already learned how to add numbers and are now ready to read the definition of addition. The operation of finding a number equal to two or more numbers taken together is called *addition*.

Thus, to add 7 and 8 is to find a number, 15, that is equal to these numbers taken together.

$$\begin{array}{r} 7 \\ + 8 \\ \hline 15 \end{array}$$

19. Sum. The result obtained by adding is called the *sum*.

Thus 15 is the sum of 7 and 8.

20. Symbol. The symbol of addition is this form of a cross, $+$. It is read *plus*, a word meaning *more*.

The expression $7 + 8 = 15$, as you have long ago learned, is read "7 plus 8 equals 15." Sometimes in school the plus sign is used to indicate addition in a column, as here shown.

$$\begin{array}{r} 7 \\ + 8 \\ \hline 15 \end{array}$$

21. Like Numbers. Numbers applied to the same unit are called *like numbers*.

Thus \$3 and \$5 have the same unit, \$1; 3 feet and 2 feet have the same unit, 1 foot; 7 and 15 have the same unit, 1.

22. Addends. The numbers added are called *addends*. *Only like numbers can be added and the sum is like the addends.*

That is, we cannot add \$2 and 3 yards, or 7 pounds and 4 books. If we add \$12 and \$13, the sum must be like the addends; that is, it must be \$25, not 25 feet. It is true that we can put 4 books with 7 crayons, but we cannot add them so as to get 11 books or 11 crayons; we have to think of them only as *things*, and say that we have 11 things.

23. Addition of Digits. You have already learned how to add digits, but the following directions will help you:

(1) *First add from the bottom upwards and write the sum.*

(2) *Then add from the top downwards, to see that you have made no mistake.*

This is called *checking* the result, and is very important in all calculations.

(3) *Try to group digits in adding, thus allowing you to work more rapidly.*

For example, in this case we may begin with 6 and add each digit in order, thinking thus: "6, 15, 18, 25, 29." It is better, however, to look at the 6 and 9 and think "15"; then look at the 7 and 3 and think of 10 as added to the 15, saying to ourselves "25"; then to look at 4 and think "29."

4
7
3
9
6

29

EXERCISE 5

Add the following, checking your work and timing yourself:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
2	3	4	3	8	9	9	8	4	2	4	2
3	4	2	9	2	8	2	8	6	1	2	8
5	7	8	7	8	7	3	8	5	4	6	7
7	6	9	7	3	6	1	2	5	7	3	6
<u>9</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>9</u>	<u>5</u>	<u>4</u>	<u>2</u>	<u>3</u>	<u>3</u>	<u>4</u>	<u>3</u>
13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
3	7	6	4	4	7	5	6	7	3	8	4
4	2	8	1	6	7	2	8	8	4	2	7
2	4	3	2	4	8	8	3	2	2	9	8
6	1	4	8	5	8	7	9	8	8	1	3
4	1	9	7	3	2	2	4	9	7	7	9
<u>3</u>	<u>2</u>	<u>2</u>	<u>3</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>3</u>	<u>9</u>	<u>3</u>	<u>2</u>

Copy the numbers in lines 25 to 35 inclusive, and add ; then add the columns numbered 36 to 43 inclusive :

	36.	37.	38.	39.	40.	41.	42.	43.
25.	2	3	2	2	3	9	8	1
26.	1	4	1	9	8	1	4	3
27.	3	7	3	3	4	2	9	5
28.	4	7	4	9	4	8	2	6
29.	2	8	3	4	1	3	7	6
30.	8	2	2	1	9	4	3	6
31.	7	3	4	2	8	1	6	9
32.	6	4	8	4	3	9	5	2
33.	4	6	6	6	2	7	8	3
34.	1	2	1	3	7	6	5	4
35.	<u>2</u>	<u>3</u>	<u>4</u>	<u>2</u>	<u>6</u>	<u>2</u>	<u>7</u>	<u>5</u>

Proceed in the same way for lines numbered 44 to 57 inclusive, and for columns numbered 58 to 65 inclusive :

	58.	59.	60.	61.	62.	63.	64.	65.
44.	1	8	6	2	4	7	3	5
45.	2	5	9	3	8	4	7	6
46.	3	4	1	5	8	6	9	7
47.	2	7	4	1	7	6	3	5
48.	7	1	5	2	8	3	9	4
49.	1	3	5	6	2	8	4	7
50.	2	4	6	3	9	5	8	7
51.	4	2	3	8	5	7	9	6
52.	3	1	2	8	6	4	7	5
53.	2	6	6	4	7	8	3	9
54.	7	9	1	6	2	4	3	5
55.	8	2	3	6	9	5	7	4
56.	1	4	5	2	8	7	3	6
57.	<u>9</u>	<u>1</u>	<u>4</u>	<u>6</u>	<u>2</u>	<u>5</u>	<u>7</u>	<u>3</u>

24. Addition of Two-Figure Numbers. You have also learned how to add two-figure numbers, but the following directions may help you to understand the work:

(1) *Write the addends so that units of the same order are in the same column.*

(2) *Add each column by itself, beginning at the right, and write the sum, if it is less than 10, under the column added.*

(3) *If the sum of any column is 10 or more, write the unit figure under the column added, and add the tens with the next column.*

$$\begin{array}{r}
 67 \\
 86 \\
 34 \\
 \hline
 17 \text{ sum of units} \\
 17 \text{ sum of tens} \\
 \hline
 187 \text{ sum}
 \end{array}$$

This is done in full in the example shown above. In practice, however, we do not write the sum of each column by itself, but proceed as here shown. We think of the work as follows: "4, 10, \$67 17," writing the 7 and adding the 1 to the tens' column; 86 then think: "1, 4, 12, 18," and write the 8 under the tens' column and the 1 in the hundreds' place.

\$187 The dollar sign before the top number in a column indicates that all the numbers in the column represent dollars.

EXERCISE 6

Add the following and check your work:

1.	2.	3.	4.	5.	6.
21	30	6	34	72	\$32
32	47	57	9	89	21
<u>46</u>	<u>22</u>	<u>61</u>	<u>68</u>	<u>67</u>	<u>49</u>
7.	8.	9.	10.	11.	12.
34	62	75	62	47	\$25
62	48	25	48	63	30
81	31	30	8	92	35
<u>92</u>	<u>43</u>	<u>7</u>	<u>75</u>	<u>81</u>	<u>40</u>

*Copy the numbers in lines 13 to 22 inclusive and add ;
then add the columns numbered 23 to 30 inclusive :*

	23.	24.	25.	26.	27.	28.	29.	30.
13.	10	11	14	34	15	20	31	12
14.	22	12	33	21	22	34	6	9
15.	13	21	20	12	31	26	15	34
16.	31	23	6	51	28	35	42	55
17.	42	31	14	62	61	77	82	69
18.	14	43	35	71	38	56	77	86
19.	31	37	7	33	46	59	68	77
20.	22	42	62	14	84	73	82	91
21.	30	61	70	68	59	38	47	65
22.	<u>50</u>	<u>12</u>	<u>15</u>	<u>73</u>	<u>77</u>	<u>82</u>	<u>39</u>	<u>46</u>

*Proceed in the same way for lines numbered 31 to 45
inclusive, and for columns numbered 46 to 53 inclusive :*

	46.	47.	48.	49.	50.	51.	52.	53.
31.	12	20	23	14	86	77	80	78
32.	6	10	15	17	83	85	90	94
33.	7	9	12	23	87	88	91	93
34.	13	15	14	36	42	23	31	46
35.	21	31	21	41	28	17	16	20
36.	42	24	33	23	14	23	41	26
37.	9	16	40	31	15	17	48	62
38.	8	33	62	14	26	33	27	15
39.	10	27	19	17	14	23	73	85
40.	23	18	28	62	17	15	52	38
41.	37	26	30	31	13	89	59	74
42.	52	58	42	48	58	77	84	80
43.	46	39	71	73	72	83	69	54
44.	51	62	16	92	86	77	9	7
45.	<u>73</u>	<u>47</u>	<u>86</u>	<u>87</u>	<u>85</u>	<u>83</u>	<u>10</u>	<u>16</u>

25. Addition of Larger Numbers. You have learned how to add numbers of more than two figures, and the directions given on page 12 for adding two-figure numbers apply to all cases.

2936	
<u>8798</u>	
14	sum of units
12	sum of tens
16	sum of hundreds
<u>10</u>	sum of thousands
11734	sum

Thus, to add 2936 and 8798 we might proceed as shown in the left-hand solution, but the process shown in the right-hand solution is better because it is shorter and allows the work to be performed more rapidly.

2936
<u>8798</u>
11734

EXERCISE 7

Add the following, checking your work and timing yourself:

1.	2.	3.	4.	5.
307	607	426	462	336
268	73	427	617	774
111	803	700	296	340
<u>702</u>	<u>727</u>	<u>99</u>	<u>287</u>	<u>760</u>
6.	7.	8.	9.	10.
4732	2732	596	4884	8492
6239	698	69	6939	3687
5268	1596	3492	4008	4009
<u>3761</u>	<u>998</u>	<u>9873</u>	<u>7227</u>	<u>698</u>
11.	12.	13.	14.	15.
64,283	41,294	28,009	43,218	52,988
2,796	58,706	77,986	37,298	13,678
12,426	14,261	20,811	42,361	60,211
<u>20,431</u>	<u>2,143</u>	<u>30,021</u>	<u>20,000</u>	<u>72,010</u>

Add the following :

16.	17.	18.	19.	20.
73,296	28,428	44,307	30,072	78,327
41,427	73,211	62,320	14,826	4,162
33,291	70,092	40,072	4,999	899
41,627	61,778	63,142	12,000	10,090
<u>20,623</u>	<u>31,732</u>	<u>40,012</u>	<u>30,200</u>	<u>41,009</u>

21. Recently we had 9196 steamers and 12,831 other American vessels engaged in trade along our coast. What was the total number?

22. In our country there were reported as built recently in one year 147 sailing vessels, 674 steam vessels, 62 canal boats, and 274 barges, not counting many smaller boats. What was the total of those reported?

23. In a recent year our country sold abroad \$32,305,412 worth of manufactured cotton, and \$2,287,936 worth of manufactured wool. What was the total value?

The following table shows certain statistics relating to our country in 1800, and the increase in 100 years. Add the increase to the column for 1800 and find the statistics for 1900.

	1800	Increase
24. Number of square miles	827,844	2,198,945
25. Number of bales of cotton grown	155,556	9,280,860
26. Receipts of post offices	\$280,804	\$102,073,775
27. Gold and silver money	\$16,000,000	\$736,856,806
28. Total circulation of money	\$26,500,000	\$2,028,650,998
29. Population	5,308,483	70,994,904

26. Addition of United States Money. In adding United States money simply proceed as with ordinary integers, keeping units of the same order in the same column as here shown.

\$12.50
0.75
23.
1.05

\$37.30

We can easily keep the units of the same order in the same column by placing the decimal points under one another.

EXERCISE 8

Add the following :

1.	2.	3.	4.	5.
\$2.34	\$1.45	\$7.40	\$5.06	\$6.92
1.35	0.75	3.80	1.04	7.21
4.26	3.00	6.20	3.07	3.87
<u>5.72</u>	<u>1.26</u>	<u>5.90</u>	<u>4.83</u>	<u>4.</u>
6.	7.	8.	9.	10.
\$2.93	\$6.42	\$5.42	\$2.50	\$23.50
3.81	7.08	0.58	3.62	7.50
4.26	3.91	0.39	0.48	18.75
5.35	0.75	3.27	4.71	32.10
0.75	0.25	0.58	3.	4.05
<u>4.</u>	<u>6.</u>	<u>1.25</u>	<u>1.96</u>	<u>5.10</u>
11.	12.	13.	14.	15.
\$6.42	\$4.08	\$5.25	\$2.93	\$14.08
7.81	2.75	1.27	4.62	0.92
3.10	3.02	3.06	8.21	26.29
2.07	6.72	4.81	4.26	53.07
0.56	8.05	2.72	8.02	62.75
0.82	9.	3.25	7.33	41.05
1.49	0.33	4.05	0.85	35.
<u>7.23</u>	<u>6.44</u>	<u>6.00</u>	<u>3.82</u>	<u>14.75</u>

CHAPTER III

SUBTRACTION OF INTEGERS

27. Subtraction. The process of finding what number must be added to one of two numbers to make the other, or of taking one number from another, is called *subtraction*.

28. Symbol. The symbol of subtraction is a short horizontal line, $-$. It is read *minus*, a word meaning *less*.

Thus $17 - 9 = 8$ is read "17 minus 9 equals 8."

29. Terms. The smaller of the two given numbers in subtraction is called the *subtrahend*; the larger one is called the *minuend*; the result of the subtraction is called the *remainder* or *difference*. To *check* the result, add the subtrahend and remainder; the result should be the minuend.

30. The Addition Method of Subtraction. This method, also called the Austrian or making-change method, is recommended in the New York State Syllabus. It consists simply in finding, in the case of $468 - 143$, what must be added to 143 to make 468. We proceed as follows:

468	minuend
143	subtrahend
325	remainder

$3 + 5 = 8$; $4 + 2 = 6$; $1 + 3 = 4$.

31. The Taking-away Method. This is, at present, a more familiar method of subtraction. Here we say: 3 from 8, 5; 4 from 6, 2; 1 from 4, 3.

Of the two methods the first is always used in making change, it is the more rapid when thoroughly learned, and it is growing in favor. When a pupil has already acquired one method, however, and works rapidly and accurately, it is not good policy to attempt to change it.

EXERCISE 9

Subtract, checking your work and timing yourself:

1. 673 <u>261</u>	2. 742 <u>420</u>	3. 396 <u>104</u>	4. 485 <u>172</u>	5. 981 <u>651</u>
6. 908 <u>702</u>	7. 659 <u>128</u>	8. 485 <u>152</u>	9. 398 <u>287</u>	10. 667 <u>421</u>
11. \$7.25 <u>1.02</u>	12. \$5.36 <u>2.15</u>	13. \$9.28 <u>1.05</u>	14. \$6.75 <u>1.25</u>	15. \$8.82 <u>2.31</u>
16. \$15.75 <u>2.20</u>	17. \$23.50 <u>2.40</u>	18. \$37.55 <u>15.20</u>	19. \$52.85 <u>21.70</u>	20. \$75.88 <u>13.52</u>
21. \$82.96 <u>71.42</u>	22. \$95.63 <u>15.02</u>	23. \$86.75 <u>14.25</u>	24. \$73.92 <u>13.92</u>	25. \$48.52 <u>24.10</u>
26. \$175.40 <u>21.30</u>	27. \$142.65 <u>21.02</u>	28. \$235.75 <u>101.20</u>	29. \$468.92 <u>107.02</u>	30. \$536.48 <u>121.25</u>

31. $62,815 - 11,702.$

32. $81,482 - 70,141.$

33. $96,348 - 62,123.$

34. $27,984 - 14,521.$

35. $31,628 - 20,407.$

36. $82,489 - 70,126.$

37. $68,925 - 61,420.$

38. $34,876 - 10,724.$

39. $42,929 - 30,009.$

40. $99,999 - 47,628.$

41. A grocer had 1360 pounds of sugar and sold 1040 pounds. How many pounds had he left?

42. A farmer had 2445 bushels of corn and sold 2120 bushels. How many bushels had he left?

43. A man's income for a year was \$1975 and his expenses were \$1630. How much had he left?

44. A dealer had 2796 tons of coal and sold 1385 tons. How many tons had he left?

45. A dry goods merchant had 1440 spools of thread and sold 1200 spools. How many spools had he left?

46. A city school had 1296 pupils of whom 184 were in the first-year class. How many were in all the other classes together?

47. A real estate dealer had 1278 acres of land for sale, and sold 1163 acres. How many acres had he left unsold?

48. The distance from New York to Albany is 143 miles, and from New York to Chicago is 975 miles. How far is it from Albany to Chicago by this route?

49. The area of Indiana is 36,350 square miles, and that of Rhode Island 1250 square miles. What is the difference in area?

50. The area of Alabama is 52,250 square miles and that of California is 158,360 square miles. What is the difference in area?

51. The area of Nebraska is 77,510 square miles, and that of South Dakota is 77,650 square miles. What is the difference in area?

52. In a recent year Pennsylvania produced 31,816,496 bushels of oats, and Massachusetts 214,472 bushels. What was the difference in the amounts?

32. General Case of Subtraction. Suppose we wish to find the difference between 652 and 476. Here we may use either of the methods stated in § 30 and § 31.

(1) The addition (Austrian) method uses this principle: *If the same number is added to both minuend and subtrahend, the remainder is not changed.* Thus $8 - 2$ gives the same result as $18 - 12$ or $108 - 102$.

Taking the case of $652 - 476$, we now consider what number added to 6 makes 2. Since it is impossible to find such a number, what number added to 6 makes 12? Evidently 6, which we write below. But we increased the minuend from 2 to 12 by adding 10, so we add 10 to the subtrahend, making the 7 into an 8. Then, since no number added to 8 makes 5, what number added to 8 makes 15? Evidently 7, which we write below. But we increased the minuend from 5 tens to 15 tens, by adding 100 (or 10 tens), so we add 100 to the subtrahend, making the 4 a 5. Then $5 + 1 = 6$, and we write the 1 below.

(2) The taking-away method. Considering the same problem, we proceed as follows: Since we cannot take 6 from 2 we change one of the 5 tens to units, and this with the 2 units makes 12 units; then $12 - 6 = 6$. Since we have used one of the tens in the 5 tens we have 4 tens left, from which we cannot take 7 tens. We change one of the 6 hundreds to tens, and this with the 4 tens makes 14 tens; then $14 \text{ tens} - 7 \text{ tens} = 7 \text{ tens}$. We now have 5 hundreds left of the 6 hundreds, and $5 \text{ hundreds} - 4 \text{ hundreds} = 1 \text{ hundred}$. The remainder is therefore 176. It will make the work clearer to see the numbers separated as here shown, but this is not done in practice.

33. Special Cases. In a case like $5000 - 2765$ we may shorten the work by thinking of the 5000 as $4990 + 10$, and of 2765 as $2760 + 5$, beginning to subtract at the left.

This amounts to taking the 2 from 4 and every other figure from 9 except the units, which we take from 10. The remainder is 2235.

$$\begin{array}{r} 4 \quad 9 \quad 9 \quad 10 \\ 2 \quad 7 \quad 6 \quad 5 \\ \hline 2 \quad 2 \quad 3 \quad 5 \end{array}$$

In subtracting United States money place the decimal points under one another and proceed as with integral numbers.

EXERCISE 10

Subtract, checking your work and timing yourself:

- | | | |
|---------------|----------------|----------------------|
| 1. 274 — 62. | 27. 638 — 126. | 53. 2782 — 120. |
| 2. 378 — 69. | 28. 427 — 127. | 54. 3648 — 439. |
| 3. 465 — 57. | 29. 636 — 318. | 55. 5276 — 168. |
| 4. 582 — 44. | 30. 342 — 225. | 56. 8496 — 229. |
| 5. 673 — 65. | 31. 596 — 379. | 57. 2349 — 256. |
| 6. 766 — 49. | 32. 758 — 449. | 58. 4872 — 380. |
| 7. 857 — 28. | 33. 342 — 207. | 59. 6463 — 291. |
| 8. 939 — 47. | 34. 467 — 208. | 60. 4486 — 395. |
| 9. 354 — 62. | 35. 621 — 416. | 61. 7286 — 193. |
| 10. 268 — 85. | 36. 733 — 329. | 62. 2872 — 385. |
| 11. 325 — 63. | 37. 570 — 444. | 63. 5342 — 264. |
| 12. 436 — 94. | 38. 392 — 288. | 64. 3353 — 296. |
| 13. 742 — 70. | 39. 340 — 225. | 65. 4374 — 287. |
| 14. 819 — 37. | 40. 628 — 419. | 66. 6681 — 599. |
| 15. 352 — 63. | 41. 492 — 284. | 67. 6720 — 535. |
| 16. 603 — 25. | 42. 462 — 373. | 68. 7872 — 5785. |
| 17. 760 — 75. | 43. 707 — 528. | 69. 3526 — 2239. |
| 18. 504 — 68. | 44. 981 — 694. | 70. 7280 — 2385. |
| 19. 320 — 33. | 45. 606 — 428. | 71. 6807 — 4938. |
| 20. 404 — 27. | 46. 702 — 513. | 72. 38,265 — 23,406. |
| 21. 570 — 92. | 47. 603 — 427. | 73. 34,635 — 25,296. |
| 22. 700 — 30. | 48. 402 — 236. | 74. 48,406 — 26,573. |
| 23. 600 — 43. | 49. 520 — 348. | 75. 62,068 — 48,679. |
| 24. 300 — 56. | 50. 630 — 469. | 76. 85,723 — 67,836. |
| 25. 400 — 90. | 51. 800 — 532. | 77. 92,631 — 75,842. |
| 26. 500 — 99. | 52. 900 — 687. | 78. 80,050 — 69,967. |

EXERCISE 11

1. The sum of two numbers is 2872 and the lesser is 929. What is the greater number?
2. The greater of two numbers is 17,214 and their difference is 2685. What is the lesser number?
3. If the minuend is 15,293 and the subtrahend 12,525, what is the remainder?
4. If the subtrahend is 32,658 and the minuend 61,229, what is the remainder?
5. If the minuend is 21,712 and the remainder 2564, what is the subtrahend?
6. If the subtrahend is 16,567 and the remainder 7543, what is the minuend?
7. A merchant sold some goods for \$2962.50, and gained \$548.75. What did they cost him?
8. What must be added to the sum of \$175.50 and \$62.75 to make \$300?
9. What must be added to the sum of \$62.30, \$41.75, and \$25, to make \$500?
10. How much greater is the sum of \$75.30 and \$21.75, than the difference between these amounts?
11. What is the difference between \$725.25 and the sum of \$15.50, \$17.92, \$25.36, \$48.75, \$19.36, and \$42.75?
12. What must be subtracted from \$1000 to make the remainder equal to the sum of \$175 and \$42.50?
13. A man borrowed \$1250 and paid back \$125 at one time and \$960 at another. How much did he still owe?
14. A man having \$375.50 in the bank deposited at different times \$175, \$320, and \$15.40. He then drew out \$645. How much had he left in the bank?

CHAPTER IV

MULTIPLICATION OF INTEGERS

34. Multiplication. The process of taking a number as many times as there are units in another number is called *multiplication*.

This means multiplication by an integer. Multiplication by a fraction is considered later.

35. Terms. The number that is multiplied is called the *multiplicand*; the number that shows how many times the multiplicand is taken is called the *multiplier*; the result of the multiplication is called the *product*.

4	multiplicand
3	multiplier
12	product

36. Multiple. The product of two or more integers is called a *multiple* of each.

For example, 15 is a multiple of 3 and 5, and 40 is a multiple of 2, 4, 5 and also of 8, 10, 20.

37. Factors. Integral numbers whose product is a given number are called *factors* of that number; the multiplicand and the multiplier, for example, are factors of the product.

38. Symbol. The symbol of multiplication is an oblique cross, $\times$. It is read *times* or *multiplied by*.

Thus $3 \times 4 = 12$ is read "3 times 4 equals 12."

The multiplier may be written either before or after the multiplicand, but the tendency in this country is to write the multiplier first, as we naturally read it. For example, $3 \times \$4$ is read "3 times \$4," and $\$4 \times 3$ is read "\$4 multiplied by 3."

39. The Multiplication Table. The following table, already learned, must be reviewed so that you can tell instantly any product when the teacher tells you the factors :

$2 \times 1 = 2$	$3 \times 1 = 3$	$4 \times 1 = 4$	$5 \times 1 = 5$
$2 \times 2 = 4$	$3 \times 2 = 6$	$4 \times 2 = 8$	$5 \times 2 = 10$
$2 \times 3 = 6$	$3 \times 3 = 9$	$4 \times 3 = 12$	$5 \times 3 = 15$
$2 \times 4 = 8$	$3 \times 4 = 12$	$4 \times 4 = 16$	$5 \times 4 = 20$
$2 \times 5 = 10$	$3 \times 5 = 15$	$4 \times 5 = 20$	$5 \times 5 = 25$
$2 \times 6 = 12$	$3 \times 6 = 18$	$4 \times 6 = 24$	$5 \times 6 = 30$
$2 \times 7 = 14$	$3 \times 7 = 21$	$4 \times 7 = 28$	$5 \times 7 = 35$
$2 \times 8 = 16$	$3 \times 8 = 24$	$4 \times 8 = 32$	$5 \times 8 = 40$
$2 \times 9 = 18$	$3 \times 9 = 27$	$4 \times 9 = 36$	$5 \times 9 = 45$
$2 \times 10 = 20$	$3 \times 10 = 30$	$4 \times 10 = 40$	$5 \times 10 = 50$
$6 \times 1 = 6$	$7 \times 1 = 7$	$8 \times 1 = 8$	$9 \times 1 = 9$
$6 \times 2 = 12$	$7 \times 2 = 14$	$8 \times 2 = 16$	$9 \times 2 = 18$
$6 \times 3 = 18$	$7 \times 3 = 21$	$8 \times 3 = 24$	$9 \times 3 = 27$
$6 \times 4 = 24$	$7 \times 4 = 28$	$8 \times 4 = 32$	$9 \times 4 = 36$
$6 \times 5 = 30$	$7 \times 5 = 35$	$8 \times 5 = 40$	$9 \times 5 = 45$
$6 \times 6 = 36$	$7 \times 6 = 42$	$8 \times 6 = 48$	$9 \times 6 = 54$
$6 \times 7 = 42$	$7 \times 7 = 49$	$8 \times 7 = 56$	$9 \times 7 = 63$
$6 \times 8 = 48$	$7 \times 8 = 56$	$8 \times 8 = 64$	$9 \times 8 = 72$
$6 \times 9 = 54$	$7 \times 9 = 63$	$8 \times 9 = 72$	$9 \times 9 = 81$
$6 \times 10 = 60$	$7 \times 10 = 70$	$8 \times 10 = 80$	$9 \times 10 = 90$

It is also useful to know the following :

$11 \times 1 = 11$	$11 \times 4 = 44$	$11 \times 7 = 77$	$11 \times 10 = 110$
$11 \times 2 = 22$	$11 \times 5 = 55$	$11 \times 8 = 88$	$11 \times 11 = 121$
$11 \times 3 = 33$	$11 \times 6 = 66$	$11 \times 9 = 99$	$11 \times 12 = 132$
$12 \times 1 = 12$	$12 \times 4 = 48$	$12 \times 7 = 84$	$12 \times 10 = 120$
$12 \times 2 = 24$	$12 \times 5 = 60$	$12 \times 8 = 96$	$12 \times 11 = 132$
$12 \times 3 = 36$	$12 \times 6 = 72$	$12 \times 9 = 108$	$12 \times 12 = 144$

40. Multiplying by a One-Figure Multiplier. Required the product of 587 by 6.

We may multiply each part of 587 by 6 and add the results, as shown in the upper solution. This method is long, so we use it only for explanation, and write the work as in the lower solution. Beginning at the right, we have $6 \times 7 = 42$; we write the 2 in the units' place and reserve the 4 tens to add to the product of the tens. Then 6×8 tens = 48 tens, which, with the 4 tens reserved, makes 52 tens, or 5 hundreds and 2 tens. We therefore write the 2 tens in

$$\begin{array}{r}
 587 \text{ multiplicand} \\
 6 \text{ multiplier} \\
 \hline
 42 \ 6 \times 7 \\
 48 \ 6 \times 8 \text{ (tens)} \\
 30 \ 6 \times 5 \text{ (hundreds)} \\
 \hline
 3522 \ 6 \times 587 \text{ product}
 \end{array}$$

the tens' place, and reserve the 5 hundreds to add to the product of the hundreds. Then 6×5 hundreds = 30 hundreds, which, with the 5 hundreds reserved, makes 35 hundreds, or 3 thousands and 5 hundreds, which we write in the thousands' and hundreds' places. The product is therefore 3522.

41. Order of the Factors. The product is the same whatever the order of the factors.

• • • • That is, $3 \times 4 = 4 \times 3$. If we read these dots by rows, we have 3×4 dots; if we read by columns, we have 4×3 dots. But the dots do not change, so $3 \times 4 = 4 \times 3$.
 • • • • This is true for any number of rows and columns.

For this reason we may always use the smaller factor as the multiplier, thus making the work easier.

Thus, instead of multiplying 3 by 478, we would multiply 478 by 3. In the same way, if we wished to find the product of $478 \times \$3$, we would take the product of $3 \times \$478$, the answer being the same.

42. Nature of the Factors. Since the multiplier shows how many times the multiplicand is taken, we have the following:

The multiplier is always an abstract number.

The multiplicand may be either abstract or concrete, and it and the product will always be like numbers.

In $3 \times \$4 = \$12$, 3 is abstract and \$4 and \$12 are like numbers.

EXERCISE 12

Find the product of:

- | | | |
|----------------------|-----------------------|-------------------------|
| 1. 2×207 . | 21. 2×6087 . | 41. $2 \times 41,203$. |
| 2. 3×409 . | 22. 3×6074 . | 42. $3 \times 26,207$. |
| 3. 4×306 . | 23. 4×8096 . | 43. $4 \times 35,705$. |
| 4. 5×704 . | 24. 5×7048 . | 44. $5 \times 42,609$. |
| 5. 6×508 . | 25. 6×3409 . | 45. $6 \times 30,926$. |
| 6. 7×802 . | 26. 7×4706 . | 46. $7 \times 40,289$. |
| 7. 8×605 . | 27. 8×6808 . | 47. $8 \times 70,443$. |
| 8. 9×909 . | 28. 9×4307 . | 48. $9 \times 80,625$. |
| 9. 2×437 . | 29. 2×2763 . | 49. $2 \times 77,234$. |
| 10. 3×526 . | 30. 3×4496 . | 50. $3 \times 82,629$. |
| 11. 4×823 . | 31. 4×8277 . | 51. $4 \times 47,726$. |
| 12. 5×715 . | 32. 5×7879 . | 52. $5 \times 83,921$. |
| 13. 6×412 . | 33. 6×9236 . | 53. $6 \times 72,277$. |
| 14. 7×813 . | 34. 7×8943 . | 54. $7 \times 63,429$. |
| 15. 8×612 . | 35. 8×9448 . | 55. $8 \times 48,296$. |
| 16. 9×730 . | 36. 9×7367 . | 56. $9 \times 36,436$. |
| 17. 2×749 . | 37. 6×9982 . | 57. $6 \times 78,879$. |
| 18. 3×684 . | 38. 7×7347 . | 58. $7 \times 89,986$. |
| 19. 8×777 . | 39. 8×8888 . | 59. $8 \times 56,789$. |
| 20. 7×648 . | 40. 9×9999 . | 60. $9 \times 99,999$. |

61. What will 48 sheep cost at \$7 each?

62. What will 480 dozen pairs of hose cost at \$5 a dozen?

63. A farmer shipped 9 cases, each containing 360 eggs.
How many eggs did he ship?

64. A farmer had 7 acres of onions, and the yield was 562 bushels per acre. How many bushels did he get in all?

43. Multiplying United States Money. The process here is the same as with other numbers. Simply keep the decimal point after the dollars.

\$23.05

$\begin{array}{r} 7 \\ \hline \$161.35 \end{array}$

Thus, to multiply \$23.05 by 7, proceed as here shown. The explanation is the same as in § 40.

EXERCISE 13

Find the product of:

- | | | |
|------------------------|-------------------------|--------------------------|
| 1. $3 \times \$1.25.$ | 22. $3 \times \$15.25.$ | 43. $3 \times \$121.50.$ |
| 2. $3 \times \$2.70.$ | 23. $3 \times \$21.70.$ | 44. $3 \times \$271.20.$ |
| 3. $4 \times \$2.65.$ | 24. $4 \times \$12.62.$ | 45. $4 \times \$125.25.$ |
| 4. $4 \times \$3.25.$ | 25. $4 \times \$32.15.$ | 46. $4 \times \$306.70.$ |
| 5. $5 \times \$2.63.$ | 26. $5 \times \$17.20.$ | 47. $5 \times \$321.51.$ |
| 6. $5 \times \$6.31.$ | 27. $5 \times \$23.11.$ | 48. $5 \times \$248.75.$ |
| 7. $6 \times \$2.48.$ | 28. $6 \times \$18.90.$ | 49. $6 \times \$328.90.$ |
| 8. $6 \times \$3.72.$ | 29. $6 \times \$37.50.$ | 50. $6 \times \$542.75.$ |
| 9. $7 \times \$4.28.$ | 30. $7 \times \$41.16.$ | 51. $7 \times \$128.75.$ |
| 10. $7 \times \$5.75.$ | 31. $7 \times \$37.75.$ | 52. $7 \times \$305.20.$ |
| 11. $8 \times \$3.62.$ | 32. $8 \times \$21.63.$ | 53. $8 \times \$211.75.$ |
| 12. $8 \times \$6.41.$ | 33. $8 \times \$82.76.$ | 54. $8 \times \$425.25.$ |
| 13. $9 \times \$3.25.$ | 34. $9 \times \$35.50.$ | 55. $9 \times \$372.72.$ |
| 14. $9 \times \$7.28.$ | 35. $9 \times \$47.63.$ | 56. $9 \times \$987.68.$ |
| 15. $452 \times \$7.$ | 36. $2142 \times \$6.$ | 57. $42,341 \times \$2.$ |
| 16. $326 \times \$6.$ | 37. $3142 \times \$5.$ | 58. $21,421 \times \$3.$ |
| 17. $281 \times \$5.$ | 38. $4126 \times \$3.$ | 59. $26,401 \times \$4.$ |
| 18. $429 \times \$8.$ | 39. $2107 \times \$2.$ | 60. $31,620 \times \$5.$ |
| 19. $532 \times \$3.$ | 40. $4340 \times \$7.$ | 61. $77,217 \times \$7.$ |
| 20. $799 \times \$9.$ | 41. $2989 \times \$9.$ | 62. $60,509 \times \$9.$ |
| 21. $867 \times \$4.$ | 42. $5842 \times \$8.$ | 63. $84,748 \times \$6.$ |

64. At \$3 each, what will 37 kegs of nails cost?
65. At \$3.25 each, what will 7 door casings cost?
66. How much will 2 doors for a house cost at \$6.75 each?
67. How much will 7 windows for a house cost at \$3.70 each?
68. At \$4.50 per thousand, what will 7 thousand shingles cost?
69. At \$17.75 per thousand, what will 8 thousand feet of lumber cost?
70. At \$42.75 per thousand, what will 4 thousand feet of pine siding cost?
71. If a man builds 4 cottages costing \$1728.75 each, what is the total cost?
72. At \$35.50 per thousand, what will 2 thousand feet of matched pine flooring cost?
73. At \$2.28 per thousand square feet, what will 3 thousand square feet of building paper cost?
74. At \$4.48 each for doors, and \$5.62 each for windows, what will 6 doors and 9 windows cost?
75. If a dealer sells 8 building lots at an average price of \$287.50 each, how much does he receive for all?
76. If a real estate dealer pays taxes of \$37.75 on each of 9 lots, what is the total tax he pays?
77. How much will it cost to give some rooms in a house 3 coats of paint, if the cost of each averages \$37.50?
78. If a builder has 7 men employed at \$2.75 a day and 3 at \$3.50 a day, how much does he pay them all a day?
79. If a contractor employs 8 men at \$3.75 a day, 6 men at \$2.90 a day, and 7 men at \$2.65 a day, how much does he pay them all a day?

44. Powers. The result of taking a number any number of times as a factor is called a *power* of the number.

For example, $3 \times 3 = 9$, and 9 is called the *second power*, or *square*, of 3. So $3 \times 3 \times 3$, or 27, is the *third power*, or *cube*, of 3. The successive powers of 10 are 10, 100, 1000, and so on.

45. Multiplying by Powers of 10. To multiply an integer by 10, annex a 0; by 100, two 0's; by 1000, three 0's.

To multiply United States money by 10, move the decimal point 1 place to the right; by 100, 2 places; by 1000, 3 places.

$$\begin{array}{r} 27 \\ 40 \\ \hline 1080 \end{array}$$

$$\begin{array}{r} \$25.16 \\ 300 \\ \hline \$7548 \end{array}$$

$$\begin{array}{r} 360 \\ 400 \\ \hline 144000 \end{array}$$

That is, to multiply 27 by 40, multiply by 4 and annex a 0.

Also to multiply \$25.16 by 300, multiply by 3 and move the decimal point 2 places to the right.

And, in general, consider the case of 400×360 . Here we see the multiplicand is 36×10 , that is, 360; the multiplier is 4×100 , that is, 400; therefore the product is 144×1000 , that is, 144,000.

Therefore, *when the multiplicand and multiplier are integers ending in zeros, or either ends in zeros, multiply without regard to the zeros, but annex to the product as many zeros as there are at the end of the multiplicand and of the multiplier.*

EXERCISE 14

Find the product of:

1. 30×59 .

8. $70 \times \$430$.

15. 300×427 .

2. 40×477 .

9. $80 \times \$3.55$.

16. 500×640 .

3. 60×238 .

10. $90 \times \$4.88$.

17. 700×4200 .

4. 50×470 .

11. $70 \times \$2.66$.

18. 900×3500 .

5. 60×930 .

12. $80 \times \$4.32$.

19. $800 \times \$3.75$.

6. 20×765 .

13. $70 \times \$7.80$.

20. $500 \times \$4.80$.

7. 50×642 .

14. $90 \times \$5.60$.

21. $900 \times 99,000$.

46. Multiplying by Larger Multipliers. Required to multiply 4674 by 649.

The multiplier is $600 + 40 + 9$. The product is obtained by multiplying by 9, then by 40, and then by 600, and adding the products.

	(1)	(2)
	4674	4674
	<u>649</u>	<u>649</u>
9 times the multiplicand =	42066	42066
40 times the multiplicand =	186960	18696
<u>600</u> times the multiplicand =	<u>2804400</u>	<u>28044</u>
649 times the multiplicand =	3033426	3033426

Zeros at the right of the partial products, since they do not affect the result of the addition, should be omitted as in (2).

Care must be taken, however, to put the right-hand figure of each partial product directly under the figure of the multiplier used to obtain it.

Find the product of 2007 times \$45.87.

The products corresponding to the zeros in the multiplier will be zero, and therefore they need not be written. With this exception the multiplication is performed as in the preceding example. The decimal point is placed before the cents as in the multiplicand.

\$45.87
2007
<u>321.09</u>
9174
<u>\$92061.09</u>

EXERCISE 15

Find the product of:

- | | | |
|-----------------------|------------------------|------------------------|
| 1. 21×2648 . | 9. 67×2987 . | 17. 54×8387 . |
| 2. 31×4872 . | 10. 74×3692 . | 18. 73×9874 . |
| 3. 42×2634 . | 11. 78×8269 . | 19. 36×7448 . |
| 4. 62×3108 . | 12. 89×4871 . | 20. 95×6877 . |
| 5. 44×2177 . | 13. 93×3642 . | 21. 68×7482 . |
| 6. 54×3623 . | 14. 96×2996 . | 22. 73×8929 . |
| 7. 47×3926 . | 15. 73×8989 . | 23. 85×4832 . |
| 8. 56×7793 . | 16. 48×7642 . | 24. 63×6309 . |

- | | | |
|-------------------------|----------------------------|-----------------------------|
| 25. 84×7470 . | 44. 407×8643 . | 63. $700 \times \$27.73$. |
| 26. 96×3998 . | 45. 502×8926 . | 64. $800 \times \$51.36$. |
| 27. 74×8647 . | 46. 708×3971 . | 65. $900 \times \$83.42$. |
| 28. 98×8766 . | 47. 637×9286 . | 66. $800 \times \$4200$. |
| 29. 99×9999 . | 48. 843×8792 . | 67. $700 \times \$7500$. |
| 30. 211×7361 . | 49. 695×5764 . | 68. $600 \times \$3200$. |
| 31. 311×4872 . | 50. 783×9871 . | 69. $500 \times \$9600$. |
| 32. 401×3664 . | 51. 847×6476 . | 70. $283 \times \$48.25$. |
| 33. 503×2763 . | 52. 972×3502 . | 71. $342 \times \$29.75$. |
| 34. 630×4876 . | 53. 866×4009 . | 72. $276 \times \$49.42$. |
| 35. 580×6924 . | 54. 707×8763 . | 73. $396 \times \$64.40$. |
| 36. 770×3871 . | 55. 5002×7983 . | 74. $428 \times \$75.19$. |
| 37. 860×3926 . | 56. 6008×3976 . | 75. $532 \times \$81.62$. |
| 38. 722×3322 . | 57. 4003×6648 . | 76. $437 \times \$29.86$. |
| 39. 421×7232 . | 58. 5009×9987 . | 77. $522 \times \$62.23$. |
| 40. 512×3211 . | 59. $300 \times \$75.35$. | 78. $729 \times \$72.30$. |
| 41. 643×7806 . | 60. $400 \times \$82.25$. | 79. $8004 \times \$91.76$. |
| 42. 428×3605 . | 61. $500 \times \$61.32$. | 80. $9006 \times \$38.92$. |
| 43. 826×8207 . | 62. $600 \times \$43.35$. | 81. $7052 \times \$47.80$. |

82. What will 175 chairs cost at \$2.35 each?

83. What will 630 barrels of salt cost at \$3.08 a barrel?

84. What will it cost to cart 168 loads of produce at \$3.15 a load?

85. What will 475 bushels of clover seed cost at \$3.12 a bushel?

86. What will a wholesale dealer have to pay for 240 yards of velvet at \$1.85 a yard, 480 yards of silk at \$1.38, 520 yards of lace at \$1.62, and 250 yards of broadcloth at \$1.75?

EXERCISE 16

REVIEW PROBLEMS

1. How much will 30 cows cost at \$35 a head?
2. A mile contains 5280 feet. How many feet in 8 miles?
3. How much will 440 bales of hops weigh at 200 pounds a bale?
4. If a square lot is 66 feet on a side, what is the distance around it?
5. If sound travels 1125 feet in 1 second, how far will it travel in 20 seconds?
6. A farmer bought 7 bushels of grass seed at \$3.40 a bushel. How much did it all cost?
7. A city school bought 20 dictionaries at \$9.75 each. How much did they all cost?
8. If a salesman receives \$25 a week and works 48 weeks a year, what is his income for the year?
9. At \$85 a month, how much will the rent of a city apartment amount to in a year?
10. A lady rents a piano for \$4.25 a month. How much is the rent for 6 months?
11. At \$250 a month, what will a man's salary amount to in a year?
12. If a man grinds 35 bushels of corn of 56 pounds each, how many pounds does he grind?
13. A farmer adds to his property a field of 65 acres, paying \$75 an acre for it. How much does the field cost?
14. A dealer sells 15 automobiles at \$925 each. How much does he receive for them all?
15. If 38 cows average 408 pounds of milk apiece a month, how many pounds do the cows produce in a month?

16. There being 12 inches in 1 foot, how many inches are there in 175 feet?

17. There being 5280 feet in 1 mile, how many feet are there in 24 miles?

18. There being 144 square inches in 1 square foot, how many square inches are there in 93 square feet?

19. There being 128 cubic feet in a cord, how many cubic feet are there in 152 cords of wood?

20. If a farm averages 28 bushels of buckwheat to the acre, how many bushels will 175 acres produce?

21. If a farm averages 18 bushels of winter rye to the acre, how many bushels will 136 acres produce?

22. A manufacturer sells 175 sets of harness at \$24 a set. How much does he receive for them all?

23. There being 640 acres in 1 square mile, how many acres are there in 135 square miles?

24. If a farm averages 2256 pounds of hay to the acre, how much hay will 275 acres produce?

25. A cubic foot of hard coal weighing 84 pounds, how much will 246 cubic feet of hard coal weigh?

26. A cubic foot of ice weighing 58 pounds, how much will 1070 cubic feet of ice weigh?

27. Allowing 5 ounces to a cubic inch, how much will a cubic foot (1728 cubic inches) of copper weigh?

28. If a man averages 2265 steps in walking a mile, how many steps will he take in walking 23 miles?

29. A barrel of flour weighing 196 pounds, how much will 144 barrels weigh?

30. Find the cost of 42 head of cattle at \$37.50 each.

31. If a farmer buys 75 acres of land at \$62.25 an acre, what is the total cost?

32. How much will 65 horses cost at \$82.75 each?
33. At \$5.75 a ton, how much will 356 tons of coal cost?
34. A book dealer sells 206 copies of a certain book at \$1.25 each. How much does he receive for them all?
35. At \$2.98 a ton at the mine, how much will 478 tons of iron cost?
36. When sound travels 1125 feet in a second, how far will it travel in 2 minutes?
37. If a cubic foot of cast iron weighs 447 pounds, how much will 12 posts weigh, each containing 3 cubic feet?
38. If a cubic foot of wrought iron weighs 486 pounds, how much will 17 beams of 2 cubic feet each weigh?
39. If a cubic foot of ice weighs 58 pounds, what will 32 cakes weigh, each containing 3 cubic feet?
40. The earth moves about the sun 101,090 feet in a second. How many feet does it move in a minute?
41. If a steam engine uses 2 tons of coal a day, how many tons will it consume in January if it runs every day?
42. If a man earns \$3.50 a day, and works 6 days in the week for 48 weeks, how much does he earn in a year?
43. A merchant bought 7 pieces of cloth of 33 yards each and paid \$1.60 a yard for it. What was the total cost?
44. A grocer sold 4 boxes of oranges, each box containing 196 oranges, at 4 cents apiece. How much did he receive for all?
45. Which will cost the more, 48 cows at \$37.50 each, or 22 horses at \$88 each? How much more?
46. If electricity travels on a wire at the rate of 288,000 miles per second, how long would a wire have to be in order that an electric current would travel from one end to the other in one minute?

CHAPTER V

DIVISION OF INTEGERS

47. Nature of Division. In multiplication two factors are given to find the product. In division the product and one of two factors are given to find the other factor.

That is, in the case of $2 \times \$4 = \$8$, we have given the factors 2 and \$4, from which to find the product \$8.

In division we have given the product \$8 and the factor 2, to find the other factor \$4; or the factor \$4, to find the other factor 2.

48. Division. If the product and one of two factors are given, the process of finding the other factor is called *division*.

49. Symbol. The symbol for division is $\div$. It is read *divided by*.

For example, $15 \div 5 = 3$. This is also indicated by another symbol, thus: $\frac{15}{5} = 3$. That is, $15 \div 5$ and $\frac{15}{5}$ have the same meaning.

50. Dividend. The given product, that is, the number to be divided, is called the *dividend*.

51. Divisor. The given factor, that is, the number by which we divide, is called the *divisor*.

52. Quotient. The required factor, that is, the number found by division, is called the *quotient*.

53. Remainder. Often a divisor does not exactly divide the dividend. Then the part of the dividend that is left is called the *remainder*.

Thus $14 \div 3 = 4$ with remainder 2. The quotient is usually written $4\frac{2}{3}$, meaning 4 with 2 still to be divided into 3 equal parts.

54. Two Kinds of Division. Since $2 \times \$4 = \$8$, therefore $\$8 \div \$4 = 2$, and $\$8 \div 2 = \$4$. That is,

If the dividend and the divisor are concrete, they must be like numbers, and the quotient is abstract.

For example, $\$8 \div \$4 = 2$. This is sometimes called *measuring* because the \$8 is measured by the \$4.

If the dividend is concrete and the divisor abstract, the quotient is like the dividend.

For example, $\$8 \div 2 = \$4$. This is sometimes called *partition*, because the \$8 is separated into 2 equal parts.

55. Check. If there is no remainder, the dividend is the product of the quotient and the divisor. To check the work in division, therefore, we multiply the quotient by the divisor; this product added to the remainder, if any, should equal the dividend.

For example, $45 \div 5 = 9$; then $5 \times 9 = 45$. Again $48 \div 5 = 9$ with remainder 3; then $5 \times 9 = 45$, and $45 + 3 = 48$.

56. Short Division. When the divisor is so small that the work may be done mentally the process is called *short division*.

(1) Required to divide 342 by 3.

$$3 \overline{)300 + 30 + 12}$$

$$100 + 10 + 4 = 114$$

If 342 is separated as shown, we can divide each part by 3. This is what we do mentally below. We write the num-

bers as shown; then 3 is contained in 3 once (that is, 100 times in 300); 3 is contained in 4 once with remainder 1 (that is, 10 times in 40 with remainder 10); this 1 is 10 units, and with the 2 makes 12.

Then 3 is contained in 12 four times. Therefore the quotient is 114. We think: "3 in 3, 1; in 4, 1; in 12, 4."

(2) Required to divide 42,364 by 7.

$$7 \overline{)42364} \\ 6052$$

Here 7 is not contained in 4, but 7 is contained in 42 six times. Then 7 is not contained in 3, so we write a 0 beneath the 3. Then 7 is contained in 36 five times with remainder 1, and in 14 twice. Therefore the quotient is 6052.

(3) Required to divide 35,282 by 9.

$$\begin{array}{r} 9 \overline{) 35282} \\ \underline{3920} \end{array}$$
 Here 9 is contained in 35 three times with remainder 8; in 82 nine times with remainder 1; in 18 twice; in 2 no times with remainder 2. Therefore the quotient is 3920 with remainder 2, and this is usually written $3920\frac{2}{9}$.

(4) Required to divide \$27.53 by 4.

$$\begin{array}{r} 4 \overline{) \$27.53} \\ \underline{\$6.88} \end{array}$$
 Dividing as before, we have $\$27 \div 4 = \$6$, with \$3 remaining; $\$3.50 \div 4 = \$0.8$ with \$0.30 remaining; $\$0.33 \div 4 = \$0.08$, with \$0.01 remaining. Therefore the quotient is $\$6.88\frac{1}{4}$.

EXERCISE 17

Find the quotient of:

- | | | |
|---------------------|------------------------|------------------------|
| 1. $232 \div 2$. | 19. $4213 \div 4$. | 37. $70,328 \div 8$. |
| 2. $345 \div 3$. | 20. $2893 \div 5$. | 38. $32,013 \div 9$. |
| 3. $416 \div 4$. | 21. $6271 \div 6$. | 39. $31,201 \div 2$. |
| 4. $325 \div 5$. | 22. $7356 \div 7$. | 40. $34,078 \div 3$. |
| 5. $738 \div 6$. | 23. $9123 \div 8$. | 41. $26,231 \div 4$. |
| 6. $924 \div 7$. | 24. $8077 \div 9$. | 42. $70,023 \div 5$. |
| 7. $592 \div 8$. | 25. $\$47.24 \div 2$. | 43. $80,335 \div 6$. |
| 8. $837 \div 9$. | 26. $\$33.15 \div 3$. | 44. $29,321 \div 7$. |
| 9. $3242 \div 2$. | 27. $\$28.72 \div 4$. | 45. $42,345 \div 8$. |
| 10. $4122 \div 3$. | 28. $\$32.75 \div 5$. | 46. $76,209 \div 9$. |
| 11. $6004 \div 4$. | 29. $\$21.30 \div 6$. | 47. $\$35.21 \div 2$. |
| 12. $3015 \div 5$. | 30. $\$29.47 \div 7$. | 48. $\$72.43 \div 3$. |
| 13. $4032 \div 6$. | 31. $\$65.68 \div 8$. | 49. $\$61.23 \div 4$. |
| 14. $3073 \div 7$. | 32. $\$83.61 \div 9$. | 50. $\$72.36 \div 5$. |
| 15. $7368 \div 8$. | 33. $81,004 \div 4$. | 51. $\$82.01 \div 6$. |
| 16. $8019 \div 9$. | 34. $29,005 \div 5$. | 52. $\$42.03 \div 7$. |
| 17. $5555 \div 2$. | 35. $31,266 \div 6$. | 53. $\$65.00 \div 8$. |
| 18. $6014 \div 3$. | 36. $40,040 \div 7$. | 54. $\$93.65 \div 9$. |

EXERCISE 18

1. If 3 head of cattle cost \$117, how much will 1 head cost ?

2. It is 108 feet around a square. What is the length of each side ?

3. There being 7 days in a week, how many weeks are there in 364 days ?

4. There being 3 feet in a yard, how many yards are there in 432 feet ?

5. There being 4 quarts in a gallon, how many gallons are there in 236 quarts ?

6. It is 132 feet around a triangle whose sides are all equal. What is the length of each side ?

7. At \$9 a dozen, how many dozen cups and saucers can be bought for \$1116 ?

8. At \$4 a dozen, how many dozen handkerchiefs can be bought for \$1152 ?

9. A dealer pays \$1224 for 9 wagons. What is the average price ?

10. A wholesale grocer pays \$1728 for a shipment of chocolate at \$3 per dozen cans. How many dozen cans does he buy ?

11. A wholesale grocer pays \$4320 for some cocoa at \$6 per dozen cans. How many dozen cans does he buy ?

12. A hotel pays \$13.20 for 8 dozen packages of breakfast food. How much does it pay per dozen ?

13. If a grocer pays \$12.15 for 9 dozen packages of breakfast cereal, how much does he pay per dozen ?

14. If a grocer pays \$19.84 for 8 dozen tins of biscuit, how much does he pay per dozen ?

57. Dividing by 10 and its Powers. To divide by 10 is merely to find how many tens there are in the number, and this we can tell at a glance.

Thus in 1230 there are 123 tens and no units. In 2373 there are 237 tens and 3 units remainder; that is, $2373 \div 10 = 237\frac{3}{10}$.

Therefore, *to divide an integer by 10 cut off the right-hand figure of the dividend. The result is the quotient, and the right-hand figure expresses the remainder.*

To divide an integer by any power of 10 cut off as many figures from the right of the dividend as there are zeros at the right of the divisor. The result is the quotient, and the right-hand figures express the remainder.

Thus $12,400 \div 100 = 124$; $43,203 \div 100 = 432\frac{3}{100}$; $624,000 \div 1000 = 624$; $736,117 \div 1000 = 736\frac{117}{1000}$.

58. Dividing by Multiples of Powers of 10. In this case we divide by the power of 10, as in § 57, and then by the number indicating the multiple.

For example, to divide 42,800 by 200 we cut off two zeros from divisor and dividend and then divide by 2.

$$\begin{array}{r} 200 \overline{)42800} \\ \underline{214} \end{array}$$

So to divide 27,407 by 300 we cut off two zeros from the divisor and two figures from the right of the dividend. We then divide by 3, and the quotient is 91, with remainder 1 in the hundreds' place, and to this we join the 07 cut off, obtaining the remainder 107.

$$\begin{array}{r} 300 \overline{)27407} \\ \underline{91107} \end{array}$$

EXERCISE 19

Find the quotient and the remainder (if any):

- | | | |
|--------------------|--------------------|---------------------|
| 1. $340 \div 10$. | 5. $750 \div 50$. | 9. $360 \div 90$. |
| 2. $720 \div 20$. | 6. $840 \div 60$. | 10. $720 \div 90$. |
| 3. $540 \div 30$. | 7. $840 \div 70$. | 11. $273 \div 10$. |
| 4. $640 \div 40$. | 8. $960 \div 80$. | 12. $423 \div 20$. |

Find the quotient and the remainder (if any) :

- | | | |
|---------------------|-------------------------|---------------------------|
| 13. $325 \div 30$. | 21. $20,300 \div 100$. | 29. $98,207 \div 900$. |
| 14. $275 \div 40$. | 22. $27,400 \div 200$. | 30. $126,000 \div 3000$. |
| 15. $236 \div 50$. | 23. $64,200 \div 300$. | 31. $328,000 \div 4000$. |
| 16. $327 \div 70$. | 24. $72,400 \div 400$. | 32. $465,000 \div 5000$. |
| 17. $542 \div 70$. | 25. $31,501 \div 500$. | 33. $564,700 \div 6000$. |
| 18. $391 \div 80$. | 26. $27,403 \div 600$. | 34. $429,820 \div 7000$. |
| 19. $278 \div 90$. | 27. $59,721 \div 700$. | 35. $742,623 \div 8000$. |
| 20. $777 \div 90$. | 28. $42,348 \div 800$. | 36. $857,542 \div 9000$. |
37. At \$90 each, how many horses can I buy for \$2160?
38. There being 60 minutes in an hour, how many hours are there in 1020 minutes?
39. There being 60 seconds in a minute, how many minutes are there in 1620 seconds?
40. At \$30 each, how many harnesses must a manufacturer sell to receive \$1110?
41. A farmer pays \$6600 for some land at \$40 an acre. How many acres does he buy?
42. A dealer sells some village lots for \$10,200, at \$600 a lot. How many lots does he sell?
43. An investor has \$87,750 with which to buy some bonds at \$500 each. What is the greatest number he can buy, and how much will he have left?
44. An automobile dealer has \$35,100 to invest in automobiles at \$2000 each. What is the greatest number he can buy for this sum, and how much will he have left?
45. A ton is 2000 pounds. If a coal pocket contains 583,260 pounds of coal, how many tons does it contain, and how many pounds are left over?

EXERCISE 20

PROBLEMS OF THE FARM

1. A farmer sowed 60 acres of land, using 780 quarts of timothy seed. How much seed did he use per acre?

2. He also sowed 70 acres, using 1050 quarts of timothy and clover mixed. How much seed did he use per acre?

3. Another farmer sowed 90 acres, using 2520 quarts of timothy and redtop mixed equally. How many quarts of each kind did he use per acre?

4. A Maryland farmer experimented with Italian rye grass, using 3250 pounds to sow 50 acres. How many pounds did he use per acre?

5. A farmer found that 30 sheep ate 11,550 pounds of dry fodder in a year. What was the average amount per sheep?

6. He found that 20 horses consumed 162,400 pounds of hay and grain in a year. What was the average amount per horse?

7. A dealer found that 60 bushels of white clover seed weighed 3780 pounds. How much did it weigh per bushel?

8. He told a customer 1080 pounds would seed 90 acres. How much is that per acre? If he mixed other seed with the clover so as to use this amount to cover 4 acres, how much clover seed would he use per acre?

9. A farmer found that it took 980 pounds of red clover for 70 acres, and 1200 pounds of crimson clover for 80 acres. What was the amount of each used per acre?

10. How many pounds of Paris green will be needed to spray 300 trees, allowing 5 gallons to a tree, if every 150 gallons contains 1 pound of Paris green?

59. Long Division. When the divisor has two or more figures, except in such cases as are given in § 57 and § 58, it is customary to use a process known as *long division*.

We shall first consider the simplest case, that in which the units' figure is 1.

Required to divide 7182 by 21.

If all the work should be written, it would appear as in the first of these forms, but for practical purposes the second is used.

(1)	(2)
$\begin{array}{r} 342 \\ 21 \overline{)7182} \\ \underline{6300} = 300 \text{ times the divisor} \\ 882 \\ \underline{840} = 40 \text{ times the divisor} \\ 42 \\ \underline{42} = 2 \text{ times the divisor} \end{array}$	$\begin{array}{r} 342 \\ 21 \overline{)7182} \\ \underline{63} \\ 88 \\ \underline{84} \\ 42 \\ \underline{42} \end{array}$

First we will consider (1), the more complete form. We see that 21 is evidently not contained in 7, so there are no thousands in the quotient. The divisor is contained 3 times in 71, and since the 71 is hundreds, the 3 is written above, over the hundreds' place. Then $300 \times 21 = 6300$, and there remains 882 still to be divided; 21 is contained in 88 four times, and since the 88 is tens, the 4 of the quotient is written over the tens' place. Then $40 \times 21 = 840$, and there remains 42 to be divided; 21 is contained in 42 twice, and 2 is written over the units' place. The quotient is therefore 342. It will be seen that 6300, 840, and 42 make up 342×21 , as should be the case.

Consider now the second form, which is always used in practice. Since at any time in this example we need only two figures of the dividend or two figures of such a partial dividend as 882, we may omit the zeros in 6300 and 840, and use only the 88 in the remainder 882, thus making the work shorter.

We may check our result by multiplying 342 by 21. Then $21 \times 342 = 7182$, the dividend.

The pupil should check his work in enough examples to become thoroughly familiar with the process.

Required to divide 6447 by 21.

$$\begin{array}{r} 307 \\ 21 \overline{)6447} \\ \underline{63} \\ 147 \\ \underline{147} \\ 0 \end{array}$$

Here we have $64 \div 21 = 3$, and since the 64 is hundreds, the 3 is written over the hundreds' place. Then $3 \times 21 = 63$, and when 63 is subtracted, there is a remainder 1. Bringing down one figure as before, we have 14 tens to be divided by 21. Since 21 is not contained in 14, we write 0 in the tens' place to show that there are no tens, and bring down the next figure 7. Then $147 \div 21 = 7$, and $7 \times 21 = 147$ exactly, so there is no remainder.

EXERCISE 21

Find the quotient of:

- | | | |
|---------------------|------------------------|------------------------|
| 1. $352 \div 11$. | 19. $1037 \div 61$. | 37. $40,953 \div 51$. |
| 2. $451 \div 11$. | 20. $1312 \div 41$. | 38. $49,349 \div 61$. |
| 3. $682 \div 11$. | 21. $1562 \div 71$. | 39. $18,696 \div 41$. |
| 4. $968 \div 11$. | 22. $2673 \div 81$. | 40. $52,582 \div 61$. |
| 5. $858 \div 11$. | 23. $3367 \div 91$. | 41. $62,267 \div 71$. |
| 6. $957 \div 11$. | 24. $2331 \div 21$. | 42. $63,048 \div 71$. |
| 7. $737 \div 11$. | 25. $2583 \div 21$. | 43. $71,280 \div 81$. |
| 8. $913 \div 11$. | 26. $3472 \div 31$. | 44. $90,090 \div 91$. |
| 9. $252 \div 21$. | 27. $2121 \div 21$. | 45. $90,909 \div 91$. |
| 10. $294 \div 21$. | 28. $6789 \div 31$. | 46. $50,337 \div 51$. |
| 11. $441 \div 21$. | 29. $8547 \div 21$. | 47. $50,898 \div 51$. |
| 12. $672 \div 21$. | 30. $3317 \div 31$. | 48. $62,937 \div 81$. |
| 13. $777 \div 21$. | 31. $10,731 \div 21$. | 49. $71,928 \div 81$. |
| 14. $819 \div 21$. | 32. $14,322 \div 31$. | 50. $60,939 \div 61$. |
| 15. $987 \div 21$. | 33. $16,461 \div 31$. | 51. $34,587 \div 61$. |
| 16. $868 \div 31$. | 34. $25,297 \div 41$. | 52. $73,629 \div 81$. |
| 17. $779 \div 41$. | 35. $18,879 \div 31$. | 53. $65,448 \div 81$. |
| 18. $765 \div 51$. | 36. $28,741 \div 41$. | 54. $73,619 \div 91$. |

60. Long Division Continued. We have considered the simple case in which the units' figure is 1, and we shall now consider the more general case of a two-figure divisor.

Required to divide 19,224 by 54.

The divisor is not contained in 1 or in 19, so there are no ten-thousands and no thousands in the quotient. But since $3 \times 54 = 162$, the divisor is contained 3 times in 192 with remainder 30. Since this is hundreds, the quotient figure 3 is written over the hundreds' place. Bringing down the next figure of the dividend, 2, we have 302 tens to be divided. Since $5 \times 54 = 270$, the divisor is contained 5 times in 302, with remainder 32. Since this is tens, the quotient figure 5 is written over the tens' place. Bringing down the next figure of the dividend, 4, we have 324 still to be divided. Since 324 is exactly 6×54 , the units' figure is 6 and there is no remainder. Hence the quotient is 356.

$$\begin{array}{r}
 356 \\
 54 \overline{)19224} \\
 \underline{162} \\
 302 \\
 \underline{270} \\
 324 \\
 \underline{324} \\
 0
 \end{array}$$

61. Product Tables. It is sometimes helpful for beginners to prepare a table of products of the divisor, as follows:

$2 \times 54 = 108$	$4 \times 54 = 216$	$6 \times 54 = 324$	$8 \times 54 = 432$
$3 \times 54 = 162$	$5 \times 54 = 270$	$7 \times 54 = 378$	$9 \times 54 = 486$

With such a table made out in advance, we can tell at once, without making several trials, what the next figure of the quotient will be.

EXERCISE 22

Find the quotient of:

- | | | |
|---------------------|------------------------|-------------------------|
| 1. $3796 \div 52$. | 8. $5929 \div 77$. | 15. $98,901 \div 99$. |
| 2. $8464 \div 92$. | 9. $2451 \div 19$. | 16. $41,831 \div 59$. |
| 3. $6510 \div 62$. | 10. $8514 \div 66$. | 17. $55,821 \div 69$. |
| 4. $4599 \div 73$. | 11. $22,311 \div 67$. | 18. $54,439 \div 77$. |
| 5. $8772 \div 43$. | 12. $31,950 \div 75$. | 19. $118,216 \div 56$. |
| 6. $2275 \div 35$. | 13. $63,204 \div 69$. | 20. $301,184 \div 64$. |
| 7. $2736 \div 57$. | 14. $74,994 \div 87$. | 21. $891,891 \div 99$. |

22. If a steamship sails 6336 miles in 18 days, what is the rate per day?

23. How many 25-pound packages can be made from 18,000 pounds of paper?

24. How many 49-pound sacks can be filled from 150,675 pounds of flour?

25. The product of two numbers is 98,441, and one of them is 49. What is the other?

26. A man paid \$28,800 for a tract of land at \$45 an acre. How many acres did he buy?

27. A man paid \$19,125 for a farm, the price being \$85 an acre. How many acres did he buy?

28. A certain book averages 29 lines to a page and contains 10,034 lines in all. How many pages are there?

29. If a desk company sells \$20,184 worth of desks in a year at \$29 each, how many desks does it sell?

30. If a factory sells \$18,648 worth of harnesses in a season at \$37 a set, how many sets does it sell?

31. A canning factory ships 30,000 cans of fruit in boxes containing 4 dozen cans each. How many boxes does it require?

32. There being 27 cubic feet in a cubic yard, how many loads (cubic yards) of earth must be removed to excavate 108,675 cubic feet?

33. One of the routes around the world is 18,424 miles long. If a man makes the trip in 56 days, what is the average distance per day?

34. If the circumference of a wagon wheel is 15 feet, and there are 5280 feet in a mile, how many times will the wheel turn in going 9 miles? How many times in going 25 miles?

62. Long Division Continued. We may now consider divisors of three or more figures, and cases of division in which there are remainders.

Required to divide 341,597 by 248.

$ \begin{array}{r} 1377 \\ 248 \overline{)341597} \\ \underline{248} \\ 935 \\ \underline{744} \\ 1919 \\ \underline{1736} \\ 1837 \\ \underline{1736} \\ 101 \end{array} $	341 once with remainder 93. Since this is thousands, the quotient figure, 1, is written over the thousands' place. Bringing down the next figure of the dividend, 5, we have 935 hundreds to be divided. It might seem as if the next figure would be 4, and here a product table (§ 61) would be helpful; but $4 \times 248 = 992$, and this is larger than 935. Trying 3 for the next figure, we have $3 \times 248 = 744$, and there is a remainder 191. Since we have divided hundreds by 248, the 3 is hundreds and is written over that place. Bringing down the next figure, we find that $1919 \div 248 = 7$, with 183 remaining. Since the 7 is tens, we write it over the tens' place. Then, bringing down the last figure, we have $1837 \div 248 = 7$, with 101 remaining. Therefore the quotient is 1377, and the remainder is 101. The complete quotient may be written $1377\frac{101}{248}$.
---	--

We may check the work by adding 101 to the product of 248×1377 , the result being 341,597, the dividend.

63. General Directions. It may assist us if we now state the general directions we have found for long division :

Write the divisor to the left of the dividend.

Take for the first partial dividend the least number of left-hand figures that contains the divisor, and write the quotient over the right-hand figure of this partial dividend.

Multiply the divisor by this quotient, and place the product under the partial dividend used.

Subtract this product, and to the remainder annex the next figure of the dividend for a new partial dividend.

Divide as before, and continue this process until all the figures of the dividend have been used.

EXERCISE 23

Find the quotient of:

- | | |
|---------------------------|-----------------------------|
| 1. $25,284 \div 301$. | 27. $7684 \div 69$. |
| 2. $51,742 \div 631$. | 28. $8522 \div 76$. |
| 3. $47,348 \div 623$. | 29. $8562 \div 23$. |
| 4. $153,186 \div 211$. | 30. $20,452 \div 56$. |
| 5. $475,950 \div 501$. | 31. $18,113 \div 64$. |
| 6. $972,360 \div 111$. | 32. $14,797 \div 57$. |
| 7. $161,463 \div 321$. | 33. $19,140 \div 46$. |
| 8. $324,360 \div 306$. | 34. $26,523 \div 47$. |
| 9. $140,850 \div 225$. | 35. $27,559 \div 59$. |
| 10. $183,750 \div 175$. | 36. $47,078 \div 69$. |
| 11. $148,555 \div 407$. | 37. $50,602 \div 82$. |
| 12. $234,901 \div 503$. | 38. $101,160 \div 232$. |
| 13. $208,575 \div 309$. | 39. $110,370 \div 124$. |
| 14. $870,672 \div 204$. | 40. $252,150 \div 412$. |
| 15. $367,848 \div 786$. | 41. $120,830 \div 653$. |
| 16. $102,465 \div 495$. | 42. $260,380 \div 422$. |
| 17. $472,932 \div 604$. | 43. $192,892 \div 232$. |
| 18. $349,206 \div 407$. | 44. $211,428 \div 5873$. |
| 19. $117,440 \div 320$. | 45. $287,712 \div 5328$. |
| 20. $419,640 \div 538$. | 46. $313,205 \div 8465$. |
| 21. $287,712 \div 216$. | 47. $411,075 \div 5481$. |
| 22. $615,000 \div 328$. | 48. $616,693 \div 8009$. |
| 23. $685,542 \div 286$. | 49. $965,496 \div 9852$. |
| 24. $997,108 \div 478$. | 50. $1,599,560 \div 2836$. |
| 25. $965,496 \div 9852$. | 51. $2,402,728 \div 6257$. |
| 26. $692,356 \div 8764$. | 52. $5,191,776 \div 5947$. |

53. If a man receives a salary of \$1728 a year, how much does he receive a month ?
54. If a man receives a salary of \$1092 for the 52 weeks of a year, how much does he receive a week ?
55. If a man pays \$8569 for 209 acres of land, how much does it cost per acre ?
56. If a book jobber pays \$3402 for 42 sets of an encyclopedia, how much does he pay per set ?
57. If a stock dealer buys some cattle for \$71,961, paying \$51 a head, how many head does he buy ?
58. If a jobber sells during the year \$82,350 worth of shoes at \$61 a case, how many cases does he sell ?
59. If a dealer sells \$142,560 worth of linseed oil during a year at \$22 a barrel, how many barrels does he sell ?
60. If a factory sells 3684 ready-made suits of a certain grade for \$92,100, what is the price per suit ?
61. If the product of two numbers is 2,137,512, and one of them is 372, what is the other number ?
62. If a state pays \$2,325,798 for constructing 471 miles of model road, what is the average cost per mile ?
63. If it costs \$5,236,602 to construct 287 miles of railroad, what is the average cost per mile ?
64. What is the smallest number that must be subtracted from 615,476 so that the result divided by 1875 gives a quotient with no remainder ?
65. If a company employs 8009 men on the same schedule of wages, and the pay roll for these men is \$6,247,020 a year, what is the annual wages of each ?
66. If a company employs 250 men on the same schedule of wages, and the pay roll for these men is \$270,000 a year, what is the monthly wages of each ?

64. Long Division of United States Money. Division of United States money does not involve any new principle. We divide as with other numbers, placing the decimal point in the quotient above that in the dividend.

Required to divide \$21,375.12 by 372.

$$\begin{array}{r}
 \$57.46 \\
 372 \overline{) \$21375.12} \\
 \underline{1860} \\
 2775 \\
 \underline{2604} \\
 1711 \\
 \underline{1488} \\
 2232 \\
 \underline{2232} \\
 0
 \end{array}$$

Since \$2, \$21, and \$213 cannot be divided by 372 with an integer for a quotient, we divide \$2137 by 372, and the quotient is \$5, with remainder \$277. Since the \$2137 is in the tens' place of dollars, we write the \$5 over that place of the dividend.

The rest of the division is as in cases already studied. The decimal point in the quotient is placed over that in the dividend, and the quotient is \$57.46.

EXERCISE 24

Find the quotient of:

- | | |
|-----------------------|----------------------------|
| 1. \$972.36 ÷ 111. | 14. \$39,832.56 ÷ 81. |
| 2. \$374.12 ÷ 199. | 15. \$15,391.25 ÷ 125. |
| 3. \$1467.50 ÷ 50. | 16. \$14,225.40 ÷ 420. |
| 4. \$6882.92 ÷ 11. | 17. \$734,986.12 ÷ 836. |
| 5. \$12,018.30 ÷ 21. | 18. \$501,738.80 ÷ 538. |
| 6. \$96,899.23 ÷ 23. | 19. \$102,465.00 ÷ 2070. |
| 7. \$36,147.06 ÷ 53. | 20. \$692,356.00 ÷ 8764. |
| 8. \$33,328.80 ÷ 54. | 21. \$616,693.00 ÷ 8009. |
| 9. \$33,747.20 ÷ 64. | 22. \$388,591.00 ÷ 9037. |
| 10. \$22,310.64 ÷ 72. | 23. \$421,203.20 ÷ 5983. |
| 11. \$30,796.80 ÷ 64. | 24. \$5,191,731.00 ÷ 5947. |
| 12. \$49,060.08 ÷ 72. | 25. \$5,236,602.00 ÷ 9123. |
| 13. \$34,689.75 ÷ 69. | 26. \$3,851,640.00 ÷ 9876. |

27. A man distributed the sum of \$17,975.10 equally among fifteen relatives. How much did each receive?

28. A man paid \$52.08 for 24 dozen cans of peas. How much did they cost him a dozen?

29. A man paid \$150.72 for 96 dozen cans of tomatoes. How much did they cost him a dozen?

30. A grocer bought 94 dozen quart jars of preserved figs for \$888.30. How much did they cost him a dozen?

31. A dealer paid \$517.50 for 250 dozen packages of shredded cocoanut. How much did he pay a dozen?

32. If \$2083.20 is divided equally among 672 men, how much does each receive?

33. If \$2652.30 is divided equally among 842 men, how much does each receive?

34. A merchant pays \$553.80 for 195 yards of silk. How much does he pay a yard?

35. A merchant pays \$865.08 for 324 yards of velvet. How much does he pay a yard?

36. A wholesale grocer sells 175 dozen quart bottles of vinegar for \$862.75. How much does he receive a dozen?

37. If \$1991 a day is paid to 724 men who have the same wages in a certain shop, how much does each receive?

38. If 624 men, working for the same wages in a certain shop, receive \$1996.80 a day, how much does each receive?

39. If 485 men, working for the same wages in a certain shop, receive \$1503.50 a day, how much does each receive?

40. A wholesale dealer purchased 144 dozen pineapple cheeses for \$1451.52. How much did he pay a dozen?

41. A shop has 664 men on its pay roll, receiving the same wages. For 6 days this pay roll amounts to \$9960. How much does each man receive a day?

65. Principles of Division. It will be seen by the following examples that some interesting relations exist among the dividend, divisor, and quotient. If we consider the case of $48 \div 6 = 8$, we see that

(1) *Multiplying the dividend multiplies the quotient by the same number.*

That is, $96 \div 6 = 16$. The dividend, 48, has been multiplied by 2, and the quotient, 8, has also been multiplied by 2.

(2) *Dividing the dividend divides the quotient by the same number.*

That is, $24 \div 6 = 4$. The dividend, 48, has been divided by 2, and the quotient, 8, has also been divided by 2.

(3) *Multiplying the divisor divides the quotient by the same number.*

That is, $48 \div 12 = 4$. The divisor, 6, has been multiplied by 2, and the quotient, 8, has been divided by 2.

(4) *Dividing the divisor multiplies the quotient by the same number.*

That is, $48 \div 3 = 16$. The divisor, 6, has been divided by 2, and the quotient, 8, has been multiplied by 2.

(5) *Multiplying or dividing both dividend and divisor by the same number does not change the quotient.*

That is, if instead of $48 \div 6$ we have $96 \div 12$ or $24 \div 3$, the quotient will still be 8.

The principle involved in (5) is a very convenient principle and is used much in arithmetic.

For example, $\frac{50}{100} = 50 \div 100 = \frac{5}{10}$. Furthermore, instead of dividing 725 by 5 we may divide 1450 by 10 because it is much easier to multiply by 2 than to divide by 5. Likewise, if we wish to divide \$11.25 by \$3.25 we may multiply each number by 100 and then divide \$1125 by \$325, thus not being troubled with decimal points; again, if we wish to divide 27,235 by 25 we may multiply each number by 4 and divide 108,940 by 100, which can be done at a glance.

**EXERCISE 25****REVIEW PROBLEMS**

1. This girl bought for her mother 7 yards of silk at \$1.35 a yard. How much did it cost?

2. Her mother sent her to buy 5 yards of serge at \$1.75 a yard. How much did it cost? How much change should she receive from a \$10 bill?

3. If the girl's purchases at another time consist of toweling at 80 cents, muslin at \$1, needles at 45 cents, thread at 16 cents, beads at 25 cents, and braid at 24 cents, how much change should she receive from \$5?

4. She saw a lady buy 13 yards of broadcloth and pay \$45.50 for it. How much did it cost a yard?

5. If the merchant charges \$34.65 for 21 yards of velvet, how much does he charge a yard?

6. If the merchant sells tweeds at \$3.25 a yard, how much will he receive for 18 yards? How much for 37 yards?

7. If a lady buys 27 yards of silk at \$1.45, 3 yards of velvet at \$1.75, and 22 yards of serge at \$1.60, how much does she pay in all?

8. A man pays \$2.52 for 12 pounds of meat. How much does it cost a pound?

9. How many inches are there in 1 foot? How many feet are there in 432 inches?

10. If the average length of a man's step is 27 inches, how many steps will he take in going 837 inches?

11. If the average length of a man's step is 27 inches, how many inches will he go in taking 275 steps?

12. If a man earns \$2.65 in each of the 27 working days of a certain month, how much does he earn in all?

13. A dealer sold 75 barrels of sweet potatoes at \$2.25 a barrel. How much did he receive?

14. A dealer sold 64 barrels of sweet potatoes at \$1.80 a barrel, and 36 barrels at \$2.40 a barrel. How much did he receive for them all?

15. If 3 farmers, having equal amounts of oats to thresh, pay \$38.10 for the use of a threshing machine, how much should each pay?

16. If the altitude of Mt. Everest is 29,002 feet, and that of Mt. McKinley 20,464 feet, how much higher is Mt. Everest than Mt. McKinley?

17. If the total railway mileage in the United States was 193,346 miles at one time, and 229,951 miles a few years later, how much was the increase in mileage?

18. If a school buys 3 dozen blackboard pointers at \$1.15 a dozen, a dictionary holder at \$6.45, a dictionary at \$6.75, and a teacher's desk at \$13.50, what is the total cost?

19. If the greatest known depth of the Atlantic Ocean is 27,366 feet, and Mt. Washington is 6279 feet high, how high does Mt. Washington stand above the bottom of the Atlantic at its greatest known depth?

20. A grocer sold 7 barrels of flour for \$38.43. How much did he receive per barrel?

21. In making the voyage from Southampton to New York, an ocean steamer consumes 2375 tons of coal in 5 days. What is the average amount consumed per day?

22. A special train made the distance from Chicago to New York in 16 hours. The distance by the route taken was 960 miles. What was the average speed per hour?

23. Two trains leave Chicago at the same time. One travels east at the rate of 48 miles an hour and the other travels west at the rate of 39 miles an hour. How far apart are they in 4 hours?

24. Two trains start at the same time, one from New York and the other from Chicago. They travel towards each other on the same line, the former at the rate of 53 miles an hour and the latter at the rate of 48 miles an hour. The distance between New York and Chicago being 980 miles by this route, how far apart are the trains in 4 hours?

25. A farmer sowed 68 acres to clover, using 13 pounds of seed to the acre. How many pounds did he use in all? If the crop averaged 4 tons to the acre, how many tons did the farmer cut in all?

26. A coal dealer buys coal at \$2.87 a ton. He pays \$0.67 a ton for railway transportation and \$0.78 a ton for delivery, and sells the coal for \$5.25 a ton. How much profit does he make per ton?

27. A watering trough holds 118 gallons. It is cracked so that water leaks out at the rate of 24 gallons an hour, but water flows in through a feed pipe at the rate of 83 gallons an hour. If the trough is empty when the water is turned on, how long will it take to fill the trough?

EXERCISE 26

WRITTEN REVIEW QUESTIONS

1. Define units and give three illustrations.
2. Define numbers and give examples of two kinds.
3. What do you mean by integers, and what other names can you give to them?
4. What are abstract numbers? concrete numbers?
5. Distinguish between notation and numeration.
6. What are the two kinds of notation known to you? Why are they so called?
7. What do you mean by numerals? by digits?
8. What is meant by the place value of a figure?
9. What is the difference between orders and periods in writing numbers? Illustrate each.
10. How are periods indicated in the writing of numbers? Illustrate in two ways.
11. In writing United States money, how are the dollars separated from the cents? Illustrate.
12. What are the characters used in writing Arabic numbers, and those used in writing Roman numbers? Write four hundred forty-nine in each system.
13. What name is given to numbers that are to be added? What is the name of the result?
14. What is meant by like numbers? Illustrate by three pairs of like numbers.
15. How do you check the work in addition?
16. What names are given to the three numbers used in subtraction? Illustrate.

17. How do you check the work in subtraction?
18. What do you mean by multiplication? Show that it is a short process of addition.
19. What names are given to the three numbers used in multiplication? Illustrate.
20. What are factors? Illustrate.
21. May the multiplicand be abstract? May it be concrete? Illustrate both cases.
22. What is the effect of changing the order of the addends? also of the factors of a number? Illustrate by the cases of $4 + 7 + 9$, and $4 \times 7 \times 9$.
23. What is meant by a power of a number? by the square of a number? by the cube of a number? Illustrate.
24. What is the short way of multiplying an integer by 100? of multiplying United States money by 100?
25. What do you mean by division? In what way does division differ from multiplication?
26. Name the terms used in division and give an example showing the use of each.
27. If the dividend and divisor are concrete, what else may be said of their nature? What is the nature of the quotient? Illustrate.
28. May the dividend be concrete and the divisor abstract? Illustrate.
29. What is the short process of dividing an integer by 10? Illustrate by dividing 4750 by 10.
30. What is the effect on the quotient of multiplying the dividend or dividing the divisor? Illustrate each case.
31. What is the effect on the quotient of dividing the dividend or multiplying the divisor? Illustrate each case.

CHAPTER VI

DECIMAL FRACTIONS

66. Meaning of Decimals. We have already seen that \$2.25 means \$2 + 25 cents, which is the same as $\$2\frac{25}{100}$ or $\$2\frac{1}{4}$.

In the same way, 2.25 miles means $2\frac{25}{100}$ miles, or $2\frac{1}{4}$ miles; 1.50 means $1\frac{50}{100}$, or $1\frac{1}{2}$; 1.75 means $1\frac{75}{100}$, or $1\frac{3}{4}$; 1.5 means $1\frac{5}{10}$, or $1\frac{1}{2}$; 2.125 means $2\frac{125}{1000}$, or $2\frac{1}{8}$; and 0.125 means $\frac{125}{1000}$, or $\frac{1}{8}$. (See § 65.)

That is, 0.5 or .5 is a fraction whose denominator is not written, it being understood to be 10 from the fact that 5 occupies the first place to the right of the decimal point.

We therefore have the following :

0.5 means $\frac{5}{10}$, for the 5 extends to the 10th's place;
 0.25 means $\frac{25}{100}$, for the 25 extends to the 100th's place;
 0.125 means $\frac{125}{1000}$, for the 125 extends to the 1000th's place.
 The names of the places are, in part, as follows :

Thousands	Hundreds	Tens	Units	(Decimal point)	Tenths	Hundredths	Thousandths	Ten-thousandths
1	3	4	5	.	2	7	6	5

This number is read "one thousand three hundred forty-five and two thousand seven hundred sixty-five ten-thousandths." The orders beyond ten-thousandths are hundred-thousandths, millionths, ten-millionths, and so on.

67. Decimal Fractions. A fraction whose denominator is not written, but is some power of 10, is called a *decimal fraction*, or more often simply a *decimal*.

Thus 0.25, which has the same value as $\frac{25}{100}$, is a decimal fraction.

An integer and a decimal together form a *mixed decimal*; for example, 2.25.

68. Decimal Point. The period written at the left of the tenths is called the *decimal point*.

For example, $0.7 = \frac{7}{10}$; $0.08 = \frac{8}{100}$, or $\frac{2}{25}$; $0.005 = \frac{5}{1000}$, or $\frac{1}{200}$.

It was not necessary to write a zero at the left of the decimal point in the above cases, for 0.5 means the same as .5. The zero is often written there to call attention more quickly to the decimal point.

69. Reading Decimals. We read a decimal precisely as if it were a whole number, and then give it the name of the lowest decimal place.

It is best to pronounce the word *and* at the decimal point only. Thus 100.023 is read "one hundred and twenty-three thousandths," while 0.123 is read "one hundred twenty-three thousandths."

EXERCISE 27

Read aloud the following:

- | | | | |
|----------|-----------|------------|--------------|
| 1. 0.2. | 9. 1.6. | 17. 6.125. | 25. 0.1225. |
| 2. 0.7. | 10. 2.9. | 18. 7.335. | 26. 0.3025. |
| 3. 0.9. | 11. 3.5. | 19. 0.475. | 27. 2.0025. |
| 4. 0.12. | 12. 4.75. | 20. 5.500. | 28. 0.0007. |
| 5. 0.65. | 13. 5.35. | 21. 5.005. | 29. 9.0009. |
| 6. 0.70. | 14. 6.40. | 22. 6.075. | 30. 200.025. |
| 7. 0.85. | 15. 7.08. | 23. 6.750. | 31. 0.225. |
| 8. 0.09. | 16. 7.10. | 24. 0.875. | 32. 220.005. |

Read aloud as dollars and decimals of a dollar:

- | | | | |
|-------------|--------------|--------------|---------------|
| 33. \$1.25. | 34. \$25.05. | 35. \$26.70. | 36. \$125.50. |
|-------------|--------------|--------------|---------------|

70. Writing Decimals. Although we write a decimal fraction without its denominator, the denominator is shown by the decimal point and is indicated in reading.

Thus 0.9 has no printed denominator, but when we say "nine tenths" we indicate the fact that the denominator is ten as well as state that the numerator is nine.

Therefore, *write only the numerator of the decimal, inserting such zeros as are necessary to indicate the denominator, and place the decimal point before the tenths.*

Thus, to write in figures two hundred one hundred-thousandths, we write 201, and prefix two zeros, preceded by a decimal point, so that the last figure is in hundred-thousandths' place. We then have 0.00201.

EXERCISE 28

Write in figures :

1. Seven tenths ; one tenth ; five tenths ; nine tenths.
2. Twelve hundredths ; fifty-five hundredths ; nine hundredths ; one hundredth ; twenty hundredths.
3. One hundred twenty-five thousandths ; forty-seven thousandths ; ten thousandths ; five thousandths.
4. One thousand two hundred fifty ten-thousandths ; seven hundred sixty-five ten-thousandths.
5. 5 and 63 thousandths ; 7 and 70 thousandths.
6. 1352 and 135 thousandths ; 60 and 60 thousandths.
7. 1000 and 1 thousandth ; 1000 and 10 thousandths.

Express in figures as dollars and cents :

8. Two and seventy-five hundredths dollars.
9. Seventy-one and two hundredths dollars.
10. One hundred and eight hundredths dollars.
11. One thousand three and three hundredths dollars.
12. Two hundred four and two hundredths dollars.

71. Reduction of Decimals. Since decimals are only a special form of fractions, we may reduce them as we reduce common fractions.

For example, since $\frac{5}{10} = \frac{50}{100}$ (§ 65), therefore $0.5 = 0.50$.

72. Annexing Zeros. Because, as just shown, $0.5 = 0.50$, therefore

Annexing zeros to a decimal does not change its value.

73. Similar Decimals. Decimals that have the same number of decimal places are called *similar decimals*.

Thus 0.75 and 0.25 are similar decimals, and so are 0.150 and 0.275; but 0.15 and 0.275 are dissimilar decimals.

74. Reduction of Dissimilar Decimals to Similar Decimals. Since we may give any decimals the same denominators (understood, not written) by annexing or cutting off zeros,

Therefore, to reduce dissimilar decimals to similar decimals, give them the same number of decimal places by annexing or cutting off zeros.

Thus 0.4, 0.72, and 0.125 may all be reduced to thousandths as follows: 0.400, 0.720, and 0.125. Similarly, 0.40 and 0.600 are equal to 0.4 and 0.6.

EXERCISE 29

Reduce to similar decimals:

- | | | |
|----------------|------------------|--------------------|
| 1. 0.2, 0.17. | 9. 0.12, 0.103. | 17. 0.001, 0.1100. |
| 2. 0.8, 0.08. | 10. 0.72, 0.008. | 18. 0.207, 0.2376. |
| 3. 0.7, 0.002. | 11. 0.69, 0.070. | 19. 0.412, 0.9320. |
| 4. 0.9, 0.130. | 12. 0.30, 0.700. | 20. 0.681, 0.6881. |
| 5. 0.6, 0.123. | 13. 0.99, 0.999. | 21. 0.900, 0.7000. |
| 6. 0.5, 0.700. | 14. 0.75, 0.630. | 22. 0.620, 0.5700. |
| 7. 0.4, 0.050. | 15. 0.50, 0.400. | 23. 0.090, 0.6550. |
| 8. 0.1, 0.357. | 16. 0.85, 0.392. | 24. 0.584, 0.5834. |

75. Reduction of Decimals to Common Fractions. We have $0.75 = \frac{75}{100}$, which may be reduced to $\frac{3}{4}$ (§ 65). In a similar way any decimal may be expressed as a common fraction.

Therefore, *to reduce a decimal to a common fraction, omit the decimal point, write the denominator of the decimal, and then reduce the common fraction to lowest terms.*

In a case like that of $0.33\frac{1}{3}$ we proceed as follows :

$$0.33\frac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{1}{3}.$$

EXERCISE 30

Reduce to common fractions in lowest terms or to mixed numbers :

1. 0.2.	11. 0.125.	21. 1.125.	31. 2.25.
2. 0.4.	12. 0.625.	22. 1.025.	32. 2.50.
3. 0.6.	13. 0.250.	23. 1.375.	33. 3.75.
4. 0.8.	14. 0.500.	24. 1.050.	34. 6.25.
5. 0.10.	15. 0.375.	25. 1.625.	35. 1.875.
6. 0.15.	16. 0.735.	26. 1.750.	36. 18.75.
7. 0.25.	17. 0.875.	27. 1.180.	37. 187.5.
8. $0.12\frac{1}{2}$.	18. $0.3\frac{1}{3}$.	28. $1.33\frac{1}{3}$.	38. $3.37\frac{1}{2}$.
9. $0.16\frac{2}{3}$.	19. $1.16\frac{2}{3}$.	29. $2.66\frac{2}{3}$.	39. 3.375.
10. $0.66\frac{2}{3}$.	20. $1.66\frac{2}{3}$.	30. $2.16\frac{2}{3}$.	40. $3.3\frac{3}{4}$.

Reduce to sixteenths :

41. $\frac{3}{4}$.	42. 0.75.	43. 0.25.	44. 0.500.
---------------------	-----------	-----------	------------

Express as mixed numbers :

45. 1.50.	47. 3.20.	49. 2.40.	51. 4.65.
46. 2.25.	48. 5.75.	50. 6.80.	52. 5.35.

76. Reduction of Common Fractions to Decimals. Since a common fraction indicates division, we may divide the numerator by the denominator and thus reduce the fraction to the decimal form. For example, consider the fraction $\frac{3}{4}$.

We have found that we may annex zeros after the decimal point without changing the value; that is, $3 = 3.00$. We may now divide exactly as in United States money, and the result is 0.75.

4 $\overline{)3.00}$ That this is allowable may be seen from the following:
 0.75 3.0 means 30 tenths, and 30 tenths $\div 4 = 7$ tenths with remainder 2 tenths. But 2 tenths = 20 hundredths ($\frac{2}{10} = \frac{20}{100}$), and 20 hundredths $\div 4 = 5$ hundredths. Therefore $\frac{3}{4} = 0.75$.

In the same way, $0.1\frac{1}{4} = 0.175$.

Therefore, *to reduce a common fraction to a decimal, place a decimal point after the numerator, and divide by the denominator as in United States money.*

A common fraction cannot always be reduced to an exact decimal. Thus $\frac{1}{3} = 0.3\frac{1}{3} = 0.33\frac{1}{3} = 0.333\frac{1}{3}$. It is customary to carry the division as far as necessary and then write a common fraction or the plus sign to indicate that the work is not complete; thus $\frac{1}{3} = 0.333\frac{1}{3}$, or $0.333 +$. Often this is written 0.333, using as many places as necessary.

No common fraction equals an exact decimal if the denominator, when the fraction is reduced to lowest terms, contains other prime factors than 2 and 5.

EXERCISE 31

Reduce to decimals, carrying the reduction only to thousandths in case of inexact decimals:

1. $\frac{1}{2}$.	7. $\frac{7}{8}$.	13. $\frac{3}{16}$.	19. $\frac{1}{3}$.	25. $1\frac{2}{5}$.
2. $\frac{1}{4}$.	8. $\frac{1}{5}$.	14. $\frac{5}{16}$.	20. $\frac{2}{3}$.	26. $3\frac{4}{5}$.
3. $\frac{1}{8}$.	9. $\frac{2}{5}$.	15. $\frac{7}{16}$.	21. $\frac{1}{6}$.	27. $0.1\frac{1}{3}$.
4. $\frac{3}{4}$.	10. $\frac{3}{5}$.	16. $1\frac{5}{16}$.	22. $\frac{5}{6}$.	28. $6.2\frac{7}{8}$.
5. $\frac{3}{8}$.	11. $\frac{4}{5}$.	17. $\frac{1}{32}$.	23. $\frac{3}{5}$.	29. $1.3\frac{1}{5}$.
6. $\frac{5}{8}$.	12. $1\frac{1}{6}$.	18. $\frac{3}{32}$.	24. $\frac{7}{5}$.	30. $6.1\frac{1}{4}$.

77. United States Money. The system of United States money is arranged on the decimal plan. Thus,

10 mills (m.) = 1 cent (ct. or ¢)

10 cents = 1 dime (d.)

10 dimes or 100 cents = 1 dollar (\$)

This table, like the common operations with United States money, is already familiar to the pupil, and is inserted only for reference.

The unit of United States money is the dollar, and the dimes and cents are written as decimal parts of the dollar.

Thus \$2.48 means 2 dollars and 48 hundredths of a dollar, or 2 dollars and 48 cents, the cent being one hundredth of a dollar. Similarly, \$2.08 means 2 dollars and 8 hundredths of a dollar, or 2 dollars and 8 cents.

78. Coins. The coins of the United States, as issued at present, are as follows:

<i>Gold</i>	<i>Silver</i>
Double eagle, \$20.00	Dollar, \$1.00
Eagle, 10.00	Half dollar, .50
Half eagle, 5.00	Quarter dollar, .25
Quarter eagle, 2.50	Dime, .10

<i>Nickel</i>	<i>Bronze</i>
5-cent piece (the "nickel")	1-cent piece

79. Paper Money. Bank bills, United States treasury notes, and gold and silver certificates are largely used in place of coins. These are issued in denominations of \$1, \$2, \$5, \$10, \$20, \$50, \$100, \$500, \$1000, \$5000, and \$10,000.

Formerly paper money of smaller denominations was issued, as well as the gold dollar, the silver 20-cent, 5-cent, and 3-cent pieces, the nickel 3-cent piece, the bronze 2-cent piece, and the copper half cent. These are no longer issued. The mill was never coined.

80. Reduction of Dollars to Cents. Required to reduce \$3.75 to cents.

Since \$1 = 100 cents, therefore \$3 = 3×100 cents, or 300 cents; then 300 cents + 75 cents = 375 cents.

Therefore, *to reduce dollars to cents, move the decimal point two places to the right.*

81. Reduction of Cents to Dollars. Required to reduce 1425 cents to dollars, or dollars and cents.

Since 1 cent = $\$ \frac{1}{100}$, therefore 1425 cents = $\$ \frac{1425}{100} = \$14.25$.

Therefore, *to reduce cents to dollars, or to dollars and cents, move the decimal point two places to the left.*

A whole number may always have a decimal point placed after it without changing its value, so if there is no decimal point given it may be placed at the right and then moved two places to the left.

EXERCISE 32

Reduce to cents :

- | | | | |
|-----------|-------------|--------------|---------------|
| 1. \$7. | 5. \$2.40. | 9. \$3.45. | 13. \$141.75. |
| 2. \$29. | 6. \$3.60. | 10. \$9.65. | 14. \$243.85. |
| 3. \$35. | 7. \$9.70. | 11. \$8.05. | 15. \$692.05. |
| 4. \$175. | 8. \$12.80. | 12. \$29.63. | 16. \$900.09. |

Reduce to dollars, or to dollars and cents :

- | | | | |
|-----------|------------|------------|----------|
| 17. 300¢. | 21. 732¢. | 25. 2642¢. | 29. 2¢. |
| 18. 450¢. | 22. 522½¢. | 26. 4900¢. | 30. 5¢. |
| 19. 575¢. | 23. 633½¢. | 27. 7275¢. | 31. 55¢. |
| 20. 606¢. | 24. 166⅔¢. | 28. 9654¢. | 32. 75¢. |
33. Reduce \$27.63 to mills.
 34. Reduce 5240 mills to dollars and cents.
 35. Reduce 2760 dimes to dollars; to cents.

82. Addition of Decimals. We add numbers containing decimals just as we add United States money.

For example, required the sum of 14.7, 3.62, 5.25, and 26.

Arranging the addends so that units of the same order are in the same column, which is easily done by placing the decimal points under one another, we add exactly as with integers, placing the decimal point in the sum directly under the decimal points in the addends.

$$\begin{array}{r} 14.7 \\ 3.62 \\ 5.25 \\ 26. \\ \hline 49.57 \end{array}$$

Therefore, *to add numbers containing decimals, write like orders under one another and add as with integers.*

If there are common fractions as well as decimals, these either may be reduced to decimals and added, or else may be added separately as is the more convenient.

EXERCISE 33

Add the following :

1.	2.	3.	4.	5.
22.05	19.26	3.	5.27 $\frac{1}{2}$	4.33 $\frac{1}{3}$
3.75	3.54	34.17	6.42 $\frac{1}{4}$	1.66 $\frac{2}{3}$
16.23	7.08	16.89	7.13 $\frac{1}{8}$	3.12 $\frac{1}{2}$
4.78	16.	17.32	5.16 $\frac{3}{4}$	5.37 $\frac{1}{2}$
9.6	32.89	4.08	4.29 $\frac{1}{4}$	4.25 $\frac{3}{4}$
<u>3.09</u>	<u>1.96</u>	<u>10.</u>	<u>2.53$\frac{3}{8}$</u>	<u>9.36$\frac{1}{4}$</u>
6.	7.	8.	9.	10.
23.41	14.02	38.92	42.17 $\frac{1}{2}$	33.33 $\frac{1}{3}$
36.98	148.36	64.87	68.13 $\frac{1}{2}$	16.66 $\frac{2}{3}$
124.75	49.79	49.81	7.14 $\frac{1}{4}$	66.66 $\frac{2}{3}$
4.92	31.64	324.06	18.21 $\frac{1}{4}$	9.95 $\frac{1}{3}$
6.07	5.50	4.27	16.82 $\frac{3}{4}$	3.37 $\frac{1}{2}$
13.53	20.00	82.46	7.16 $\frac{1}{2}$	16.52 $\frac{1}{2}$
<u>4.81</u>	<u>1.97</u>	<u>73.20</u>	<u>8.23</u>	<u>5.23$\frac{1}{5}$</u>

Add the following :

11.	12.	13.	14.	15.	16.
2.75	1.92	5.77	2.07	2.70	6.342
3.40	3.48	2.83	4.81	3.00	4.921
4.62	8.08	4.69	6.72	5.02	8.713
5.96	6.52	8.36	3.96	0.79	6.984
7.25	2.09	1.64	5.83	0.62	9.862
6.60	8.76	5.31	4.89	7.88	8.691
5.38	7.91	7.17	5.72	5.55	4.447
<u>4.04</u>	<u>1.24</u>	<u>4.23</u>	<u>6.08</u>	<u>9.99</u>	<u>1.206</u>

17.	18.	19.	20.	21.	22.
0.75	4.44	2.27	4.44	1.29	2.723
0.62	0.26	7.73	5.62	3.42	4.681
3.08	8.75	4.81	3.81	8.71	2.807
0.04	5.92	5.19	7.29	6.58	9.087
2.72	3.27	6.42	8.84	4.44	4.620
3.86	8.02	3.58	2.73	3.33	3.557
4.41	5.72	9.27	1.46	5.56	7.277
2.89	6.00	0.93	2.98	6.67	5.319
8.64	9.87	5.81	3.27	1.11	7.193
5.23	3.81	1.11	4.52	2.22	0.913
<u>8.37</u>	<u>4.04</u>	<u>3.08</u>	<u>2.00</u>	<u>6.67</u>	<u>1.823</u>

23. $1.2 + 3.9 + 6.8 + 5.7 + 0.7 + 6.75 + 9.81.$

24. $4.82 + 5.18 + 6.97 + 3.03 + 4.44 + 5.56.$

25. $1.04 + 3.08 + 9.09 + 6.67 + 5.70 + 9.87.$

26. $4.035 + 2.817 + 3.628 + 4.926 + 5.2.$

27. $1.009 + 0.007 + 0.809 + 0.209 + 0.007.$

28. $12.006 + 3.07 + 9.001 + 4.7 + 9.00 + 40 + 3.25 + 2.06.$

29. $54.476 + 28.927 + 39.824 + 91.643 + 27.123 + 92.877.$

30. $127.635 + 298.436 + 481.723 + 4.590 + 3.2706 + 2.8.$

31. $96.45 + 88.34\frac{3}{4} + 2.91\frac{1}{8} + 6.33\frac{1}{8}$.

32. $72.63 + 19.81\frac{1}{3} + 4.66\frac{2}{3} + 2.87\frac{2}{3}$.

33. $24.75 + 16.37\frac{1}{2} + 6.33\frac{1}{3} + 5.44\frac{1}{8}$.

34. $75\frac{1}{8} + 25.16\frac{2}{3} + 17\frac{2}{3} + 250 + 0.33\frac{1}{3}$.

35. $42\frac{1}{2} + 67.80 + 19.75 + 135\frac{1}{4} + 0.08 + 100$.

36. $33\frac{1}{3} + 25\frac{1}{2} + 17.66\frac{2}{3} + 128.50 + 0.16\frac{2}{3} + 0.83\frac{1}{3}$.

37. $82.75 + 14\frac{1}{4} + 19\frac{1}{2} + 16.50 + 67 + 0.66\frac{2}{3} + 0.33\frac{1}{3}$.

38. The distance from Canajoharie to Canastota is 67.3 miles, and from Canastota to Syracuse 20.8 miles. How far is it from Canajoharie to Syracuse by way of Canastota?

39. The distance from Syracuse to Lyons is 45.8 miles, and from Lyons to Buffalo 102.3 miles. How far is it from Syracuse to Buffalo by way of Lyons?

40. Taking the distances as given in Exs. 38 and 39, find how far it is from Canajoharie to Buffalo by this route.

41. The distance from Buffalo to Albany is 296.64 miles, and from Albany to Boston 202 miles. How far is it from Buffalo to Boston by way of Albany?

42. The distance from Weehawken to Newburgh is 56.7 miles, from Newburgh to Kingston 31.4 miles, and from Kingston to Albany 53.2 miles. How far is it from Weehawken to Albany by this route? from Newburgh to Albany? from Weehawken to Kingston?

43. If a man has 168.41 acres of land and buys $22\frac{1}{2}$ acres more of one man, and $16\frac{2}{3}$ acres of another, how many acres does he then have?

44. A man has a deed to a farm that states the area to be $198\frac{3}{4}$ acres. He has the five fields in the farm measured and finds that their areas are 21.25 acres, 33.4 acres, 82.6 acres, 58 acres, and $3\frac{1}{2}$ acres. Is the area stated in the deed correct?

83. Subtraction of Decimals. We subtract numbers containing decimals just as we subtract United States money.

Required the difference between 26.32 and 17.754.

26.320 We first reduce the numbers to similar decimals by
17.754 annexing zeros as may be necessary. Then arranging the
8.566 numbers so that units of the same order are in the same
column, which is easily done by placing the decimal points
under one another, we subtract exactly as with integers.

Therefore, *to subtract one number from another when either contains or both contain decimals, reduce to similar decimals, write like orders under one another, and subtract as with integers.*

Any common fractions may be subtracted separately if they cannot easily be reduced to decimals. Thus $4.6\frac{2}{3} - 1.3\frac{1}{3} = 3.3\frac{1}{3}$.

EXERCISE 34

Subtract :

1.	5.	9.	13.	17.
26.75	71.81	27.6	36.91	29.433
<u>3.96</u>	<u>4.68</u>	<u>13.01</u>	<u>17.935</u>	<u>17.5</u>
2.	6.	10.	14.	18.
43.15	24.8	61.3	82.	48.224
<u>7.23</u>	<u>4.32</u>	<u>27.25</u>	<u>16.273</u>	<u>9.0005</u>
3.	7.	11.	15.	19.
62.81	34.9	34.75	48.007	72.2
<u>9.75</u>	<u>6.43</u>	<u>16.08</u>	<u>23.25</u>	<u>65.025</u>
4.	8.	12.	16.	20.
63.42	48.7	42.27	83.422	0.72834
<u>4.73</u>	<u>4.08</u>	<u>17.752</u>	<u>16.307</u>	<u>0.1976</u>
21. 7 — 0.2698.	23. 6 — 1.0273.	25. 8.4 — 1.66 $\frac{2}{3}$.		
22. 3 — 0.2709.	24. 6.2 — 1.33 $\frac{1}{3}$.	26. 9.5 — 4.12 $\frac{1}{2}$.		

27. A man having a farm of 175.25 acres sells a lot of $12\frac{1}{2}$ acres. How much has he left?

28. The English sovereign is worth \$4.8665. How much does this lack of \$5?

29. If $629.33\frac{1}{3}$ barrels of water are in a tank that holds 948.25 barrels, how much does it lack of being full?

30. The United States \$5 gold piece contains 116.1 grains of pure gold, and the English sovereign 113.0003 grains. Find the difference in the weight of the pure gold.

31. A man has three adjacent lots with a frontage of 148.5 feet. One of the lots has a frontage of 52.75 feet and another of 49.5 feet. What is the frontage of the third lot?

32. A man who had a farm of 215.8 acres bought an adjacent farm of $192\frac{3}{4}$ acres. He sold a field of $72\frac{1}{3}$ acres and another of 68.25 acres. How much land had he left?

33. The perimeter of a triangle is 27.25 feet, and two of the sides are $8.33\frac{1}{3}$ feet and $9.75\frac{2}{3}$ feet. What is the length of the third side?

34. The distance from Niagara Falls to Buffalo is 22 miles, and from Niagara Falls to North Tonawanda 10.9 miles. How far is it from Buffalo to North Tonawanda? In this and the following examples the cities are on the same line.

35. The distance from Chicago to Lincoln is 561.4 miles, and from Chicago to Colorado Springs 1072.2 miles. How far is it from Lincoln to Colorado Springs?

36. The distance from Salt Lake City to Riverside, California, is 723.7 miles, and to Pasadena by way of Riverside 791.2 miles. How far is it from Riverside to Pasadena?

37. The distance from Louisville to Greencastle is 146.5 miles, and from Louisville to Chicago 324.3 miles. How far is it from Greencastle to Chicago?

84. Multiplication of a Decimal by an Integer. In multiplying a decimal by an integer we proceed exactly as in multiplying United States money.

For example, required to multiply 2.375 by 7.

$\begin{array}{r} 2.375 \\ 7 \\ \hline 16.625 \end{array}$	Since 5 thousandths multiplied by 7 is 35 thousandths, we know that the 5 will go in the thousandths' place. Proceeding with the multiplication in the ordinary way, we finally place the decimal point before the 625 so as to have the 5 come in the thousandths' place as above stated.
--	---

85. Multiplication of a Decimal by a Mixed Number. Multiplication of a decimal by a mixed number may be illustrated by the case of $3\frac{1}{2} \times 2.44$.

First taking $\frac{1}{2}$ of 2.44, we know that $\frac{1}{2}$ of $\frac{244}{100} = \frac{122}{100}$, or 1.22. Then 3×4 hundredths = 12 hundredths, so we write the 2 in the hundredths' place. Proceeding in the ordinary way, the sum of the partial products is 8.54.

$$\begin{array}{r} 2.44 \\ 3\frac{1}{2} \\ \hline 1.22 \\ 7.32 \\ \hline 8.54 \end{array}$$

EXERCISE 35

Multiply :

- | | | |
|--------------------------------|-----------------------------------|-------------------------------------|
| 1. 2×6.7 . | 14. $3 \times 3.33\frac{1}{3}$. | 27. 25×0.125 . |
| 2. 4×9.8 . | 15. $6 \times 1.66\frac{2}{3}$. | 28. $24 \times 0.166\frac{2}{3}$. |
| 3. 7×0.9 . | 16. $6 \times 6.66\frac{2}{3}$. | 29. $63 \times 3.333\frac{1}{3}$. |
| 4. 8×0.6 . | 17. 8×4.125 . | 30. 70×1.4314 . |
| 5. 5×3.25 . | 18. 9×3.627 . | 31. 143×7.007 . |
| 6. 7×2.75 . | 19. 8×5.328 . | 32. 125×1.022 . |
| 7. $6 \times 3.3\frac{1}{3}$. | 20. 7×6.132 . | 33. 225×3.123 . |
| 8. $9 \times 6.6\frac{2}{3}$. | 21. 8×2.312 . | 34. $16\frac{1}{2} \times 3.025$. |
| 9. 8×2.125 . | 22. $9 \times 0.033\frac{1}{3}$. | 35. $35\frac{1}{4} \times 2.376$. |
| 10. 4×6.375 . | 23. $7 \times 0.277\frac{1}{7}$. | 36. $64\frac{1}{3} \times 2.032$. |
| 11. 7×5.025 . | 24. $5 \times 0.044\frac{2}{5}$. | 37. $48\frac{3}{4} \times 2.372$. |
| 12. 8×7.009 . | 25. $9 \times 6.666\frac{2}{3}$. | 38. $75\frac{1}{2} \times 4.2162$. |
| 13. 5×6.285 . | 26. $7 \times 1.425\frac{5}{7}$. | 39. $12\frac{1}{2} \times 8.8888$. |

40. What is the perimeter of a triangle if each side is 19.25 inches? if each side is $16.33\frac{1}{3}$ inches?

41. What is the perimeter of a square if each side is $6.12\frac{1}{2}$ inches? if each side is $5.31\frac{1}{3}$ inches?

42. At $\$0.33\frac{1}{3}$ a yard, how much will 15 yards of cloth cost? How much will 27 yards cost?

43. If a French meter is 39.37 inches, how many inches in 12 meters? in 25 meters? in 37 meters?

44. If a French kilogram is 2.204 lb., how many pounds in 75 kilograms? in 127 kilograms?

45. If a French franc is worth $\$0.193$, what are 125 francs worth in our money?

46. If a square foot is 0.0929 of a square meter, how many square meters in 35 square feet?

47. If an English sovereign is equivalent to $\$4.8665$, what is the value in our money of 27 sovereigns?

48. At the rate of 49.74 miles an hour, how far will an express train travel in $6\frac{2}{3}$ hours?

49. At the rate of $52.66\frac{2}{3}$ miles an hour, how far will an express train travel in 3 hours?

50. If the thickness of a book is 1.15 inches, how many inches will 2 dozen such books occupy on a dealer's shelf?

51. If two sides of a rectangular field are respectively 96.8 feet and 137.6 feet, what is the perimeter of the field?

52. If a man cuts a piece of land into 17 building lots, each having an area of 0.72 of an acre, what is the area of the whole property?

53. It was estimated recently that the average cost of the actual necessities of life for each person in this country was $\$108.955$ a year. At this rate what would be the cost for 75 persons?

86. Multiplication of a Decimal by Multiples of 10. In multiplying 3.1416 by 300 we may proceed as in § 84, but there is a shorter process that may be used when the multiplier is any multiple of 10.

Since moving the decimal point one place to the right changes tenths into units, hundredths into tenths, and so on, it multiplies the number

by 10. Similarly, moving the decimal point two places to the right multiplies by 100, and so on.

Hence, to multiply by 300 we need only to multiply by 3 and move the decimal point two places to the right.

When necessary, zeros must be annexed to the product, as in the case of 300×3141.6 .

EXERCISE 36

Multiply:

- | | | |
|------------------------|--------------------------|-------------------------------------|
| 1. 200×3.42 . | 6. 700×4.062 . | 11. 400×0.1250 . |
| 2. 300×6.28 . | 7. 800×0.317 . | 12. $600 \times 0.166\frac{2}{3}$. |
| 3. 500×9.36 . | 8. 600×5.214 . | 13. $800 \times 0.012\frac{1}{2}$. |
| 4. 200×2.48 . | 9. 600×52.14 . | 14. 700×7.9775 . |
| 5. 200×24.8 . | 10. 600×521.4 . | 15. 8000×674.3 . |

16. A meter being 39.37 inches, how many inches are there in 4000 meters?

17. At \$37.75 a front foot, what will a piece of land of 300 feet front cost?

18. If a cubic foot of water weighs 62.5 pounds, what will 2500 cubic feet weigh?

19. If the thickness of each board is 0.875 of an inch, what is the height of a pile of 80 boards?

20. The circumference of a circle being 3.1416 times the diameter, what is the circumference of a circular water tank 40 feet in diameter?

87. Multiplication by a Decimal. Let us first consider the case of 0.5 of 0.7, or 0.5×0.7 .

0.7 We learned in primary arithmetic that $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$, the numerators being multiplied together for a new numerator, and the denominators for a new denominator.

0.5
0.35 In the same way, writing the given decimals as common fractions, we have

$$\frac{5}{10} \times \frac{7}{10} = \frac{35}{100}.$$

Therefore $0.5 \times 0.7 = 0.35$.

This we see without using common fractions, for $5 \times 7 = 35$, and a tenth of a tenth is a hundredth, so that we have 35 hundredths.

3.24 Next consider the case of 0.21×3.24 .
0.21 We multiply in the ordinary way, and notice that 1 (hundredth) $\times$ 4 (hundredths) = 4 (ten-thousandths), and that the
324 4 of the product must therefore go in the ten-thousandths'
648 place.
0.6804

Therefore, *to multiply by a decimal, multiply as if by an integer, and point off as many decimal places in the product as there are decimal places in both factors together.*

Zeros should be prefixed as necessary to give the required number of places; that is, $0.07 \times 0.069 = 0.00483$.

EXERCISE 37

Multiply:

- | | | |
|-----------------------|--------------------------|------------------------------------|
| 1. 0.7×0.7 . | 10. 0.6×0.31 . | 19. 0.7×0.127 . |
| 2. 0.9×0.9 . | 11. 0.8×0.72 . | 20. 0.8×0.125 . |
| 3. 0.8×1.7 . | 12. 0.3×0.88 . | 21. $0.3 \times 0.33\frac{1}{3}$. |
| 4. 0.6×3.9 . | 13. 0.6×1.29 . | 22. $0.6 \times 0.66\frac{2}{3}$. |
| 5. 0.5×7.5 . | 14. 0.4×3.21 . | 23. 0.9×2.735 . |
| 6. 0.8×6.7 . | 15. 3.17×4.22 . | 24. 4.35×5.006 . |
| 7. 2.3×4.1 . | 16. 7.39×5.81 . | 25. 8.25×9.901 . |
| 8. 3.2×5.7 . | 17. 9.75×6.92 . | 26. 7.37×0.073 . |
| 9. 8.7×9.6 . | 18. 3.82×9.89 . | 27. 8.24×0.0009 . |

28. A cubic foot of water weighs 62.5 pounds. Ice is 0.92 as heavy as water. What does a cubic foot of ice weigh?

29. Taking the weight of water as in Ex. 28, and knowing that cork is 0.24 as heavy as water, find the weight of a cubic foot of cork.

30. Taking the weight of water as in Ex. 28, and knowing that a certain kind of granite is 2.78 times as heavy as water, find the weight of a cubic foot of the granite.

31. Taking the weight of water as in Ex. 28, and knowing that copper is 8.95 times as heavy as water, what is the weight of a cubic foot of copper?

32. The circumference of a circle is 3.1416 times the diameter. What is the circumference of a wheel whose diameter is 42 inches?

33. From the statement made in Ex. 32, find the circumference of the earth, assuming its diameter to be 8000 miles.

34. From the statement made in Ex. 32, find the circumference of the sun, assuming its diameter to be 866,500 miles.

35. In a year when the average yield of oats in the United States was 31.2 bushels per acre, and the average price on the farm was 31.7 cents per bushel, what was the average value on the farm of the oat crop per acre?

36. In a year when the average yield of corn in the United States was 30.3 bushels per acre, and the average price on the farm was 39.9 cents per bushel, what was the average value on the farm of the corn crop per acre?

37. In a year when the average yield of wheat in the United States was 15.5 bushels per acre, and the average price on the farm was 66.7 cents per bushel, what was the average value on the farm of the wheat crop per acre?

EXERCISE 38

DRILL WORK IN MULTIPLICATION

- | | | |
|-----------------------------------|--------------------------|--------------------------------------|
| 1. 2×0.73 . | 27. 20.6×1.27 . | 53. 5.31×3.15 . |
| 2. 4×1.25 . | 28. 0.75×0.84 . | 54. 2.97×3.51 . |
| 3. 5×7.20 . | 29. 7.5×0.84 . | 55. 5×1.278 . |
| 4. 7×3.42 . | 30. 7.5×8.40 . | 56. 0.5×1.278 . |
| 5. $9 \times 3.3\frac{1}{3}$. | 31. 1.25×240 . | 57. 0.005×0.007 . |
| 6. 12×8.75 . | 32. 12.5×2.4 . | 58. 0.003×0.123 . |
| 7. 16×9.25 . | 33. 0.125×24 . | 59. 4.72×1.0001 . |
| 8. 25×8.80 . | 34. 123×23.1 . | 60. 5.27×2.1111 . |
| 9. $50 \times 7.12\frac{1}{2}$. | 35. 12.3×2.31 . | 61. 65.5×48.88 . |
| 10. 75×6.38 . | 36. 4.06×4.06 . | 62. 6.55×4.888 . |
| 11. 86×0.07 . | 37. 40.6×40.6 . | 63. 275×5600 . |
| 12. 88×12.5 . | 38. 2.73×4.80 . | 64. 27.5×5600 . |
| 13. $96 \times 33.3\frac{1}{3}$. | 39. 27.3×0.48 . | 65. 2.75×5.600 . |
| 14. 0.3×48.75 . | 40. 4.51×2.36 . | 66. 5.43×1.0271 . |
| 15. 0.7×6.281 . | 41. 57.2×0.96 . | 67. 6.28×2.0002 . |
| 16. 0.9×351.1 . | 42. 43.4×0.27 . | 68. 1.01×6.0005 . |
| 17. 1.2×4.211 . | 43. 2.73×0.26 . | 69. 3.04×40.002 . |
| 18. 2.1×3.922 . | 44. 0.98×0.98 . | 70. 5.11×2.0101 . |
| 19. 75×44.88 . | 45. 5.27×63.5 . | 71. 7.75×6.4480 . |
| 20. 7.5×448.8 . | 46. 4.88×0.75 . | 72. 5.5×3.1416 . |
| 21. 32×2.111 . | 47. 2.36×0.83 . | 73. 0.55×4.7238 . |
| 22. 3.2×21.11 . | 48. 0.98×9.80 . | 74. 2.75×4.8888 . |
| 23. 3.2×2.111 . | 49. 17.9×62.8 . | 75. $5.33\frac{1}{3} \times 6.666$. |
| 24. 4.5×8.275 . | 50. 2.43×2.43 . | 76. $2.66\frac{2}{3} \times 3.333$. |
| 25. 0.45×2.45 . | 51. 6.39×1.62 . | 77. $8.12\frac{1}{2} \times 4.888$. |
| 26. 0.07×0.095 . | 52. 89.1×6.21 . | 78. $1.16\frac{2}{3} \times 36.36$. |

88. Division of a Decimal by an Integer. In dividing a decimal by an integer we proceed exactly as in dividing United States money.

For example, required to divide 1.561 by 7.

Here $1 \div 7$ gives no units in the quotient, so we write 0 in the units' place; $15 \text{ tenths} \div 7 = 2 \text{ tenths}$, and we write 2 in the tenths' place, considering the remainder (1 tenth) as 10 hundredths. Then $16 \text{ hundredths} \div 7 = 2 \text{ hundredths}$, and we write this 2 in the hundredths' place, considering the remainder as 20 thousandths. Then $21 \text{ thousandths} \div 7 = 3 \text{ thousandths}$. Hence the quotient is 0.223.

Zeros may be annexed as needed, to the right of the decimal point, to enable the division to proceed. Thus $6.1 \div 5 = 6.10 \div 5 = 1.22$.

Therefore, *to divide a decimal by an integer divide as with integers, writing the decimal point in the quotient as soon as the decimal point in the dividend is reached.*

EXERCISE 39

Divide :

$$1. 4.52 \div 2. \quad 6. 7.3425 \div 25. \quad 11. 32.4714 \div 46.$$

$$2. 5.58 \div 9. \quad 7. 81.675 \div 75. \quad 12. 8838.80 \div 19.$$

$$3. 3.15 \div 7. \quad 8. 92.100 \div 25. \quad 13. 2.94996 \div 39.$$

$$4. 31.5 \div 7. \quad 9. 211.428 \div 36. \quad 14. 9.65496 \div 98.$$

$$5. 0.315 \div 7. \quad 10. 215.016 \div 34. \quad 15. 0.40715 \div 85.$$

16. If 12 dozen eggs cost \$5.16, what does 1 dozen cost?

17. A train goes from New York to Philadelphia, 90.3 miles, in 2 hours. What is the average rate per hour?

18. A train leaves Poughkeepsie at 9 A.M. and reaches Hopewell Junction, 13.2 miles distant, at 9:29 A.M. What is the average rate per hour?

19. There being 231 cubic inches in a gallon, how many gallons in a tank containing 14,437.5 cubic inches?

89. Division by a Decimal. Since we may multiply both divisor and dividend by the same number without affecting the quotient (§ 65), we may, if the divisor contains decimal places, multiply both divisor and dividend by such a power of 10 as shall make the divisor an integer.

For example, the quotient of $1.25 \div 0.5$ is the same as that of 12.5 by 5 , and this we may find as in § 88.

Therefore, *to divide by a divisor containing decimal places, multiply both divisor and dividend by such a power of 10 as shall make the divisor an integer. Then divide as with integers, writing the decimal point in the quotient as soon as the decimal point in the dividend is reached.*

$$\begin{array}{r} 14.4 \\ 12 \overline{) 172.8} \\ \underline{12} \\ 52 \\ \underline{48} \\ 48 \\ \underline{48} \\ 0 \end{array}$$

Thus, in dividing 17.28 by 1.2 , we multiply both divisor and dividend by 10 , and then have $172.8 \div 12$. The divisor is now an integer and we proceed as shown, finding the quotient to be 14.4 . It should be noted that the decimal point in the quotient is placed exactly above that in the dividend. This arrangement of the work is sometimes called "the Austrian method" of division.

90. Division with a Remainder. If the divisor is not contained in the dividend without a remainder, zeros may be annexed to the dividend and the division continued.

Required to divide 5.273 by 0.24 to four decimal places.

$$\begin{array}{r} 21.9708 \\ 24 \overline{) 527.3000} \\ \underline{48} \\ 47 \\ \underline{24} \\ 233 \\ \underline{216} \\ 170 \\ \underline{168} \\ 200 \\ \underline{192} \\ 8 \end{array}$$

Multiplying both divisor and dividend by 100 , we have $527.3 \div 24$. Dividing, the quotient is 21.9708 with remainder 8 , and $\frac{8}{24} = \frac{1}{3}$. Therefore the quotient is $21.9708\frac{1}{3}$, or simply 21.9708 to four decimal places.

If the next figure of the quotient had been 5 or more, we should call the quotient 21.9709 to four decimal places, this being nearer the correct result than 21.9708 . But since the fraction is only $\frac{1}{3}$, and we are required to go only to four decimal places, we drop the fraction and take 21.9708 as the result.

91. Division by Multiples of 10. If the divisor is an integer and ends in one zero or more than one zero, we may shorten the work by dividing both divisor and dividend by such a power of 10 as shall cut off the zeros.

For example, $42.08 \div 8000$ gives the same quotient as $0.04208 \div 8$. That is, we have divided both numbers by 1000 by cutting off three zeros from the divisor and moving the decimal point three places to the left in the dividend.

EXERCISE 40

Divide :

- | | | |
|-----------------------|-------------------------|--------------------------|
| 1. $1.12 \div 0.4$. | 11. $2.72 \div 0.02$. | 21. $36.08 \div 3.28$. |
| 2. $6.25 \div 0.5$. | 12. $5.34 \div 0.03$. | 22. $1.036 \div 0.56$. |
| 3. $5.04 \div 0.7$. | 13. $3.64 \div 0.04$. | 23. $62.32 \div 0.82$. |
| 4. $5.84 \div 0.8$. | 14. $7.15 \div 0.05$. | 24. $115.6 \div 0.34$. |
| 5. $7.02 \div 0.9$. | 15. $27.6 \div 0.06$. | 25. $3.509 \div 12.1$. |
| 6. $7.26 \div 0.6$. | 16. $36.4 \div 0.07$. | 26. $1.9044 \div 9.2$. |
| 7. $8.44 \div 200$. | 17. $0.425 \div 2.5$. | 27. $1.0989 \div 3.3$. |
| 8. $96.4 \div 400$. | 18. $0.288 \div 1.2$. | 28. $0.8118 \div 1.23$. |
| 9. $70.2 \div 300$. | 19. $0.057 \div 1.9$. | 29. $19.536 \div 44.4$. |
| 10. $0.85 \div 500$. | 20. $8.12 \div 0.029$. | 30. $17.139 \div 59.1$. |

31. A man pays \$113 for coal at \$5.65 a ton. How many tons does he buy ?

32. The product of two numbers is 156.25 and one of the numbers is 5000. What is the other number ?

33. The product of two numbers is 6.51021 and one of the numbers is 3.207. What is the other number ?

34. If wind blows in a hurricane at the rate of 8796.6 feet per minute, what is the rate per second ?

35. A knot, used at sea, is 1.152 miles. When a steamship is traveling 23.04 miles an hour, what is its rate in knots ?

36. A contractor bought a shipment of bricks for \$176.40, paying \$7.20 a thousand. How many bricks did he buy?

37. A dealer paid \$54.40 freight on some oil at \$.068 per thousand pounds. How many pounds were there?

38. A contractor bought a lot of shingles for \$717.75, paying \$4.12 $\frac{1}{2}$ a thousand. How many thousand did he buy?

39. Wheat that cost 98 cents a bushel is sold for \$1.04 a bushel. How much must be sold to realize a profit of \$132?

40. How many times will a steam roller 4.25 feet in circumference turn in rolling the length of a lawn 136 feet long?

41. How many lengths of fence, each 12.75 feet long, will be required to inclose a square field that is 522.75 feet on a side?

42. A dealer receives a shipment of stoves, the total weight being 9.75 thousand pounds. He pays \$70.20 freight. What is the freight per thousand pounds?

43. A wheel is 28 inches in diameter, and the circumference is 3.1416 times as much. Find the number of times the wheel turns in going a mile, 63,360 inches.

44. A cubic foot of a certain kind of granite weighs 173.75 pounds, and a cubic foot of water weighs 62.5 pounds. The granite is how many times as heavy as water?

45. Cedar is 0.61 as heavy as water, and oak is 1.17 times as heavy as water. Oak is how many times as heavy as cedar? Answer to the nearest hundredth.

46. Find to the nearest hundredth the weight of a cubic foot of air, water being 830 times as heavy as air, and a cubic foot of water weighing 1000 ounces.

47. A gallon contains 231 cubic inches. Alcohol is 0.84 as heavy as water. Find the number of cubic inches of alcohol that will weigh as much as a gallon of water.

EXERCISE 41

DRILL WORK IN DIVISION

- | | | |
|-----------------------|------------------------|---------------------------|
| 1. $62.5 \div 25.$ | 27. $1.156 \div 3.4.$ | 53. $7.375 \div 2.5.$ |
| 2. $62.5 \div 2.5.$ | 28. $7.14 \div 4.2.$ | 54. $0.279 \div 0.09.$ |
| 3. $6.25 \div 2.5.$ | 29. $71.4 \div 4.2.$ | 55. $12,325 \div 25.$ |
| 4. $62.9 \div 37.$ | 30. $7.14 \div 0.42.$ | 56. $1232.5 \div 2.5.$ |
| 5. $6.29 \div 37.$ | 31. $62.32 \div 7.6.$ | 57. $1.2325 \div 2.5.$ |
| 6. $62.9 \div 3.7.$ | 32. $88.44 \div 7.5.$ | 58. $62.75 \div 125.$ |
| 7. $6.29 \div 3.7.$ | 33. $8190 \div 0.9.$ | 59. $6.275 \div 1.25.$ |
| 8. $80.5 \div 35.$ | 34. $61.20 \div 1.2.$ | 60. $36.08 \div 32.8.$ |
| 9. $80.5 \div 3.5.$ | 35. $7.200 \div 1.2.$ | 61. $3.608 \div 0.328.$ |
| 10. $8.05 \div 3.5.$ | 36. $78.4 \div 0.08.$ | 62. $1.036 \div 0.560.$ |
| 11. $46.2 \div 22.$ | 37. $3.12 \div 0.04.$ | 63. $882.32 \div 0.82.$ |
| 12. $46.2 \div 2.2.$ | 38. $7.81 \div 1.1.$ | 64. $4.4116 \div 0.41.$ |
| 13. $4.62 \div 2.2.$ | 39. $8.91 \div 0.11.$ | 65. $11,300 \div 5.65.$ |
| 14. $600 \div 0.6.$ | 40. $4.55 \div 0.07.$ | 66. $6.51021 \div 3.207.$ |
| 15. $600 \div 0.06.$ | 41. $2.73 \div 0.03.$ | 67. $651.021 \div 320.7.$ |
| 16. $6 \div 0.0006.$ | 42. $8.96 \div 12.8.$ | 68. $176.4 \div 0.0072.$ |
| 17. $627 \div 0.3.$ | 43. $4.41 \div 0.21.$ | 69. $5.44 \div 0.0068.$ |
| 18. $6.27 \div 0.3.$ | 44. $17.28 \div 14.4.$ | 70. $717.75 \div 4.125.$ |
| 19. $62.7 \div 0.03.$ | 45. $6.25 \div 0.25.$ | 71. $4.41 \div 0.002.$ |
| 20. $305 \div 6.1.$ | 46. $6.25 \div 2.50.$ | 72. $4.752 \div 2.2.$ |
| 21. $3.05 \div 6.1.$ | 47. $2.002 \div 0.22.$ | 73. $4.752 \div 0.022.$ |
| 22. $62.32 \div 8.2.$ | 48. $3.003 \div 7.7.$ | 74. $32.4714 \div 4.6.$ |
| 23. $623.2 \div 8.2.$ | 49. $2.65 \div 0.53.$ | 75. $883.88 \div 0.19.$ |
| 24. $6.232 \div 8.2.$ | 50. $60.06 \div 0.33.$ | 76. $32.4714 \div 2.3.$ |
| 25. $1156 \div 3.4.$ | 51. $72.5 \div 0.25.$ | 77. $324.714 \div 9.2.$ |
| 26. $11.56 \div 3.4.$ | 52. $71.25 \div 2.5.$ | 78. $27.75 \div 0.25.$ |

EXERCISE 42

MISCELLANEOUS PROBLEMS

1. If a family uses 3 barrels of flour a year, how much more will it spend by purchasing by the bag (of $\frac{1}{8}$ of a barrel) at 65¢, than by the barrel at \$5?

2. When butter sells at 35¢ a pound, and at \$2.80 per box of 10 pounds, how much will be saved on 10 pounds by purchasing by the box?

3. If a family uses 4 pounds of butter a week costing 35¢ per pound, how much would it save in 50 weeks by purchasing butter by the 10-pound box at \$2.80 a box?

4. If a sponge cake is made in March, when eggs cost 30¢ a dozen, what will be the cost of the 4 eggs needed for the cake?

5. If the same kind of cake is made in December, when eggs are 66¢ a dozen, what will the eggs cost?

6. If flour costs \$6.70 a barrel (196 pounds), and in smaller quantities 4¢ a pound, how much is saved in purchasing by the barrel, by a family that buys 196 pounds?

7. If a family's ice bill is \$1.50 a month, and ice costs 30¢ per 100 pounds, how many pounds does the family use? If by having a better ice box it uses only 0.9 as much ice, how many pounds does it use?

8. In July a family used 1500 pounds of ice, paying 30¢ per 100 pounds, and would have used only 0.8 as much by taking care of the ice box. How much could it have saved?

9. A family with an income of \$2000 spent \$460 on food. What decimal of the income was spent on food?

10. A family with an income of \$800 spent 0.45 of it on food. How much did it spend on food?

11. With an income of \$1100 a family spent \$275 for food. This was what decimal fraction of the income?

12. With an income of \$900 a family spent \$252 for food. This was what decimal fraction of the income?

13. A man whose income is \$2800 a year spends 0.26 of it for rent. How much does he spend for rent?

14. A man whose income is \$2800 a year spends \$728 for rent. This is what decimal fraction of the income?

15. A man spends 0.23 of his income for food, 0.26 of it for rent, and 0.1 of it for clothing. If his income is \$2400 a year, how much does he spend for each? for all three?

16. A man earns \$200 a month and spends 0.22 of it for rent. How much does he pay for rent in a year?

17. A grocer bought 5 barrels of sugar of respectively 313 pounds, 311 pounds, 307 pounds, 316 pounds, and 317 pounds, at $5\frac{5}{16}$ ¢ a pound. How much did the sugar cost?

18. The sides of a field are respectively 141.25 feet, 136.75 feet, 158.6 feet, and 149.8 feet. How many feet are there in the perimeter of the field?

19. A man owned 0.60 of a certain manufacturing plant, and sold $0.33\frac{1}{3}$ of his share for \$3000. At this rate, what was the whole plant worth?

20. Cedar is 0.61 as heavy as water, and marble is 2.7 times as heavy as water. Marble is how many times as heavy as cedar? Answer to the nearest hundredth.

21. White oak is 0.83 as heavy as water, and granite is 2.72 times as heavy as water. Granite is how many times as heavy as oak? Answer to the nearest hundredth.

22. A gallon contains 231 cubic inches. Gasoline is 0.64 as heavy as water. Find the number of cubic inches of gasoline that will weigh as much as a gallon of water.

23. A man bought 6250 (6.25 thousand) tiles at \$7.20 a thousand. What was the total cost?

24. A man bought 6500 tiles at \$7.50 a thousand. What was the total cost?

25. A man bought 13,750 shingles for \$49.50. What did they cost per thousand?

26. A man bought 20,500 shingles for \$73.80. What did they cost per thousand?

27. A dealer paid \$0.74 per hundred pounds for a shipment of 4800 pounds of salt. How much did he pay in all?

28. A dealer paid \$0.79 per hundred pounds on a shipment of 5900 pounds of salt. How much did he pay in all?

29. What will 320 barrels of apples cost at \$2.12 $\frac{1}{2}$ a barrel? at \$2.33 $\frac{1}{3}$ a barrel?

30. What will 640 pounds of sugar cost at \$0.04 $\frac{1}{8}$ a pound? at \$0.04 $\frac{5}{8}$ a pound?

31. A merchant bought 320 yards of silk for \$790.40, and sold it at \$2.85 a yard. How much did he gain?

32. The sides of a field are respectively 374.75 feet, 369.5 feet, 348.8 feet, and 353.25 feet. What is the perimeter of the field?

33. If the sides of a triangle are 148.3 feet, 134.75 feet, and 179.6 feet respectively, how much less is the sum of the first two sides than the sum of the second two?

34. If the sides of a field are 271.3 feet, 268.5 feet, and 283.7 feet respectively, how much less is the sum of the first two of these sides than the sum of the second two?

35. A manufacturer pays 35 men at the rate of \$1.50 a day, 48 at the rate of \$1.75, 30 at the rate of \$2.66 $\frac{2}{3}$, and 10 at the rate of \$3.30. How much is the pay roll for a week of 6 days?

36. Divide the sum of 4.78 and 5.22 by $3\frac{1}{3}$.

37. Divide the product of 2.25 and 48 by the sum of 2.25 and 1.75.

38. Divide the product of 800.7 and 0.5 by the difference between 10.91 and 3.06.

39. Divide the product of 22.5 and 37.5 by the product of 36.25 and 21.25.

40. Divide the product of $1.33\frac{1}{3}$ and 87.5 by the product of $\frac{1}{3}$ and $3\frac{3}{4}$.

41. Divide the product of 36.25 and 21.25 by the product of 22.5 and 37.5.

42. If we know, in an example in division, that the divisor is 2.3 and the quotient is 5.7, what is the dividend?

43. If we know, in an example in division, that both the divisor and the quotient are 25.25, what is the dividend?

44. If the area of a triangle is represented by half the product of the base and altitude, and the base is represented by 17.25 and the altitude by 8.4, what number represents the area?

45. Mr. A owned 0.75 of a manufacturing plant and sold $0.33\frac{1}{3}$ of his share to Mr. B, who in turn sold 0.2 of what he bought to Mr. C. What part of the plant did Mr. C buy?

46. A man bought three barrels of granulated sugar containing respectively 312.75 pounds, 314.75 pounds, and 316.5 pounds. He paid 5.375¢ a pound for the sugar. How much did the sugar cost him?

47. A man has to make a railway journey of 254.7 miles. After he has gone $0.33\frac{1}{3}$ of the distance he receives a telegram calling him back to a place half the distance he has already gone. After he gets back there, how many miles is it to the end of his journey?

EXERCISE 43

QUESTIONS ABOUT DECIMALS

1. What is the unit of United States money, and what sign is used to indicate it?

2. How many places of decimals do cents (including dimes) occupy? How many places do dimes occupy?

3. Write two decimals showing the part of a dollar a cent is worth, and the part of a dollar a mill is worth.

4. Write three decimals showing the part of a dollar a dime is worth, the part of a dime a cent is worth, and the part of a cent a mill is worth.

5. How are dollars reduced to cents? How are dollars reduced to mills?

6. How are cents reduced to dollars? How are mills reduced to dollars?

7. How are dollars and cents arranged for addition or subtraction? Illustrate each operation.

8. In multiplying dollars and cents by an integer, how do you know where to place the decimal point in the product?

9. In dividing dollars and cents by an integer, how do you know where to place the decimal point in the quotient?

10. A unit consists of how many tenths? of how many hundredths? of how many thousandths?

11. A tenth consists of how many hundredths? of how many thousandths? of how many ten-thousandths?

12. How is the value of a figure in a number changed if it is moved one place to the left? one place to the right?

13. What decimal place is occupied by tenths? by hundredths? by thousandths? by ten-thousandths? by hundred-thousandths? by millionths?

14. How should you read a number that has an integral part and a decimal part? Illustrate by reading 200.007.

15. What effect does it have on a decimal to annex zeros to it? Explain this in the case of 0.5 and 0.50.

16. In adding and subtracting decimals, how do you arrange the decimal points? Why is this?

17. In subtraction of decimals, if the minuend has a smaller number of decimal places than the subtrahend how do you proceed? What right have you to do this?

18. What effect does it have on the value of a number if the decimal point is moved one place to the right? one place to the left?

19. What effect does it have on the value of a number if the decimal point is moved two places to the right? two places to the left?

20. In the multiplication of decimals, how many decimal places must the product have? Illustrate by the case of 0.7 and 0.25.

21. Explain why the product of 0.6 and 0.4 should have two decimal places, by reducing both to common fractions and then multiplying.

22. In division, if the divisor is not an integer, how may you make it such without altering the quotient?

23. If the divisor is an integer, how can you tell where to put the decimal point in the quotient? Illustrate by dividing 41.44 by 4.

24. What is the reason for writing the quotient over the dividend? Give an example to assist in your explanation.

25. Why do you not have to write the denominators in decimal fractions, while you do have to write them in common fractions?

92. Aliquot Parts. An exact divisor of a number is called an *aliquot part* of that number.

Thus 5, 10, 25, $12\frac{1}{2}$, $16\frac{2}{3}$, $33\frac{1}{3}$, are aliquot parts of 100.

We may also have aliquot parts of numbers besides 100. For example, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are important aliquot parts of 10; 125, $333\frac{1}{3}$, and 250 are important aliquot parts of 1000, and so on. The aliquot parts of 100 and \$10 are, however, the ones most frequently used.

93. Important Aliquot Parts. The following fractional parts of \$1 should be memorized:

$5\text{¢} = \$\frac{1}{20} = \$0.05$	$40\text{¢} = \$\frac{2}{5} = \$0.40$
$6\frac{1}{4}\text{¢} = \$\frac{1}{16} = \$0.06\frac{1}{4}$	$50\text{¢} = \$\frac{1}{2} = \$0.50$
$8\frac{1}{3}\text{¢} = \$\frac{1}{12} = \$0.08\frac{1}{3}$	$60\text{¢} = \$\frac{3}{5} = \$0.60$
$10\text{¢} = \$\frac{1}{10} = \$0.10$	$62\frac{1}{2}\text{¢} = \$\frac{5}{8} = \$0.625$
$12\frac{1}{2}\text{¢} = \$\frac{1}{8} = \$0.125$	$66\frac{2}{3}\text{¢} = \$\frac{2}{3} = \$0.66\frac{2}{3}$
$16\frac{2}{3}\text{¢} = \$\frac{1}{6} = \$0.16\frac{2}{3}$	$70\text{¢} = \$\frac{7}{10} = \$0.70$
$20\text{¢} = \$\frac{1}{5} = \$0.20$	$75\text{¢} = \$\frac{3}{4} = \$0.75$
$25\text{¢} = \$\frac{1}{4} = \$0.25$	$80\text{¢} = \$\frac{4}{5} = \$0.80$
$30\text{¢} = \$\frac{3}{10} = \$0.30$	$87\frac{1}{2}\text{¢} = \$\frac{7}{8} = \$0.875$
$33\frac{1}{3}\text{¢} = \$\frac{1}{3} = \$0.33\frac{1}{3}$	$90\text{¢} = \$\frac{9}{10} = \$0.90$
$37\frac{1}{2}\text{¢} = \$\frac{3}{8} = \$0.375$	$100\text{¢} = \$1 = \$1.00$

The most important aliquot parts of 100, including those that should be memorized, are as follows:

$50 = \frac{1}{2}$ of 100	$33\frac{1}{3} = \frac{1}{3}$ of 100	$20 = \frac{1}{5}$ of 100
$25 = \frac{1}{4}$ of 100	$16\frac{2}{3} = \frac{1}{6}$ of 100	$10 = \frac{1}{10}$ of 100
$12\frac{1}{2} = \frac{1}{8}$ of 100	$8\frac{1}{3} = \frac{1}{12}$ of 100	$5 = \frac{1}{20}$ of 100
$6\frac{1}{4} = \frac{1}{16}$ of 100	$4\frac{1}{6} = \frac{1}{24}$ of 100	$2 = \frac{1}{50}$ of 100

Other important fractional parts of 100, but not aliquot parts, are as follows:

$75 = \frac{3}{4}$ of 100	$62\frac{1}{2} = \frac{5}{8}$ of 100	$66\frac{2}{3} = \frac{2}{3}$ of 100
$37\frac{1}{2} = \frac{3}{8}$ of 100	$87\frac{1}{2} = \frac{7}{8}$ of 100	$83\frac{1}{3} = \frac{5}{6}$ of 100

As in the case of aliquot parts, these should be memorized.

94. Multiplication by Aliquot Parts. We may simplify multiplication by 10, 100, and 1000, and by their aliquot parts, as follows:

(1) *To multiply by 10, move the decimal point one place to the right, annexing zero if necessary.*

$$\text{Thus } 10 \times 46.75 = 467.5; 10 \times 75 = 750.$$

(2) *To multiply by 100, move the decimal point two places to the right, annexing zeros if necessary.*

$$\text{Thus } 100 \times 0.275 = 27.5; 100 \times 6.5 = 650; 100 \times 72 = 7200.$$

(3) *To multiply by 1000, move the decimal point three places to the right, annexing zeros if necessary.*

$$\text{Thus } 1000 \times 0.02365 = 23.65; 1000 \times 147 = 147,000.$$

(4) *To multiply by 50, multiply by 100 and divide by 2; to multiply by 0.5, divide by 2.*

$$\text{Thus } 50 \times 84 = \frac{100}{2} \times 84 = \frac{8400}{2} = 4200; 0.5 \times 8 = \frac{1}{2} \text{ of } 8 = 4.$$

(5) *To multiply by 25, multiply by 100 and divide by 4; to multiply by 0.25, divide by 4.*

$$\text{Thus } 25 \times 84 = \frac{100}{4} \times 84 = \frac{8400}{4} = 2100; 0.25 \times 8 = \frac{1}{4} \text{ of } 8 = 2.$$

(6) *To multiply by $12\frac{1}{2}$, multiply by 100 and divide by 8; to multiply by $0.12\frac{1}{2}$, divide by 8.*

$$\text{Thus } 12\frac{1}{2} \times 64 = \frac{100}{8} \times 64 = 800; 0.12\frac{1}{2} \times 24 = \frac{1}{8} \text{ of } 24 = 3.$$

(7) *To multiply by 125, multiply by 1000 and divide by 8; to multiply by 0.125, divide by 8.*

$$\text{Thus } 125 \times 72 = \frac{1000}{8} \times 72 = 9000; 0.125 \times 16 = \frac{1}{8} \text{ of } 16 = 2.$$

(8) *To multiply by $33\frac{1}{3}$, multiply by 100 and divide by 3; to multiply by $0.33\frac{1}{3}$, divide by 3.*

$$\text{Thus } 33\frac{1}{3} \times 69 = \frac{100}{3} \times 69 = 2300; 0.33\frac{1}{3} \times 15 = \frac{1}{3} \text{ of } 15 = 5.$$

(9) *To multiply by $16\frac{2}{3}$, multiply by 100 and divide by 6; to multiply by $0.16\frac{2}{3}$, divide by 6.*

(10) *To multiply by 5, multiply by 10 and divide by 2.*

EXERCISE 44

Multiply:

- | | | |
|--------------------------------------|-------------------------------------|--|
| 1. $10 \times \$0.25$. | 12. $25 \times \$144$. | 23. $33\frac{1}{3} \times 36$. |
| 2. $10 \times \$8251$. | 13. $25 \times \$640$. | 24. $33\frac{1}{3} \times 48$. |
| 3. $10 \times \$6.25$. | 14. $25 \times \$325$. | 25. $33\frac{1}{3} \times 144$. |
| 4. $10 \times \$5.50$. | 15. $675 \times \$25$. | 26. $33\frac{1}{3} \times \$312.30$. |
| 5. $100 \times \$0.37$. | 16. $25 \times \$25.50$. | 27. $16\frac{2}{3} \times 12,630$. |
| 6. $100 \times \$2.87\frac{1}{2}$. | 17. $25 \times \$75.25$. | 28. $16\frac{2}{3} \times \$636.30$. |
| 7. $1000 \times \$0.25$. | 18. $12\frac{1}{2} \times \$6400$. | 29. $5 \times 47,664.80$. |
| 8. $1000 \times \$6.37\frac{1}{2}$. | 19. $12\frac{1}{2} \times \$4880$. | 30. $5 \times \$210,650$. |
| 9. $50 \times \$64,224$. | 20. $125 \times \$640$. | 31. $125 \times \$84,480$. |
| 10. $50 \times \$752.20$. | 21. $125 \times \$6400$. | 32. $125 \times \$844.80$. |
| 11. $50 \times \$861.55$. | 22. $125 \times \$2800$. | 33. $16\frac{2}{3} \times 4,800,000$. |

34. What will 25 books cost at \$1.20 each ?
35. What will 25 wagons cost at \$116 each ?
36. What will 50 cows cost at \$54 a head ?
37. What will 100 books cost at $62\frac{1}{2}\%$ each ?
38. What will 25 tons of hay cost at \$12.40 a ton ?
39. What will $12\frac{1}{2}$ yards of cloth cost at 36¢ a yard ?
40. What will 25 dozen pencils cost at \$0.48 a dozen ?
41. What will 48 office desks cost at \$25 each ?
42. At \$5 an acre, what is a tract of 14,264 acres of mountain land worth ?
43. How much will a man earn in 36 weeks at \$25 a week ? at $\$33\frac{1}{3}$ a week ?
44. How much will a dozen tables weigh at 25 pounds each ? at 50 pounds each ?
45. How much will 615 men earn in a week if each man earns $\$12\frac{1}{2}$? if each man earns $\$16\frac{2}{3}$?

95. Multiplication by Fractional Parts. We may simplify multiplication by fractional parts as follows:

(1) *To multiply by 0.75 , take $\frac{3}{4}$ of the number*

For example, $0.75 \times \$64.40 = \frac{3}{4}$ of $\$64.40 = \$48.30$. Similarly, to multiply by 75 , multiply by 100 and take $\frac{3}{4}$ of the product.

(2) *To multiply by $0.37\frac{1}{2}$, take $\frac{3}{8}$ of the number.*

For example, $0.37\frac{1}{2} \times \$72 = \frac{3}{8}$ of $\$72 = \$27$. Similarly, to multiply by $37\frac{1}{2}$, multiply by 100 and take $\frac{3}{8}$ of the product. Proceed in the same way in (3), (4), and (5), following.

(3) *To multiply by $0.62\frac{1}{2}$, take $\frac{5}{8}$ of the number.*

For example, $0.62\frac{1}{2} \times 640 = \frac{5}{8}$ of $640 = 400$.

(4) *To multiply by $0.87\frac{1}{2}$, take $\frac{7}{8}$ of the number.*

(5) *To multiply by $0.66\frac{2}{3}$, take $\frac{2}{3}$ of the number.*

EXERCISE 45

Multiply:

1. 0.75×684 . 7. 0.75 of 6884 . 13. $744 \times 0.87\frac{1}{2}$.
2. 0.75×816 . 8. $736 \times \$0.37\frac{1}{2}$. 14. $0.66\frac{2}{3} \times 3366$.
3. $0.75 \times \$1.48$. 9. $736 \times \$0.62\frac{1}{2}$. 15. $0.66\frac{2}{3} \times 288.6$.
4. $0.75 \times \$3.20$. 10. $824 \times \$0.87\frac{1}{2}$. 16. $0.66\frac{2}{3} \times 28.86$.
5. $248 \times \$0.75$. 11. $366 \times \$0.66\frac{2}{3}$. 17. $48,800 \times 7.5$.
6. $432 \times \$0.75$. 12. $174 \times \$0.16\frac{2}{3}$. 18. $0.12\frac{1}{2} \times 0.16$.
19. How much will 75 cows cost at $\$48$ each?
20. How much will 75 cows cost at $\$55$ each?
21. How much will 72 horses cost at $\$75$ each?
22. How much will 88 horses cost at $\$125$ each?
23. How much will 288 whalebones cost at 25¢ each?
24. How much will 275 packing boxes cost at $62\frac{1}{2}\text{¢}$ each?
25. How much will 484 yards of silk cost at 75¢ a yard?
26. How much will 364 yards of cloth cost at 75¢ a yard?

96. Division by Fractional Parts. There are certain divisors, some of them aliquot parts of 1, 10, 100, or 1000, that are used so often that it is convenient to know short methods of dividing by them.

The principal ones are 10, 100, 1000, and other powers of 10; 5, 25, 125, $12\frac{1}{2}$, and $33\frac{1}{3}$; and 0.5, 0.25, 0.125, and $0.33\frac{1}{3}$.

Since division is the inverse of multiplication, we easily derive the following from the cases already considered:

(1) *To divide by 10, move the decimal point one place to the left; by 100, two places; by 1000, three places; and so on.*

Thus $4936 \div 10 = 493.6$; $\$4876 \div 100 = \$48.76$; $29.5 \div 1000 = 0.0295$. This has already been explained in division of decimals

(2) *To divide by 5, multiply by 2 and divide by 10; to divide by 0.5, multiply by 2.*

For to divide by 5 is to multiply by $\frac{1}{5}$ or $\frac{2}{10}$. To divide 32,305 by 5 we may multiply by 2, obtaining 64,610, and cut off the zero, obtaining 6461, a process somewhat easier than actual division.

(3) *To divide by 25, multiply by 4 and divide by 100; to divide by 0.25, multiply by 4.*

For $\frac{1}{25} = \frac{4}{100}$. It is much easier to multiply by 4 and move the decimal point two places than to divide by 25. Furthermore, $0.25 = \frac{1}{4}$, and to divide by $\frac{1}{4}$ we merely multiply by 4.

(4) *To divide by 125, multiply by 8 and divide by 1000; to divide by 0.125, multiply by 8.*

Why is this true? To what common fraction is 0.125 equal? How do we divide by this common fraction?

(5) *To divide by $33\frac{1}{3}$, multiply by 3 and divide by 100; to divide by $0.33\frac{1}{3}$, multiply by 3.*

Why is this true? To what common fraction is $0.33\frac{1}{3}$ equal?

(6) *To divide by $0.66\frac{2}{3}$, multiply by 3 and divide by 2.*

Why is this true?

(7) *To divide by 0.75, multiply by 4 and divide by 3.*

EXERCISE 46

Divide :

- | | | |
|--------------------------|----------------------------------|--------------------------------|
| 1. $\$7463 \div 10.$ | 7. $\$9125 \div \$25.$ | 13. $76 \div 0.33\frac{1}{3}.$ |
| 2. $\$2896 \div 100.$ | 8. $12,325 \div 25.$ | 14. $87 \div 0.33\frac{1}{3}.$ |
| 3. $\$4960 \div 1000.$ | 9. $31,325 \div 125.$ | 15. $96 \div 0.66\frac{2}{3}.$ |
| 4. $\$2215.50 \div 5.$ | 10. $62,750 \div 125.$ | 16. $68 \div 0.66\frac{2}{3}.$ |
| 5. $\$3122.35 \div 5.$ | 11. $\$1525 \div 33\frac{1}{3}.$ | 17. $96 \div 0.75.$ |
| 6. $\$172,232 \div \$5.$ | 12. $\$3275 \div 33\frac{1}{3}.$ | 18. $69 \div 0.75.$ |

19. At 25¢ a yard, how many yards of gingham can a dealer buy for \$124?

20. At 25¢ a yard, how many yards of percaline can a dealer buy for \$160?

21. At 25¢ a dozen, how many dozen spools of darning cotton can a dealer buy for \$180?

22. At $33\frac{1}{3}$ ¢ a yard, how many yards of canvas can a dealer buy for \$135?

23. At $33\frac{1}{3}$ ¢ a yard, how many yards of jute burlap can a dealer buy for \$165?

24. At $\$3.33\frac{1}{3}$ a day, how many days must a man work in order to earn \$130?

25. At $3\frac{1}{3}$ ¢ a yard, how many yards of cheesecloth can a dealer buy for \$115?

26. At $2\frac{1}{2}$ ¢ a skein, how many skeins of cotton must a manufacturer sell in order to receive \$125.75?

27. At \$1.25 a box, how many boxes of fancy candies can a dealer buy for \$180?

28. At \$125 each, how many horses can a dealer buy for \$1750? for \$1375?

29. At \$5 each, how many rocking-chairs must a factory sell in order to receive \$3375?

30. How much will 144 boxes cost at $37\frac{1}{2}$ ¢ each ?
31. How much will 22 typewriters cost at $\$62\frac{1}{2}$ each ?
32. How much will 16 acres of land cost at $\$87\frac{1}{2}$ an acre ?
33. How much will 375 mules cost at $\$125$ each ?
34. If 25 tons of coal cost $\$187.50$, what will be the cost of 99 tons ?
35. How much will 288 dozen sewing-machine needles cost at $37\frac{1}{2}$ ¢ a dozen ?
36. How much will 250 dozen yards of millinery braid cost at $62\frac{1}{2}$ ¢ a dozen ?
37. There are 44,000 yards in 25 miles. How many yards are there in one mile ?
38. How much will 18 thousand feet of oak flooring cost at $\$125$ a thousand feet ?
39. How many bushels of oats will 81 acres produce, if an average acre produces $33\frac{1}{3}$ bushels ?
40. How many bushels of potatoes will 32 acres produce, if an average acre produces 125 bushels ?
41. A tailor paid $\$210$ for some broadcloth at $\$3.33\frac{1}{3}$ a yard. How many yards did he buy ?
42. A dealer sold some overcoats for $\$2900$, at the rate of $\$33\frac{1}{3}$ apiece. How many overcoats did he sell ?
43. How much will 16 thousand feet of mixed red and white birch flooring cost at $\$50$ a thousand feet ?
44. A manufacturer sells $\$1810$ worth of silk at $\$1.25$ a yard. How many yards does he sell ?
45. A manufacturer sells $\$2777.50$ worth of velvet at $\$2.50$ a yard. How many yards does he sell ?
46. A farmer owns 250 acres of land covered with a growth of white pine which he values at $\$166$ an acre. At what amount does he value the property ?

EXERCISE 47

DRILL WORK IN FRACTIONAL PARTS

Multiply :

- | | | |
|-------------------------------------|-------------------------------------|------------------------------------|
| 1. $0.25 \times 488.$ | 17. $0.37\frac{1}{2} \times 424.$ | 33. $0.87\frac{1}{2} \times 728.$ |
| 2. $2.5 \times 488.$ | 18. $3.7\frac{1}{2} \times 424.$ | 34. $0.87\frac{1}{2} \times 7.28$ |
| 3. $25 \times 488.$ | 19. $37\frac{1}{2} \times 424.$ | 35. $8.75 \times 72.8.$ |
| 4. $0.25 \times 4.88.$ | 20. $37\frac{1}{2} \times 4.24.$ | 36. $8.7\frac{1}{2} \times 7.28.$ |
| 5. $0.75 \times 644.$ | 21. $0.62\frac{1}{2} \times 584.$ | 37. $875 \times 728.$ |
| 6. $7.5 \times 644.$ | 22. $6.25 \times 584.$ | 38. $0.66\frac{2}{3} \times 546.$ |
| 7. $75 \times 644.$ | 23. $6.2\frac{1}{2} \times 5.84.$ | 39. $6.6\frac{2}{3} \times 546.$ |
| 8. $0.75 \times 6.44.$ | 24. $0.62\frac{1}{2} \times 0.584.$ | 40. $6.66\frac{2}{3} \times 546.$ |
| 9. $0.75 \times 2436.$ | 25. $592 \times \$0.87\frac{1}{2}.$ | 41. $2952 \times 0.05.$ |
| 10. $3574 \times 0.05.$ | 26. $368 \times \$0.12\frac{1}{2}.$ | 42. $248 \times 0.05.$ |
| 11. $0.25 \times 536.$ | 27. $336 \times \$0.33\frac{1}{3}.$ | 43. $24.8 \times 0.5.$ |
| 12. $0.25 \times 5.36.$ | 28. $297 \times \$0.33\frac{1}{3}.$ | 44. $2.48 \times 0.5.$ |
| 13. $0.25 \times 0.536.$ | 29. $372 \times \$0.66\frac{2}{3}.$ | 45. $24 \times 7.5.$ |
| 14. $704 \times \$0.25.$ | 30. $492 \times \$0.66\frac{2}{3}.$ | 46. $2.4 \times 7.5.$ |
| 15. $172 \times \$0.25.$ | 31. $558 \times \$0.16\frac{2}{3}.$ | 47. $2.4 \times 0.75.$ |
| 16. $776 \times \$0.12\frac{1}{2}.$ | 32. $174 \times \$0.16\frac{2}{3}.$ | 48. $0.12\frac{1}{2} \times 0.16.$ |

Divide :

- | | | |
|---------------------------------|---------------------------------|---------------------------------|
| 49. $485 \div 0.25.$ | 56. $328 \div 1.25.$ | 63. $424 \div 6.66\frac{2}{3}.$ |
| 50. $526 \div 0.12\frac{1}{2}.$ | 57. $564 \div 0.125.$ | 64. $520 \div 0.6\frac{1}{4}.$ |
| 51. $17,216 \div 5.$ | 58. $275 \div 33\frac{1}{3}.$ | 65. $390 \div 0.5.$ |
| 52. $436 \div 25.$ | 59. $326 \div 3.3\frac{1}{3}.$ | 66. $4.5 \div 0.05.$ |
| 53. $528 \div 2.5.$ | 60. $290 \div 0.33\frac{1}{3}.$ | 67. $288 \div 0.75.$ |
| 54. $232 \div 125.$ | 61. $428 \div 0.66\frac{2}{3}.$ | 68. $2.88 \div 7.5.$ |
| 55. $232 \div 12.5.$ | 62. $524 \div 6.6\frac{2}{3}.$ | 69. $666 \div 0.75.$ |

97. Cancellation. The process of shortening work in division by dividing both divisor and dividend by the same number is called *cancellation*.

For example, what is the quotient when 17.8×45.39 is divided by 0.267 ?

Multiplying both divisor and dividend by 1000, to make the divisor integral, we have for the dividend 178×4539 (i.e. 10 times 17.8 and 100 times 45.39). Canceling, we have $2 \times 1513 = 3026$.

$$\begin{array}{r} 2 \times \cancel{89} \quad 1513 \\ \underline{17\cancel{8} \times 4\cancel{5}39} \\ 267 \\ \underline{3 \times 89} \end{array}$$

EXERCISE 48

1. $\frac{3.9 \times 0.84}{15.6}$

4. $\frac{5.5 \times 0.42}{0.77}$

7. $\frac{2.1 \times 0.4}{0.015}$

2. $\frac{1.4 \times 0.6}{0.63}$

5. $\frac{4.5 \times 6.3}{8.1}$

8. $\frac{0.21 \times 0.6}{0.126}$

3. $\frac{2.2 \times 2.4}{4 \times 7.7}$

6. $\frac{5.6 \times 0.3}{0.63 \times 0.8}$

9. $\frac{1.8 \times 5.5}{9.9 \times 0.1}$

10. The product of 6.3 , 1.4 , and 5.7 is how many times as great as the product of 17.1 , 0.06 , and 7 ?

11. The product of 0.2 , 0.21 , and 7.5 is how many times as great as the product of 0.14 , 0.3 , and 1.5 ?

12. How many times is the product of 1.6 , 0.01 , and 1.5 contained in the product of 0.032 , 2.25 , and 3 ?

13. $\frac{0.8 \times 1.5 \times 0.6}{3.6 \times 0.2 \times 10}$

15. $\frac{0.2 \times 3.6 \times 1.43}{9.9 \times 0.4 \times 2.6}$

14. $\frac{1.2 \times 0.39 \times 1.4}{2.6 \times 0.42 \times 2}$

16. $\frac{6.3 \times 4.2 \times .17}{2.7 \times 3.4 \times .49}$

17. The volume of a box whose interior dimensions are 2.4 inches, 5 inches, and 1.7 inches, is how many times as great as that of one whose interior dimensions are 3 inches, 1.6 inches, and 0.85 inches?

EXERCISE 49

DRILL WORK IN DECIMALS

1. Why will there be three decimal places in the product of 0.75 multiplied by 0.3? of 2.325 multiplied by 7?

Exs. 1-9 illustrate the oral work that should be given frequently.

Without multiplying, state the number of decimal places in each of the following products:

2. 2.8×7.9 . 4. 2.007×6.09 . 6. 1.203×2.67 .
 3. 1.7×1.42 . 5. 1.234×3.456 . 7. 3.421×0.713 .
 8. Multiply 1.21 by 2; by 3; by 4; by 10; by 100.
 9. Divide 1.05 by 5; by 10; by 2; by 3; by 0.1.

Add the following:

10.	11.	12.	13.
4.028	0.387	1.043	0.2909
0.396	0.0426	0.427	0.0628
0.285	0.0539	6.091	0.0703
4.72	0.2064	0.5	0.5076
3.942	0.0708	0.709	0.0289
0.409	0.5029	6.273	0.0079
<u>6.029</u>	<u>0.0627</u>	<u>0.108</u>	<u>0.0403</u>

Subtract:

14.	15.	16.	17.
1.035	0.2809	82.481	64.035
<u>0.876</u>	<u>0.1589</u>	<u>2.909</u>	<u>37.029</u>
18.	19.	20.	21.
86.431	102.98	62.035	80.289
<u>28.902</u>	<u>72.89</u>	<u>51.975</u>	<u>79.939</u>

Add the following, timing yourself:

22.	23.	24.	25.
\$1000.00	\$824.63	\$209.09	\$802.35
289.00	981.42	32.78	608.72
40.27	127.69	45.67	402.81
6.35	842.38	682.73	96.43
42.07	327.29	102.93	87.29
<u>189.27</u>	<u>420.63</u>	<u>41.52</u>	<u>289.62</u>

Copy and subtract, timing yourself:

- | | |
|----------------------------|----------------------------|
| 26. \$640.00 — \$298.29. | 29. \$702.53 — \$129.65. |
| 27. \$902.63 — \$127.49. . | 30. \$1000.00 — \$298.75. |
| 28. \$3476.42 — \$1029.29. | 31. \$2025.30 — \$1627.92. |

Multiply the following:

- | | |
|-----------------------|------------------------------------|
| 32. 234.5 by 8.001. | 35. 0.500 by 0.600. |
| 33. \$683.45 by 0.2. | 36. \$281.42 by 6.50. |
| 34. \$283.75 by 42.8. | 37. \$333.33 $\frac{1}{3}$ by 0.3. |

38. At \$17.52 a front foot, what will a piece of land of 28.33 $\frac{1}{3}$ feet front cost?

39. A tank when full holds 42.8 gallons of water. How much does it contain when 0.37 full?

40. A certain rectangle is 0.78 as wide as it is long. It is 4.9 inches long. What is the perimeter?

41. A certain rectangle is 3.42 times as long as it is wide. It is 4.8 feet wide. What is the perimeter?

42. A city lot is 45.8 feet by 76.3 feet. What is the area of the lot? What is it worth at \$1.50 a square foot?

43. If the circumference of a circle is 3.1416 times the diameter, what is the circumference of a circle the diameter of which is 42.27 feet?

44. At \$1.75 each, what will 17 books cost?
 45. Find the perimeter of a square 34.7 feet on a side.
 46. At \$75.50 an acre, what will 25.5 acres of land cost?
 47. At \$0.75 a yard, what will 32.2 yards of silk cost?
 48. At \$125 a front foot, how much will a city lot cost having a frontage of 72.3 feet?

Multiply :

- | | | |
|---------------------------|----------------------------|----------------------------|
| 49. 1.4×6.64 . | 59. 6.81×7.92 . | 69. 30.5×57.2 . |
| 50. 3.6×19.72 . | 60. 8.35×6.423 . | 70. 8.25×6.37 . |
| 51. 43.2×98.6 . | 61. 89.1×49.3 . | 71. 68.7×40.5 . |
| 52. 0.823×2946 . | 62. 72.9×87.09 . | 72. 53.6×2.889 . |
| 53. 48.1×32.5 . | 63. 20.5×60.7 . | 73. 43.7×74.1 . |
| 54. 26.8×80.8 . | 64. 20.2×20.2 . | 74. 1.6×140.64 . |
| 55. 60.9×90.6 . | 65. 73.7×73.7 . | 75. 6.2×53.134 . |
| 56. 4.72×6.93 . | 66. 42.3×5.37 . | 76. 17.2×26.35 . |
| 57. 2.87×5.492 . | 67. 1.5×553.35 . | 77. 2.175×6.73 . |
| 58. 16.4×17.25 . | 68. 5.8×439.002 . | 78. 0.273×0.625 . |

Divide :

- | | | |
|--------------------------|-------------------------|---------------------------|
| 79. $17.25 \div 500$. | 84. $\$32.48 \div 8$. | 89. $20.720 \div 0.28$. |
| 80. $817.6 \div 700$. | 85. $\$26.25 \div 25$. | 90. $38.8692 \div 5.4$. |
| 81. $0.7364 \div 7000$. | 86. $\$21.42 \div 18$. | 91. $825.468 \div 4.2$. |
| 82. $3.248 \div 200$. | 87. $\$99.54 \div 14$. | 92. $888.16 \div 2.8$. |
| 83. $82.48 \div 8000$. | 88. $\$82.20 \div 12$. | 93. $1131.264 \div 4.8$. |

In the following divisions, carry the quotient to three decimal places :

- | | | |
|---------------------------|--------------------------|--------------------------|
| 94. $17.78 \div 1.5$. | 97. $167.8 \div 3.7$. | 100. $19.26 \div 3.1$. |
| 95. $423.6 \div 0.27$. | 98. $62.23 \div 0.23$. | 101. $8.348 \div 2.68$. |
| 96. $976.34 \div 0.125$. | 99. $483.62 \div 42.7$. | 102. $826.3 \div 12.5$. |

CHAPTER VII

DENOMINATE NUMBERS

98. Denominate Number. A concrete number that expresses measure is called a *denominate number*.

For example, \$7 and 2 feet 3 inches are denominate numbers, but not 5 chairs or 8 rivers.

99. Compound Number. A denominate number involving two or more different units is called a *compound number*.

For example, 2 feet 3 inches, or 3 hours 20 minutes. We do not usually speak of \$2.50 as a compound number because it is written as an integer and a decimal fraction. If we should write $2\frac{1}{4}$ feet or 2.25 feet instead of 2 feet 3 inches, we should not be using compound numbers.

Some work in compound numbers is done in the primary grades, and many of the common measures are therefore already known.

100. Measures of Length. The table of common measures of length is as follows :

12 inches (in.) = 1 foot (ft.)

3 feet = 1 yard (yd.)

$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)

320 rods, or 5280 feet = 1 mile (mi.)

A *hand* (4 in.) is used in measuring the height of horses ; a *fathom* (6 ft.) and *cable length* (120 fathoms) in measuring depths of water ; a *knot* (nautical mile, 1.152 common or statute miles, or 6080.27 ft.) in measuring distances at sea. Carpenters and mechanics usually write 2' 6" for 2 ft. 6 in.

101. Reduction. The process of changing the unit of a number without changing the value is called *reduction*.

Thus 2 ft. 6 in. = 30 in. = 2.5 ft.

102. Reduction Descending. Reduction from a higher to a lower denomination is called *reduction descending*.

For example, required to reduce 2 ft. 3 in. to inches.

$$1 \text{ ft.} = 12 \text{ in.,}$$

$$2 \text{ ft.} = 2 \times 12 \text{ in.}$$

$$= 24 \text{ in.}$$

Therefore $24 \text{ in.} + 3 \text{ in.} = 27 \text{ in.}$

ft.	in.
2	3
	<u>12</u>
	24
	<u>3</u>
	27

For convenience the actual multiplications and additions are carried out as shown at the right.

In practical life we seldom have reductions involving more than two different units (denominations), and in this arithmetic other cases will not in general be considered.

Therefore, *in reduction descending, multiply the number of the highest denomination given, by the number showing how many units of the next lower denomination are equal to one of the higher. To the product add the given number of this lower denomination.*

Proceed thus with each successive result until the required denomination is reached.

EXERCISE 50

Reduce :

- | | |
|---|----------------------------------|
| 1. 54 ft. 7 in. to inches. | 10. 160 rd. to yards. |
| 2. 72 ft. 9 in. to inches. | 11. 5 mi. to rods. |
| 3. 28 yd. to feet; to inches. | 12. 7 mi. to yards. |
| 4. $5\frac{1}{2}$ yd. to feet; to inches. | 13. $22\frac{1}{2}$ mi. to rods. |
| 5. 26 rd. to yards; to feet. | 14. 3.25 mi. to feet. |
| 6. 320 rd. to yards; to feet. | 15. 0.5 mi. to feet. |
| 7. 41.8 mi. to miles and feet. | 16. 0.6 mi. to feet. |
| 8. 25.75 mi. to miles and feet. | 17. 0.65 mi. to feet. |
| 9. 126.7 mi. to miles and feet. | 18. 0.01 mi. to yards. |

19. A knot being 1.152 miles, how many miles does a ship sail in going 480 knots?

103. Reduction Ascending. Reduction from a lower to a higher denomination is called *reduction ascending*.

For example, required to reduce 104 in. to higher units.

For practical purposes the work may be arranged as shown at the left, but the explanation is better understood from the following:

$$\begin{array}{rcl}
 12 \overline{)104} \text{ (in.)} & \text{Since } 1 \text{ in.} = \frac{1}{12} \text{ ft.,} & \\
 \underline{3} \overline{)8} \text{ (ft.)} \dots 8 \text{ in.} & 104 \text{ in.} = 104 \times \frac{1}{12} \text{ ft.} = 8\frac{8}{12} \text{ ft.} = 8 \text{ ft. } 8 \text{ in.} & \\
 2 \text{ (yd.)} \dots 2 \text{ ft.} & \text{Since } 1 \text{ ft.} = \frac{1}{3} \text{ yd.,} & \\
 & 8 \text{ ft.} = 8 \times \frac{1}{3} \text{ yd.} = 2\frac{2}{3} \text{ yd.} = 2 \text{ yd. } 2 \text{ ft.} &
 \end{array}$$

Therefore the answer is 2 yd. 2 ft. 8 in.

Therefore, in reduction ascending,

Divide the given number of units by the number of units required to make one of the next higher denomination.

Divide this quotient and each successive quotient in like manner until the required denomination is reached.

The last quotient with the several remainders arranged in order is the answer sought.

EXERCISE 51

Reduce to higher units:

- | | | | |
|------------|-----------|-----------|--------------|
| 1. 96 in. | 4. 5 ft. | 7. 11 yd. | 10. 700 rd. |
| 2. 100 in. | 5. 9 ft. | 8. 15 yd. | 11. 960 rd. |
| 3. 115 in. | 6. 25 ft. | 9. 35 yd. | 12. 1280 rd. |

Reduce to feet, or to feet and inches:

- | | | | |
|-------------|-------------|-------------|-------------|
| 13. 150 in. | 15. 360 in. | 17. 600 in. | 19. 960 in. |
| 14. 165 in. | 16. 400 in. | 18. 720 in. | 20. 972 in. |

Reduce to yards, or to yards and one lower unit:

- | | | | |
|------------|-------------|------------|-----------------|
| 21. 60 ft. | 23. 500 ft. | 25. 72 in. | 27. 50 in. |
| 22. 65 ft. | 24. 750 ft. | 26. 84 in. | 28. 6 ft. 8 in. |

EXERCISE 52

Express as inches :

- | | | |
|------------------|-----------------------|------------------|
| 1. 9 ft. 6 in. | 5. 2 ft. 6 in. | 9. 3 yd. 2 in. |
| 2. 36 ft. 4 in. | 6. $5\frac{1}{2}$ ft. | 10. 4 yd. 15 in. |
| 3. 42 ft. 9 in. | 7. 3 ft. 8 in. | 11. 6 yd. 22 in. |
| 4. 28 ft. 11 in. | 8. $4\frac{2}{3}$ ft. | 12. 8 yd. 30 in. |

Express as feet :

- | | | |
|------------------|-------------------------|-------------------|
| 13. 4 yd. 2 ft. | 17. $33\frac{1}{3}$ yd. | 21. 5 rd. |
| 14. 22 yd. 1 ft. | 18. $66\frac{2}{3}$ yd. | 22. 51 rd. 7 ft. |
| 15. 33 yd. 2 ft. | 19. 66 yd. 2 ft. | 23. 3 mi. 720 ft. |
| 16. 41 yd. 1 ft. | 20. 75 yd. 2 ft. | 24. 4 mi. 8 rd. |

Express as yards :

- | | | |
|------------|------------------|------------------|
| 25. 22 rd. | 27. 3 mi. | 29. 2 mi. 57 rd. |
| 26. 28 rd. | 28. 1 mi. 10 rd. | 30. 3.25 mi. |

Express as rods :

- | | | |
|-----------------------|-------------|--------------|
| 31. $\frac{1}{2}$ mi. | 32. 1.2 mi. | 33. 3.75 mi. |
|-----------------------|-------------|--------------|

Express as feet, or as feet and inches :

- | | | |
|-------------|-------------|--------------|
| 34. 48 in. | 37. 275 in. | 40. 635 in. |
| 35. 480 in. | 38. 325 in. | 41. 590 in. |
| 36. 360 in. | 39. 450 in. | 42. 1250 in. |

Express as yards, or as yards and one lower unit :

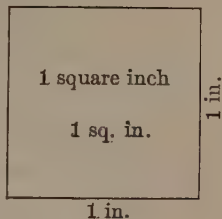
- | | | |
|--------------|--------------|-------------------|
| 43. 1280 ft. | 45. 5280 ft. | 47. 425 in. |
| 44. 7650 ft. | 46. 1795 ft. | 48. 72 ft. 27 in. |

Express as rods, or as rods and feet :

- | | | |
|-------------|------------|------------------|
| 49. 66 ft. | 51. 33 yd. | 53. 5 yd. 18 in. |
| 50. 330 ft. | 52. 75 ft. | 54. 350 yd. |

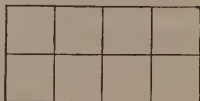
104. Square. A flat surface that has four equal sides and four equal angles is called a *square*.

If each side is 1 in. long, the square is 1 square inch; if each side is 1 ft. long, the square is 1 square foot; if each side is 1 yd. long, the square is 1 square yard; and so on.



105. Area. The number of square units in a surface is called its *area*.

For example, if this figure represents a surface 4 yd. long and 2 yd. wide, the area is $2 \times 4 \times 1$ square yard, or 8 square yards.



106. Square Measure. The common table of square measure is as follows:

144 square inches (sq. in.) = 1 square foot (sq. ft.)

9 square feet = 1 square yard (sq. yd.)

$30\frac{1}{2}$ square yards = 1 square rod (sq. rd.)

160 square rods = 1 acre (A.)

640 acres = 1 square mile (sq. mi.)

EXERCISE 53

Reduce to the next lower units:

1. 13 sq. ft. 4. 43 sq. yd. 7. 6 A. 10. 5 sq. mi.
2. 28 sq. ft. 5. 62 sq. yd. 8. 15 A. 11. 25 sq. mi.
3. 60 sq. ft. 6. 124 sq. rd. 9. 35 A. 12. 150 sq. mi.
13. Reduce 75 sq. ft. 36 sq. in. to square inches.
14. Reduce 2 sq. yd. 58 sq. in. to square inches.
15. Reduce 52 sq. yd. 8 sq. ft. to square feet.
16. Reduce 67,592 sq. in. to higher units.
17. Reduce 47,916 sq. ft. to higher units.
18. Reduce 22,471 sq. yd. to higher units.

EXERCISE 54

Express as square inches :

- | | | |
|---------------------------|--------------------------|-------------------------|
| 1. 1.2 sq. ft. | 4. $\frac{1}{2}$ sq. yd. | 7. 3 sq. ft. 75 sq. in. |
| 2. 1.7 sq. ft. | 5. 2.5 sq. yd. | 8. 2 sq. ft. 80 sq. in. |
| 3. $1\frac{3}{4}$ sq. ft. | 6. 0.7 sq. yd. | 9. 4 sq. ft. 72 sq. in. |

Express as square feet :

- | | | |
|------------------|------------------------------|----------------------------|
| 10. 288 sq. in. | 14. 59 sq. yd. | 18. 5 sq. rd. |
| 11. 720 sq. in. | 15. 8.7 sq. yd. | 19. $6\frac{1}{2}$ sq. rd. |
| 12. 864 sq. in. | 16. $42\frac{1}{2}$ sq. yd. | 20. 7.25 sq. rd. |
| 13. 1008 sq. in. | 17. $0.3\frac{1}{3}$ sq. yd. | 21. 130 sq. rd. |

Express as square yards :

- | | | |
|------------------|------------------|--------------------------|
| 22. 1296 sq. in. | 26. 60.5 sq. rd. | 30. 1 A. |
| 23. 6480 sq. in. | 27. 121 sq. rd. | 31. 0.5 A. |
| 24. 108 sq. ft. | 28. 605 sq. rd. | 32. 0.75 A. |
| 25. 801 sq. ft. | 29. 242 sq. rd. | 33. $0.33\frac{1}{3}$ A. |

Express as square rods :

- | | | |
|------------------------------|------------------------------|-----------------------|
| 34. $272\frac{1}{4}$ sq. ft. | 38. 121 sq. yd. | 42. 7 A. |
| 35. 1089 sq. ft. | 39. 242 sq. yd. | 43. $1\frac{1}{2}$ A. |
| 36. 5445 sq. ft. | 40. 605 sq. yd. | 44. $1\frac{3}{4}$ A. |
| 37. 10,890 sq. ft. | 41. $302\frac{1}{2}$ sq. yd. | 45. $1\frac{5}{8}$ A. |

Express as acres :

- | | | |
|------------------|--------------------|----------------------------|
| 46. 480 sq. rd. | 49. 4840 sq. yd. | 52. 7 sq. mi. |
| 47. 800 sq. rd. | 50. 24,200 sq. yd. | 53. $2\frac{1}{2}$ sq. mi. |
| 48. 1120 sq. rd. | 51. 26,620 sq. yd. | 54. 3.75 sq. mi. |

Express as square miles :

- | | | |
|-------------|-------------|-------------|
| 55. 1280 A. | 57. 320 A. | 59. 4480 A. |
| 56. 1920 A. | 58. 2240 A. | 60. 5760 A. |

107. Measurement of a Rectangle. If this figure represents a rectangle 4 in. long by 2 in. wide, there are evidently 4 sq. in. in the lower row of squares, and since there are 2 rows, there are 2×4 sq. in., or 8 sq. in., in the rectangle.

If we know the area and one dimension, we may find the other dimension; for, since 8 sq. in. $= 2 \times 4$ sq. in., then $8 \text{ sq. in.} \div 4 \text{ sq. in.} = 2$, the number of units in the other dimension.



Express the length and breadth of a rectangle in the same linear unit; the product of these two numbers will express the area in square units of the same name as the linear unit.

The number of square units in a rectangle divided by the number of linear units in one dimension gives the number of linear units in the other dimension.

This means that the numbers are to be considered as abstract. The complete solutions with concrete numbers are given above.

The expression $4'' \times 7''$ means and is read 4 inches by 7 inches.

EXERCISE 55

Find the areas of rectangles with these bases and altitudes:

1. 127 ft., 46 ft. 5. 6 ft. 4 in., 3 ft.
2. 152 in., 35 in. 6. 7 ft. 2 in., 5 in.
3. 225 yd., 82 yd. 7. 3 ft. 6 in., 1 ft. 4 in.
4. 240 in., 11 ft. 8. 9 yd. 2 ft., 3 ft. 6 in.

9. How many square rods in a rectangular field 32 rd. long and 16 rd. wide?

10. How many acres in a rectangular field 126 rd. long and 38 rd. wide?

11. A rectangular city lot is $129\frac{1}{2}$ ft. long and 25 ft. 9 in. wide. Find the number of square feet in the lot.

12. A rectangular field containing 4.05 A. is 36 rd. long. How wide is it?

108. Triangle. A plane figure bounded by three straight lines is called a *triangle*.

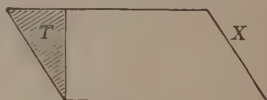
The perpendicular distance from the vertex to the line of the base is called the *altitude* or *height* of the triangle.

109. Parallelogram. A plane figure bounded by two pairs of parallel lines is called a *parallelogram*.

The *base* of a parallelogram is the side on which it rests. The *altitude* is the perpendicular distance from the opposite side to the base. The opposite parallel sides of a parallelogram are equal.

If we cut off the triangle *T* and place it at *X*, the parallelogram will be changed to a rectangle of the same base and the same altitude.

Therefore, *the area of a parallelogram is equal to the product of the base and altitude.*



For example, if the base of a parallelogram is 3 ft. 8 in. and the altitude is 3 ft., what is the area?

$$3 \times 3\frac{2}{3} \text{ sq. ft.} = 11 \text{ sq. ft.}$$

If the area of a parallelogram is 15 sq. ft. and the base is 5 ft., required the altitude.

$$15 \text{ sq. ft.} \div 5 \text{ sq. ft.} = 3, \text{ the number of feet of altitude (§107).}$$

EXERCISE 56

Find the areas of parallelograms with the following bases and altitudes :

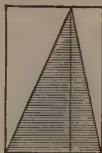
- | | |
|---|------------------------------|
| 1. 157 ft., 96 ft. | 3. 148 rd., 100 rd. |
| 2. $32\frac{1}{2}$ ft., $15\frac{1}{2}$ ft. | 4. 27 ft. 6 in., 3 ft. 4 in. |

Find the altitudes of parallelograms with the following areas and bases :

- | | |
|-------------------------|-------------------------|
| 5. 972 sq. ft., 36 ft. | 7. 1728 sq. ft., 18 yd. |
| 6. 1170 sq. ft., 45 ft. | 8. 40 A., 100 rd. |

110. Measurement of a Triangle. We see that this triangle may be cut into two parts, each of which is half of some rectangle, and that therefore the whole triangle is half of a rectangle of the same base and altitude.

Whatever the shape of the triangle, it may, in a similar way, be shown to be half of a parallelogram of the same base and altitude, and therefore (§ 109) equal to half of a rectangle of the same base and altitude.



Therefore, *the area of a triangle equals half the product of the numbers representing the base and altitude.*

EXERCISE 57

Find the areas of triangles with the following bases and altitudes :

- | | |
|--------------------------------|-------------------------------|
| 1. 47 ft., 28 ft. | 6. 3 yd. 18 in., 2 ft. |
| 2. 75 ft., 25 ft. | 7. 7 ft. 2 in., 3 ft. 1 in. |
| 3. $82\frac{1}{2}$ ft., 35 ft. | 8. 8 ft. 6 in., 4 ft. 8 in. |
| 4. 160 rd., 25 rd. | 9. 12 ft. 9 in., 7 ft. 6 in. |
| 5. 6 ft. 9 in., 27 in. | 10. 4 yd. 27 in., 2 ft. 4 in. |

Find the altitudes of triangles with the following areas and bases :

- | | |
|-------------------------|--------------------------|
| 11. 92 sq. ft., 23 ft. | 13. 594 sq. ft., 54 ft. |
| 12. 480 sq. ft., 48 ft. | 14. 1053 sq. in., 39 in. |
15. At \$5.50 a square foot, what is the value of a triangular city lot with base 82 ft. and altitude 45 ft. 6 in. ?
16. What is the area of a triangle with sides all equal, whose perimeter is 18 in. and altitude 5.19 in. ?
17. A triangular gable to a house has a base 30 ft. and an altitude 10 ft. 6 in. Find the area of the gable.

18. What is the altitude of a triangle with area $3\frac{1}{8}$ sq. in. and base $\frac{3}{4}$ in. ?

19. What is the altitude of a triangle with area $7\frac{5}{16}$ sq. in. and base $3\frac{1}{4}$ in. ?

20. What is the base of a triangle with area $4\frac{7}{12}$ sq. ft. and altitude $2\frac{3}{4}$ ft. ?

21. The gable of a house is a triangle with base $32\frac{1}{2}$ ft. and altitude $9\frac{1}{2}$ ft. What is the area of the gable ?

22. In laying out a triangular flower bed the sides are all made equal. The perimeter is found to be 33 ft. and the altitude 9.5 ft. What is the area ?

23. The four faces of a church spire are triangles, each with base 24 ft. and altitude 53 ft. How many square feet in each triangle ? in the whole surface of the spire ?

24. A man buys a 54-acre lot that is a parallelogram in shape. The frontage along the road is 144 rd. What is the distance to the rear line ?

25. For the panels of a staircase 20 marble slabs are needed. They are parallelograms, each with base 1 ft. 6 in. and altitude 4 ft. 3 in. What is the total area of all the slabs ?

26. A marble floor is made of 975 equilateral triangles, each with base 10 in. and altitude 8.6 in. What is the area of the floor space in square inches, making no allowance for cement ?

27. A corner lot, where two streets meet at oblique angles, is a parallelogram in shape. Its length along one street is $63\frac{1}{2}$ ft., and the shortest distance from that front to the side parallel to it is $98\frac{1}{2}$ ft. What is the area ?

28. A triangle, a square, a rectangle, and a parallelogram all have the same base and the same altitude. The side of the square is $4\frac{1}{4}$ in. What is the area of the square ? of the triangle ? of the rectangle ? of the parallelogram ?

111. Plastering, Painting, and Calcimining. The unit of plastering, painting, and calcimining is usually the square yard. A common business rule for estimating such work is as follows:

From the total area of the wall or ceiling deduct half the area of the doors, windows, and other openings, and express the result to the nearest square yard.

It would not be fair to deduct the full area of the openings because some allowance should be made for the difficulty of plastering or painting around them.

The rule varies in different places, and an exact agreement in advance is necessary when work is to be done. In some places it is the custom to deduct even the full area of the openings, while in others a certain number of square feet is allowed.

Technical rules of this nature need not be memorized. They should be referred to when necessary. Teachers will find it helpful to use local rules in matters of this kind.

In giving dimensions, architects and contractors write first the length, then the width, and then the height.

At 21¢ a square yard, find the cost of plastering a room $18' \times 16' \times 10'$, if the room has a baseboard 10 in. high, two doors $7' 4'' \times 4'$, and three windows $6' 6'' \times 4'$.

Perimeter of room	$= 2 \times (18' + 16') = 68'$	
Height above baseboard	$= 10' - 10'' = 9\frac{1}{2}'$	
Total wall area	$= 9\frac{1}{2} \times 68 \text{ sq. ft.}$	$= 623\frac{1}{2} \text{ sq. ft.}$
Area of ceiling	$= 18 \times 16 \text{ sq. ft.}$	$= 288 \text{ sq. ft.}$
Total area		$= 911\frac{1}{2} \text{ sq. ft.}$

$$\text{Height of doors above baseboard} = 7' 4'' - 10'' = 6' 6''$$

Area of 2 doors	$= 2 \times 4 \times 6\frac{1}{2} \text{ sq. ft.} = 52 \text{ sq. ft.}$	
Area of 3 windows	$= 3 \times 4 \times 6\frac{1}{2} \text{ sq. ft.} = 78 \text{ sq. ft.}$	
Total area of openings	$= 130 \text{ sq. ft.}$	
Half area of openings		$= 65 \text{ sq. ft.}$
Area to be paid for		$= 846\frac{1}{2} \text{ sq. ft.}$
		$= 94\frac{1}{27} \text{ sq. yd.}$

At 21¢ a square yard, 94 sq. yd. will cost $94 \times 21\text{¢} = \$19.74$.

EXERCISE 58

Find the number of square yards to be covered in ceilings having the following dimensions :

- | | | | |
|-----------------------|-----------------------|-----------------------|------------------------|
| 1. $17' \times 15'$. | 4. $16' \times 14'$. | 7. $25' \times 20'$. | 10. $32' \times 29'$. |
| 2. $18' \times 16'$. | 5. $17' \times 16'$. | 8. $32' \times 25'$. | 11. $34' \times 26'$. |
| 3. $15' \times 13'$. | 6. $20' \times 19'$. | 9. $35' \times 28'$. | 12. $37' \times 25'$. |

If the cost of plastering is 22¢ per square yard, find the cost of plastering ceilings having the following dimensions :

- | | | | |
|------------------------|------------------------|------------------------|------------------------|
| 13. $16' \times 15'$. | 16. $20' \times 18'$. | 19. $16' \times 16'$. | 22. $17' \times 14'$. |
| 14. $20' \times 15'$. | 17. $18' \times 15'$. | 20. $16' \times 13'$. | 23. $22' \times 20'$. |
| 15. $19' \times 15'$. | 18. $18' \times 14'$. | 21. $17' \times 13'$. | 24. $25' \times 22'$. |

Allowing $1\frac{1}{2}$ ¢ per square yard for labor and $\frac{1}{2}$ ¢ per square yard for material, find the cost of calcimining ceilings having the following dimensions :

- | | | | |
|------------------------|------------------------|------------------------|------------------------|
| 25. $13' \times 12'$. | 28. $16' \times 12'$. | 31. $20' \times 17'$. | 34. $35' \times 30'$. |
| 26. $14' \times 12'$. | 29. $19' \times 16'$. | 32. $25' \times 21'$. | 35. $40' \times 35'$. |
| 27. $15' \times 14'$. | 30. $21' \times 17'$. | 33. $30' \times 25'$. | 36. $50' \times 40'$. |

37. How much will it cost at $33\frac{1}{3}$ ¢ per square yard to plaster a room whose walls and ceiling have an area of 117 sq. yd., and have openings amounting to 9 sq. yd.?

38. A room is $18' \times 14' \times 9'$, and has a baseboard 12 in. high, two doors $7' \times 3' 6''$, and two windows $6' \times 3'$. How much will it cost to plaster the room at 30¢ per square yard?

39. A room is $22' \times 16' \times 10'$, and has a baseboard 10 in. high, three doors $7' 6'' \times 4'$, and four windows $6' 6'' \times 4'$. How much will it cost to plaster the room at 25¢ per square yard?

112. Carpeting Rooms. Carpet is usually either 27 in. or 1 yd. wide, and in estimating the amount needed for a room, allowance must be made for matching the pattern.

In allowing for the waste in matching the pattern, it is not necessary to allow for loss on the first strip, since the other strips are cut to match that one. In the following examples, unless otherwise directed, the strips are to run lengthwise of the room.

For example, how much will it cost to carpet a room $13\frac{1}{2}' \times 16'$, the strips to run lengthwise, and the carpet being plain color, 27 in. wide, and worth 90¢ a yard?

$$\begin{aligned} & 13\frac{1}{2} \text{ ft.} = 4\frac{1}{2} \text{ yd., the width,} \\ \text{and} & 16 \text{ ft.} = 5\frac{1}{3} \text{ yd., the length.} \\ & 4\frac{1}{2} \text{ yd.} \div \frac{2}{3} \text{ yd.} = 6, \text{ the number of strips.} \\ & 6 \times 5\frac{1}{3} \text{ yd.} = 32 \text{ yd.} \\ & 32 \times 90¢ = \$28.80. \end{aligned}$$

EXERCISE 59

Find the number of yards of plain carpet 27 in. wide that will be required for rooms of the following dimensions:

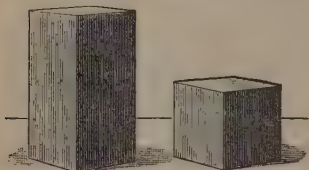
- | | | |
|-------------------|-------------------|-------------------|
| 1. 24 ft., 15 ft. | 4. 15 ft., 12 ft. | 7. 25 ft., 18 ft. |
| 2. 20 ft., 15 ft. | 5. 17 ft., 12 ft. | 8. 26 ft., 18 ft. |
| 3. 22 ft., 18 ft. | 6. 19 ft., 15 ft. | 9. 32 ft., 21 ft. |

10. What will it cost to carpet a room 19 ft. 8 in. long and 15 ft. wide with plain carpet 27 in. wide, at 75¢ a yard?

11. A floor 16 ft. wide is to be covered with 27-inch carpet, the strips running lengthwise. How many strips will be needed, and how much of the width of one strip will be lost by turning under?

12. What will it cost to cover a floor 18 ft. 6 in. long and 14 ft. 9 in. wide with carpet 1 yd. wide, worth \$0.85 a yard, if the strips run lengthwise and $\frac{1}{8}$ yd. is allowed on every strip except the first for matching the pattern?

113. Rectangular Solid. A solid that is bounded by six rectangles is called a *rectangular solid*.



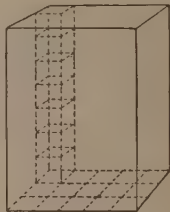
The six rectangles are called the *faces* of the solid, and their sides form the *edges* of the solid.

A rectangular solid has length, breadth, and thickness, and these are called the three *dimensions* of the solid.

114. Cube. A rectangular solid all of whose faces are squares is called a *cube*.

115. Volume. The number of cubic units a body contains is called its *volume*.

If this figure represents a rectangular solid 5 in. long, 3 in. wide, and 7 in. high, it is evident that in the column of cubes shown there are 7 cu. in. It is also evident that on the base we can place 3×5 such columns. Hence the volume is $3 \times 5 \times 7$ cu. in., or 105 cu. in. Hence, if the dimensions are all expressed in the same unit,



The volume of a rectangular solid equals the product of its three dimensions.

We read 2 ft. $\times$ 3 ft. $\times$ 4 ft. "two feet by three feet by four feet," and this is a proper method of expressing dimensions, but not of expressing multiplication. The multiplier always should be an abstract number, as in $3 \times 5 \times 7$ cu. in.

116. Cubic Measure. The common table of cubic measure is as follows:

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)

27 cubic feet = 1 cubic yard (cu. yd.)

128 cubic feet = 1 cord (cd.)

A cubic yard of earth is considered a load.

EXERCISE 60

Reduce to the next lower units :

- | | | | |
|---------------|----------------------------|----------------|-------------|
| 1. 2 cu. ft. | 4. 125 cu. ft. | 7. 25 cu. yd. | 10. 25 cd. |
| 2. 25 cu. ft. | 5. $33\frac{1}{3}$ cu. ft. | 8. 36 cu. yd. | 11. 125 cd. |
| 3. 75 cu. ft. | 6. $66\frac{2}{3}$ cu. ft. | 9. 125 cu. yd. | 12. 500 cd. |

Reduce to higher units :

- | | | |
|--|------------------|------------------|
| 13. 3456 cu. in. | 15. 2457 cu. ft. | 17. 2000 cu. in. |
| 14. 8640 cu. in. | 16. 4900 cu. ft. | 18. 4260 cu. in. |
| 19. Reduce 25 cu. ft. 65 cu. in. to cubic inches. | | |
| 20. Reduce 15 cu. yd. $10\frac{1}{2}$ cu. ft. to cubic feet. 2150.42 cu | | |
| 21. Reduce 1290 cu. ft. to cords ; to cubic yards. in 1 cu 16. | | |

Find the volumes of rectangular solids having the following dimensions :

- | | |
|---------------------------------|-------------------------------------|
| 22. 17", 19", 32". | 28. 124 ft., 6 ft., 1 ft. 9 in. |
| 23. 16", 22", 27". | 29. 135 ft., 5 ft., 2 ft. 2 in. |
| 24. 15", 16", 34". | 30. 275 ft., 4 ft., 1 ft. 10 in. |
| 25. 17", 19", 29". | 31. 2 ft. 6 in., 3 ft., 4 ft. 8 in. |
| 26. 15', 29', 12'. | 32. 6 ft. 2 in., 2 ft., 3 ft. 9 in. |
| 27. 127', 6', $1\frac{1}{2}$ '. | 33. 4 ft. 3 in., 7 ft., 6 ft. 2 in. |

34. In the school is an aquarium 16 in. long, $8\frac{1}{2}$ in. wide, and $9\frac{1}{2}$ in. deep. How many cubic inches does it contain ?

35. A cellar 24 ft. by 39 ft. by 6 ft. is to be excavated. How much will it cost at 40¢ a load (cubic yard) ?

36. A cellar $23\frac{1}{2}$ ft. by $32\frac{1}{2}$ ft. by $6\frac{1}{2}$ ft. is to be excavated. How much will it cost at $37\frac{1}{2}$ ¢ a load ?

37. The front of a school building is 40 ft. long, $30\frac{1}{2}$ ft. high, and $1\frac{1}{2}$ ft. thick. It is built of brick. Making no allowance for openings, how many cubic feet of masonry does it contain ?

38. What is the volume of a cube 16 in. on an edge?

39. How many cubic feet of air are there in a schoolroom 28 ft. long, 20 ft. wide, and 14 ft. high?

40. What is the total area of the six faces of a rectangular solid 8 in. by 7 in. by 5 in.?

41. What is the total area of the six faces of a rectangular solid 3 ft. 6 in. by 2 ft. 4 in. by 1 ft. 9 in.?



42. These children are measuring an excavation, and find it to be 6 ft. 2 in. wide, 8 ft. 4 in. long, and 4 ft. 3 in. deep. Assuming it to be practically rectangular, how many cubic feet of earth have been removed?

43. What is the weight of the water in an aquarium 1 ft. 4 in. long and 1 ft. wide, the water being 9 in. deep, if a cubic foot of water weighs $62\frac{1}{2}$ lb.?

44. What is the weight of a block of marble 4 ft. 6 in. long, 2 ft. wide, and 1 ft. 4 in. thick, marble being 2.7 times as heavy as water, and a cubic foot of water weighing $62\frac{1}{2}$ lb.?

45. A rectangular solid is 4 ft. 6 in. long and 2 ft. 8 in. wide, and the volume is 72 cu. ft. What is the height?

46. A rectangular solid is 6 ft. 4 in. long and 3 ft. 9 in. wide, and the volume is 285 cu. ft. What is the height?

47. A rectangular solid is 2 ft. 6 in. high and the volume is 240 cu. in. What is the area of the base?

117. Capacity of Bins. In finding the capacity of bins we seek either the number of cubic feet or the number of bushels. A bushel (bu.) contains 2150.42 cu. in., or approximately $\frac{5}{4}$ cu. ft., and 1 cu. ft. contains approximately $\frac{4}{5}$ of a bushel.

Therefore, *to find the approximate capacity, in bushels, of a bin, multiply $\frac{4}{5}$ bu. by the number of cubic feet in the bin.*

For example, how many bushels in a bin $10' \times 8' \times 5'$?

$$10 \times 8 \times 5 \times \frac{4}{5} \text{ bu.} = 320 \text{ bu.}$$

To find the approximate space, in cubic feet, required for a given number of bushels of grain, multiply $\frac{5}{4}$ of a cubic foot by the number of bushels.

For example, how many cubic feet will be required for 1000 bu.?

$$1000 \times \frac{5}{4} \text{ cu. ft.} = 1250 \text{ cu. ft.}$$

EXERCISE 61

Find the number of bushels in bins of the following dimensions :

1. $12' \times 6' \times 5'$. 4. $16' \times 10' \times 6'$. 7. $12\frac{1}{2}' \times 6' \times 4'$.

2. $12' \times 7' \times 6'$. 5. $18' \times 10' \times 8'$. 8. $15\frac{1}{2}' \times 8\frac{1}{2}' \times 5'$.

3. $15' \times 9' \times 5'$. 6. $18' \times 12' \times 7'$. 9. $16\frac{3}{4}' \times 9\frac{1}{4}' \times 5\frac{1}{2}'$.

Find the number of cubic feet in bins of the following capacities :

10. 600 bu. 14. 1625 bu. 18. 7500 bu. 22. 37,500 bu.

11. 850 bu. 15. 1860 bu. 19. 16,000 bu. 23. 66,960 bu.

12. 975 bu. 16. 3200 bu. 20. 22,320 bu. 24. 59,200 bu.

13. 1500 bu. 17. 5600 bu. 21. 28,000 bu. 25. 32,975 bu.

26. At 95¢ a bushel, how much is the wheat worth that fills a bin $10' \times 9' \times 4'$?

27. At \$1.02 a bushel, how much is the wheat worth that fills a bin $30' \times 20' \times 10'$?

Allowing 35 cu. ft. to a ton of hard coal, find the number of tons that can be placed in bins of the following dimensions :

- | | |
|----------------------------------|--|
| 28. $10' \times 7' \times 6'$. | 31. 9 ft. 6 in. $\times$ 6 ft. $\times$ 5 ft. |
| 29. $12' \times 10' \times 7'$. | 32. 8 ft. 4 in. $\times$ 7 ft. $\times$ 6 ft. |
| 30. $13' \times 7' \times 5'$. | 33. 10 ft. 6 in. $\times$ 6 ft. $\times$ 6 ft. |

Allowing 42 cu. ft. to a ton of soft coal, find the number of tons that can be placed in bins of the following dimensions :

- | | |
|---------------------------------|---|
| 34. $10' \times 9' \times 7'$. | 37. 8 ft. 4 in. $\times$ 7 ft. $\times$ 5 ft. |
| 35. $12' \times 8' \times 6'$. | 38. 9 ft. 2 in. $\times$ 8 ft. $\times$ 6 ft. |
| 36. $14' \times 7' \times 5'$. | 39. 7 ft. 3 in. $\times$ 9 ft. $\times$ 4 ft. |

40. Taking a bushel as $1\frac{1}{4}$ cu. ft., and a ton of hard coal as measuring 35 cu. ft., how much should a bushel of hard coal weigh?

41. Taking a ton of soft coal as measuring 42 cu. ft., how much should a bushel of soft coal weigh?

42. How many loads in a rectangular cut for a mill flume 300 ft. long, 15 ft. wide, and 10 ft. deep?

43. How many loads of sand in a gravel train of 24 cars, each car being $15' 6'' \times 5' 8''$, and the sand being 2' deep?

44. How many loads will be removed in preparing the ground for macadamizing a street 1500' long, 40' wide, and $2\frac{1}{2}'$ deep?

45. What is the value of the bricks in a pile 60 ft. long, 25 ft. wide, and 9 ft. high, each brick being 8 in. by 4 in. by 2 in., at \$9 a thousand?

46. Allowing 231 cu. in. to the gallon, how many gallons in a watering trough that is 6 ft. long and 16 in. wide, the depth being 0.75 of the width?

118. Circumference of a Circle. If we measure around and across any circular object, we shall find that the circumference is about $3\frac{1}{7}$ times the diameter. It is shown in geometry that 3.1416 is more nearly correct than $3\frac{1}{7}$.

It is sufficient for most ordinary purposes to use $3\frac{1}{7}$ instead of 3.1416.

Thus, if the diameter of a wheel is 7 ft., the circumference is $3\frac{1}{7} \times 7$ ft., or 22 ft. If we use the multiplier 3.1416, the result is 3.1416×7 ft., or 21.9912 ft.

We need to consider the circle in finding the capacity of cisterns. Little or no explanation should be required of the pupil in connection with §§ 118-120.



EXERCISE 62

Using $3\frac{1}{7}$, find the circumference, given the diameter :

- | | | | |
|-------------|-------------|--------------|--------------|
| 1. 105 ft. | 3. 25.9 ft. | 5. 0.364 in. | 7. 4970 in. |
| 2. 18.9 ft. | 4. 2.94 ft. | 6. 4270 in. | 8. 51.10 in. |

Using $3\frac{1}{7}$, find the circumference, given the radius :

- | | | | |
|------------|-------------|--------------|--------------|
| 9. 63 in. | 11. 98 in. | 13. 61.6 in. | 15. 85.4 ft. |
| 10. 77 in. | 12. 210 in. | 14. 68.6 in. | 16. 8.82 ft. |

Using 3.1416, find the circumference, given the diameter :

- | | | | |
|------------|-------------|--------------|--------------|
| 17. 27 ft. | 19. 32 yd. | 21. 0.62 ft. | 23. 2.25 ft. |
| 18. 43 ft. | 20. 4.7 yd. | 22. 0.75 ft. | 24. 1.27 in. |

Using 3.1416, find the circumference, given the radius :

- | | | | |
|------------|-------------|-------------|--------------|
| 25. 27 in. | 27. 61 in. | 29. 1.9 yd. | 31. 0.48 ft. |
| 26. 38 in. | 28. 4.8 ft. | 30. 2.7 yd. | 32. 0.62 ft. |

Using 3.1416, find the diameter, given the circumference :

- | | | |
|----------------|-----------------|-----------------|
| 33. 62.832 ft. | 34. 345.576 in. | 35. 502.656 in. |
|----------------|-----------------|-----------------|

119. Area of a Circle. If we know the circumference and the radius of a circle, we can find the area.

The area of a circle equals half the product of the numbers representing the circumference and radius.

It is necessary to give this rule at this time, on account of its use in finding the capacity of cisterns. Teachers who wish to give some proof of it may refer to page 370.

For example, if the radius is 2 in. and the circumference is 12.5664 in., the area is $\frac{1}{2}$ of 2×12.5664 sq. in., or 12.5664 sq. in.

120. Area in Terms of Radius. We can also find the area of a circle if we know merely the radius.

The area of a circle equals nearly 3.1416 times the square of the radius.

The *square* of a number is the product of the number multiplied by itself.

For example, if the radius is 7 in., the area is $3\frac{1}{7} \times 7 \times 7$ sq. in., or 154 sq. in.

Instead of 3.1416 we may use $3\frac{1}{7}$, which gives the area somewhat less exactly.

A proof of this rule is given on page 370, where it will be better understood by the pupil.

In Exercise 63, use 3.1416 instead of $3\frac{1}{7}$.

EXERCISE 63

1. Find the area of the circle with radius 1 in. and circumference 6.2832 in.

2. Find the area of the circle with radius 3 in. and circumference 18.8496 in.

3. Find the area of the circle with radius $2\frac{1}{2}$ in. and circumference 15.708 in.

4. Find the area of the circle with diameter 8 in. and circumference 25.1328 in.

5. Find the area of the circle of which the radius is 8 in.

6. Find the area of the circle of which the radius is 6.5 in.

Find the areas of circles with the following radii and circumferences:

- | | |
|-------------------------------------|-------------------------------------|
| 7. 5 in., 31.416 in. | 11. $5\frac{1}{2}$ in., 34.5576 in. |
| 8. $4\frac{1}{2}$ in., 28.2744 in. | 12. 10 in., 62.832 in. |
| 9. 4 in., 25.1328 in. | 13. 20 in., 125.664 in. |
| 10. $3\frac{1}{2}$ in., 21.9912 in. | 14. 30 in., 188.496 in. |

Find the areas of circles with the following radii:

- | | | | | |
|-----------|------------|-----------------------|-----------------------|------------------------|
| 15. 1 in. | 18. 5 in. | 21. $\frac{1}{2}$ in. | 24. $\frac{1}{4}$ in. | 27. $2\frac{1}{2}$ in. |
| 16. 4 in. | 19. 10 in. | 22. $\frac{1}{3}$ in. | 25. $\frac{3}{4}$ in. | 28. $3\frac{1}{4}$ in. |
| 17. 3 in. | 20. 20 in. | 23. 0.1 in. | 26. $\frac{2}{3}$ in. | 29. 2.3 in. |

30. What is the circumference of a water pipe that is 4 in. in diameter?

31. What is the area of a cross section of a water pipe that is 3 in. in diameter?

32. A water pipe is 4 in. in external diameter. If the iron is $\frac{1}{4}$ in. thick, what is the internal circumference?

33. A school globe is 15 in. in diameter. How long is the equator on the globe?

34. The equator on a school globe is 62.832 in. What is the radius of the globe?

35. What is the circumference of the largest circle that can be drawn in a square 10 ft. on a side?

36. What is the area inclosed between the circle and the sides of the square in Ex. 35?

37. What is the diameter of a circular plot of ground of which the circumference is 691.152 ft.? What is the area?

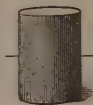
38. A pipe is 3.75 in. in external diameter. If the iron is $\frac{1}{4}$ in. thick, what is the area of an internal cross section?

39. A pipe has an internal diameter of 3.5 in. The iron is $\frac{1}{8}$ in. thick. What is the area of an external cross section?

121. Cylinder. A solid formed by the revolution of a rectangle about one of its sides is called a *cylinder*.

The two circles that are thus formed are called the *bases*, and the curved surface is called the *lateral surface* of the cylinder.

The distance between the two bases is called the *altitude* of the cylinder.



122. Volume of a Cylinder. We can think of 1 cu. in. on each square inch of the base. Therefore, if the base contains 4 sq. in., and if the cylinder is 1 in. high, the volume will be 4×1 cu. in., or 4 cu. in., and if the cylinder is 5 in. high, the volume will be 5×4 cu. in., or 20 cu. in.

Therefore, to find the volume of a cylinder,

Multiply the number of cubic units corresponding to the number of square units of the base, by the number of units of altitude.

That is, if the base has 9 sq. in., and the altitude is 12 in., the volume is 12×9 cu. in., or 108 cu. in.

If the base has a diameter of 2 in., and the altitude is 5 in., then the area of the base is 3.1416×1 sq. in. (§ 120), and the volume is $5 \times 3.1416 \times 1$ cu. in., or 15.708 cu. in.

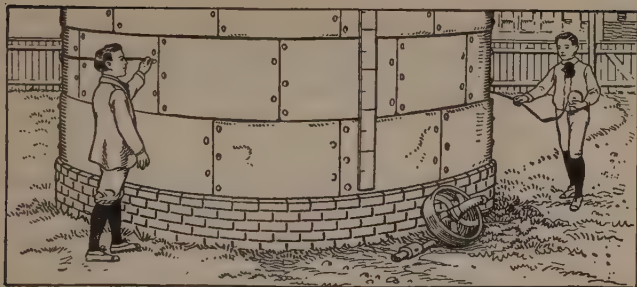
123. Lateral Area of a Cylinder. We may think of the surface of a cylinder as unrolled, like a paper reaching once around a pencil. In this case it would unfold into a rectangle, one side of which would be the circumference of the base and the other the altitude.

Therefore, to find the lateral area of a cylinder,

Multiply the number of units in the circumference by the number of units in the altitude.

For example, if the circumference is 6 in. and the altitude 4 in., the number of square units of lateral area is 4×6 , or 24. We may express the work thus :

$$4 \times 6 \text{ sq. in.} = 24 \text{ sq. in.}$$



EXERCISE 64

Find the volumes of cylinders whose altitudes and areas of base are as follows :

- | | |
|------------------------|--|
| 1. 13 in., 27 sq. in. | 4. 3 ft. 7 in., $8\frac{1}{2}$ sq. in. |
| 2. 17 in., 36 sq. in. | 5. 4 ft. 6 in., $7\frac{1}{2}$ sq. in. |
| 3. 38 in., 125 sq. in. | 6. 8 ft. 7 in., 4 sq. ft. 24 sq. in. |

Find the altitudes of cylinders whose volumes and areas of base are as follows :

7. 20 cu. ft. 324 cu. in., 8 sq. ft. 72 sq. in.
8. 190 cu. ft. 1305 cu. in., 30 sq. ft. 75 sq. in.

Find the areas of base of cylinders whose volumes and altitudes are as follows :

- | | |
|-------------------------|---|
| 9. 324 cu. ft., 18 ft. | 11. 28 cu. ft. 288 cu. in., 2 ft. 2 in. |
| 10. 323 cu. ft., 17 ft. | 12. 81 cu. ft. 576 cu. in., 5 ft. 4 in. |

Find the volumes of cylinders whose altitudes and radii of base are as follows :

- | | | |
|-------------------|-------------------|------------------------|
| 13. 27 in., 4 in. | 15. 32 in., 8 in. | 17. 2 ft. 6 in., 2 in. |
| 14. 25 in., 6 in. | 16. 36 in., 9 in. | 18. 3 ft. 4 in., 1 ft. |

124. Capacity of Cisterns. The following measurements are used in computing the capacity of cisterns:

$$\begin{aligned} 1 \text{ gal.} &= 231 \text{ cu. in.} \\ 1 \text{ cu. ft.} &= 7.48 \text{ gal.} = \text{nearly } 7\frac{1}{2} \text{ gal.} \\ 31\frac{1}{2} \text{ gal.} &= 1 \text{ bbl.} = \text{nearly } 4\frac{1}{5} \text{ cu. ft.} \end{aligned}$$

In Exercise 65 use $7\frac{1}{2}$ gal. for 1 cu. ft., $4\frac{1}{5}$ cu. ft. for 1 bbl., and $3\frac{1}{2}$ as the multiplier in cases requiring the area of base, given the diameter.

Find the number of gallons of water in a cylindrical tank 40 ft. in diameter, the water being 35 ft. deep.

$$\begin{aligned} \text{Radius of base} &= \frac{1}{2} \text{ of } 40 \text{ ft.} = 20 \text{ ft.} \\ \text{Area of base} &= 3\frac{1}{2} \times 20 \times 20 \text{ sq. ft. (§ 120).} \\ \text{Volume} &= 35 \times 3\frac{1}{2} \times 20 \times 20 \text{ cu. ft. (§ 122)} \\ &= 35 \times 3\frac{1}{2} \times 20 \times 20 \times 7\frac{1}{2} \text{ gal.} \\ &= 330,000 \text{ gal.} \end{aligned}$$

EXERCISE 65

Find the number of gallons in these rectangular tanks:

1. $4' \times 2' \times 1'$. 3. $6' \times 1\frac{1}{2}' \times \frac{3}{4}'$. 5. $12\frac{1}{2}' \times 12\frac{1}{2}' \times 6'$.
2. $5' \times 2' \times 1'$. 4. $6' \times 2\frac{1}{4}' \times 1'$. 6. $15\frac{1}{2}' \times 12\frac{1}{2}' \times 6\frac{1}{2}'$.

Find the number of barrels in the following tanks:

7. $12' \times 11' \times 9'$. 8. $15' \times 14' \times 8'$. 9. $15\frac{1}{2}' \times 12\frac{1}{2}' \times 5\frac{3}{4}'$.

Find the number of gallons in cylindrical tanks of the following diameter and depth of water respectively:

10. 6', 10'. 12. 16', 14'. 14. $48\frac{1}{2}'$, 56'.
11. 15', 12'. 13. 35', 40'. 15. $48\frac{3}{4}'$, $45\frac{1}{2}'$.

Find the number of gallons of water in wells of the following diameter and depth of water respectively:

16. 3 ft., 8 ft. 18. 4 ft., 8 ft. 20. $3\frac{3}{4}$ ft., $5\frac{1}{4}$ ft.
17. 3 ft., 6 ft. 19. $3\frac{1}{2}$ ft., 6 ft. 21. $3\frac{3}{4}$ ft., $8\frac{3}{4}$ ft.

125. Brick Work and Stone Work. The number of bricks required for walls is estimated by the thousand, and 22 bricks of ordinary size laid in mortar are allowed to each cubic foot of wall.

A brick of ordinary size is $2'' \times 4'' \times 8''$, and its volume is therefore 64 cu. in., or $\frac{1}{27}$ cu. ft. Allowing for mortar, however, it is reckoned as $\frac{1}{22}$ cu. ft., as stated.

Stone work is reckoned by the cubic foot or cubic yard, and sometimes by the perch.

A perch is $16\frac{1}{2}' \times 1\frac{1}{2}' \times 1'$, and therefore equals $24\frac{3}{4}$ cu. ft. Practically, however, it is taken as 25 cu. ft., although this varies.

In estimating the labor for brick and stone work, masons measure the outside of the walls, thus reckoning the corners twice. This is considered fair because of the extra trouble in laying a corner. Openings are generally not considered, although sometimes an allowance for half of the openings is made, the gain to the mason being offset by the extra labor in building around them.

In estimating the material they will need, however, masons consider the corners and openings. The custom varies in these matters.

For example, find the number of bricks required for a wall $42' \times 25'$ and 1 ft. thick.

$$42 \times 25 \times 1 \times 22 \text{ bricks} = 23,100 \text{ bricks.}$$

How many cubic feet should be allowed in estimating the labor on a cellar wall $40' \times 30'$, 6' high, and 2' thick?

Perimeter of cellar, outside the wall, is $2 \times (40 + 30)$ ft., or 140 ft. Therefore the number is $6 \times 2 \times 140$ cu. ft., or 1680 cu. ft.

EXERCISE 66

Find the number of cubic feet in walls of these dimensions:

- | | |
|-------------------------------|-------------------------------|
| 1. 62 ft., 4 ft., 1 ft. | 4. 75 ft., 6 ft., 2 ft. |
| 2. 78 ft., 5 ft., 1 ft. 6 in. | 5. 82 ft., 6 ft., 2 ft. 2 in. |
| 3. 49 ft., 5 ft., 1 ft. 8 in. | 6. 96 ft., 6 ft. 6 in., 2 ft. |

Find the number of cubic yards in walls having these dimensions:

7. 4 rd., 6 ft., 15 in.

9. 4 rd., 5 ft., 18 in.

8. 120 ft., 2 yd., 1 ft.

10. 150 ft., 6 ft., 1 ft. 6 in.

11. At \$8 a thousand, what will the bricks cost for a wall 120 ft. long, 5 ft. high, and 1 ft. thick?



12. These children measure this house and find it to be 30 ft. by 20 ft. They learn that the brick walls are to be 20 ft. high and 1 ft. thick, and the openings are to amount to 250 cu. ft. How many bricks will be required? How much will they cost at \$8.25 a thousand?

13. How many cubic feet of stone in the cellar walls of a house $42' \times 25'$, if the walls are 18" thick and 6' high? How many perches of 25 cu. ft. each? What will it cost at \$9 a perch?

14. How many bricks will be required for the walls of a flat-roofed house $42' \times 36' \times 21'$, if the walls are 1 ft. thick and if 400 cu. ft. is deducted for doors and windows? How much will they cost at \$8.30 a thousand?

15. How many bricks will be required for the walls of a flat-roofed building $100' \times 60' \times 22'$, if the walls are $1\frac{1}{2}$ ft. thick, and if 800 cu. ft. is deducted for doors and windows?

126. Weight. The common weight used in the United States is called *avoirdupois weight*.

The table is as follows:

16 ounces (oz.) = 1 pound (lb.)

2000 pounds = 1 ton (T.)

A barrel of flour weighs 196 lb. (Memorize.)

100 lb. is sometimes called a *hundredweight* (cwt.). The ton of 2000 lb. is sometimes called the *short ton*, there being an old ton of 2240 lb., known as the *long ton*, which is used in the customhouse and in some wholesale transactions in mining products. There is also the *long hundredweight* of 112 lb., but it is rarely used except in English trade. A ton of loose hay fills about $8 \times 8 \times 8$ cu. ft., or 512 cu. ft., of space, but this varies greatly with the condition of the hay. A keg of nails, a cental of corn, or a quintal of fish weighs 100 lb. A barrel of salt weighs 280 lb., and a barrel of pork or of beef, 200 lb. A cask of lime weighs 240 lb. A cubic foot of water weighs $62\frac{1}{2}$ lb.

The laws of the several states specify the weight of a bushel of various products. In New York the following are the standards:

Barley . . .	48 lb.	Oats	32 lb.
Buckwheat	48 "	Potatoes	60 "
Corn (shelled)	56 "	Wheat	60 "

These weights should be memorized.

A *carat* weight, used in weighing diamonds, is commonly taken in the United States as 3.2 grains. The word *carat*, meaning $\frac{1}{4}$, is used also in speaking of the purity of gold. Hence 18 carats fine means $\frac{18}{24}$ pure gold and $\frac{6}{24}$ alloy.

EXERCISE 67

- Express 19,000 lb. as tons.
- Express 255 T. as pounds.
- What is the weight of 25 barrels of salt?
- What is the weight of 250 bu. of barley?
- What is the weight of 750 bu. of wheat?
- What is the weight of 1250 bu. of oats?
- What is the weight of 1675 bu. of potatoes?

8. What do 100 bu. of buckwheat weigh? What do 275 bu. weigh?

9. What do 100 bu. of barley weigh? What do 25 bu. weigh? What do 2500 bu. weigh?

10. What do 100 bu. of shelled corn weigh? What do 25 bu. weigh? What do 2500 bu. weigh?

11. What do 25 bu. of oats weigh? What do 250 bu. weigh? What do 2500 bu. weigh?

12. How many tons to 1000 bu. of wheat? of oats?

13. How many bushels of potatoes to the ton?

14. How many bushels of oats to the ton?

15. The weight of a bushel of shelled corn is what part of the weight of a bushel of wheat? of a bushel of potatoes?

16. Barley is what fraction heavier than oats? Oats are what fraction lighter than barley?

17. What fraction more does 1 bu. of shelled corn weigh than 1 bu. of barley? than 1 bu. of buckwheat?

18. Buckwheat is what fraction heavier than oats? Oats are what fraction lighter than buckwheat?

19. Potatoes are what fraction heavier than shelled corn? Barley is what fraction lighter than shelled corn?

20. Barley is what fraction lighter than wheat? Oats are what fraction lighter than wheat?

21. At \$14.50 a ton how much will 25 tons of hay cost?

22. At \$15.75 a ton how much will 62.5 tons of hay cost?

23. At \$15.80 a ton how much will $48\frac{3}{4}$ tons of hay cost?

24. If $9\frac{1}{2}$ tons of hay cost \$139.65, what is the price per ton?

25. Taking the weight of 1 cu. ft. of water as $62\frac{1}{2}$ lb., what is the weight of 6 cu. ft. of water? of 70 cu. ft.?



26. In this load of hay there are 2150 lb. Express this as tons and a common fraction.

27. If there were 1850 lb. in the load, what decimal of a ton would there be?

Reduce to the next lower units:

28. 16 lb. 29. $2\frac{1}{2}$ lb. 30. 16 T. 31. $28\frac{1}{2}$ T.

Reduce to higher units:

32. 176 oz. 33. 200 oz. 34. 32,000 oz. 35. 10,000 lb.

Reduce to ounces:

36. $29\frac{3}{4}$ lb. 37. $56\frac{5}{8}$ lb. 38. $54\frac{3}{8}$ lb. 39. 27 lb. 5 oz.

Reduce to pounds:

40. 27 T. 41. 2.7 T. 42. 144 oz. 43. 3 T. 700 lb.

44. What is the weight of the water in an aquarium 1 ft. 6 in. long and 1 ft. 4 in. wide, the water being $9\frac{1}{2}$ in. deep, if a cubic foot of water weighs $62\frac{1}{2}$ lb.?

45. What is the weight of a block of stone 3 ft. 4 in. long, 3 ft. 1 in. wide, and 1 ft. 2 in. thick, the stone being 2.7 times as heavy as water?

46. What is the weight of $\frac{1}{2}$ cu. ft. of water? of 0.1 cu. ft.?
of 0.8 cu. ft.?

47. What part of 1 cu. ft. of water weighs $12\frac{1}{2}$ lb.? 25 lb.?
50 lb.? $31\frac{1}{4}$ lb.?

48. Silver is about ten times as heavy as water. About
how much does 1 cu. ft. of silver weigh? 3 cu. ft.?

49. How much does 864 cu. in. of water weigh? 432 cu. in.?
216 cu. in.?

50. Water is half as heavy as sulphur. How much does
1 cu. ft. of sulphur weigh?

51. The weight of cork is $\frac{6}{25}$ of the weight of an equal
volume of water. How much does 1 cu. ft. of cork weigh?

52. A certain quality of gold is 19.3 times as heavy as
water. How much does 1 cu. ft. weigh?

53. If silver is $10\frac{1}{2}$ times as heavy as water, how much is
1 cu. ft. of silver worth at 50¢ an ounce?

54. If good milk is 0.03 heavier than water, what is the
weight of a can of such milk containing 1 cu. ft., the can
itself weighing 5 lb. 10 oz.?

55. Ice is 0.92 as heavy as water. I wish 100 lb. of ice
to fill my refrigerator. If the iceman sends me 2 cu. ft., he
sends me how many pounds too much?

56. A man building a house wishes to put a tank for water
in his attic. The tank is to be 5 ft. long, 4 ft. wide, and is to
allow for $3\frac{1}{2}$ ft. depth of water. Allowing 600 lb. for the
weight of the tank, how much will it weigh when full?

57. A tree sparrow eats $\frac{1}{4}$ oz. of weed seed a day. Allow-
ing 10 birds to 1 sq. mi., and supposing them to average 200
days a year in the state of New York, which has an area of
49,170 sq. mi., how many tons of weed seed are consumed
a year by these birds alone?



127. Dry Measure. The common table of dry measure is as follows:

2 pints (pt.) = 1 quart (qt.)

8 quarts = 1 peck (pk.)

4 pecks = 1 bushel (bu.)

A bushel, as we have learned, contains 2150.42 cu. in., or about $1\frac{1}{4}$ cu. ft. A dry quart contains 67.2 cu. in., while the liquid quart contains only 57.75 cu. in. In measuring grain, seeds, and small fruits the measure must be *even* full. In measuring apples, potatoes, and other large articles the measure must be *heaping* full.

The various states prescribe the weight of certain farm products per bushel. Some of these weights have been given on page 125.

EXERCISE 68

Reduce to the next lower units:

- | | | | |
|------------|-------------------------|--------------------------|--------------------------|
| 1. 37 qt. | 5. $16\frac{1}{2}$ qt. | 9. 48 pk. | 13. 47 bu. |
| 2. 49 qt. | 6. $37\frac{1}{2}$ qt. | 10. 59 pk. | 14. $56\frac{1}{2}$ bu. |
| 3. 99 qt. | 7. $87\frac{1}{2}$ qt. | 11. $67\frac{1}{4}$ pk. | 15. $92\frac{3}{4}$ bu. |
| 4. 256 qt. | 8. $248\frac{1}{2}$ qt. | 12. $268\frac{3}{4}$ pk. | 16. $427\frac{1}{4}$ bu. |

Reduce to higher units:

- | | | | |
|------------|------------|------------|-------------|
| 17. 72 pt. | 20. 64 qt. | 23. 96 pk. | 26. 156 pt. |
| 18. 76 pt. | 21. 60 qt. | 24. 89 pk. | 27. 156 qt. |
| 19. 98 pt. | 22. 98 qt. | 25. 75 pk. | 28. 156 pk. |

Reduce to pints:

- | | | | |
|-------------------------|-------------------------|-------------------------|-----------------|
| 29. 72 qt. | 32. 49 pk. | 35. 2 bu. | 38. 2 pk. 3 qt. |
| 30. $85\frac{1}{2}$ qt. | 33. $53\frac{1}{2}$ pk. | 36. 35 bu. | 39. 5 pk. 7 qt. |
| 31. $68\frac{1}{2}$ qt. | 34. $75\frac{3}{4}$ pk. | 37. $28\frac{1}{2}$ bu. | 40. 2 bu. 1 pk. |

Reduce to quarts:

- | | | | |
|-------------------------|-------------------------|-------------------------|-----------------|
| 41. 38 pk. | 43. $59\frac{3}{4}$ pk. | 45. 75 bu. | 47. 2 bu. 3 pk. |
| 42. $48\frac{1}{2}$ pk. | 44. $78\frac{1}{8}$ pk. | 46. $87\frac{1}{2}$ bu. | 48. 9 bu. 1 pk. |

Reduce to pecks:

- | | | | |
|------------|-------------------------|-------------|-------------|
| 49. 72 bu. | 50. $89\frac{1}{2}$ bu. | 51. 144 qt. | 52. 576 pt. |
|------------|-------------------------|-------------|-------------|

Reduce to bushels:

- | | | | |
|-------------|-------------|--------------|--------------|
| 53. 336 pk. | 55. 772 pk. | 57. 352 qt. | 59. 1984 pt. |
| 54. 516 pk. | 56. 956 pk. | 58. 1952 qt. | 60. 6336 pt. |

61. A bushel contains 2150.42 cu. in. How many bushels of wheat can be put in a bin containing 645,126 cu. in.?

62. Allowing $1\frac{1}{4}$ cu. ft. to a bushel, how many bushels of wheat can be put in a bin containing 900 cu. ft.?

63. If a bushel of wheat weighs 60 lb., what will be the weight of the wheat in Ex. 62?

64. A dry quart contains 67.2 cu. in. How many quarts in a measure containing 336 cu. in.?

65. If a bushel of oats weighs 32 lb., what will be the weight of the oats required to fill a bin that contains 875 cu. ft., allowing $1\frac{1}{4}$ cu. ft. to a bushel?

66. There are 231 cu. in. in a gallon and 537.6 cu. in. in a peck. From these measures verify the number of cubic inches in a liquid quart and in a dry quart as given in § 127 below the table.



128. Liquid Measure. The common table of liquid measure is as follows :

4 gills (gi.) = 1 pint (pt.)

2 pints = 1 quart (qt.)

4 quarts = 1 gallon (gal.)

These measures also are in common use :

$31\frac{1}{2}$ gallons = 1 barrel (bbl.)

63 gallons = 1 hogshead (hhd.)

Casks holding from 28 gal. to 43 gal. are called barrels, and casks holding from 54 gal. to 63 gal. are called hogsheads. Whenever barrels or hogsheads are used as *measures*, a barrel means $31\frac{1}{2}$ gallons, and a hogshead means 63 gallons. A barrel contains about $4\frac{1}{2}$ cu. ft. A gallon of water weighs about 8.3 lb., and contains 231 cu. in.

EXERCISE 69

Reduce to the next lower units :

- | | | | |
|-----------|-----------|------------|-------------------------|
| 1. 27 pt. | 3. 69 qt. | 5. 89 gal. | 7. $64\frac{1}{4}$ gal. |
| 2. 46 pt. | 4. 87 qt. | 6. 68 gal. | 8. $62\frac{1}{2}$ qt. |

Reduce to higher units :

- | | | | |
|------------|------------|-------------|-------------------------|
| 9. 96 gi. | 11. 35 gi. | 13. 300 qt. | 15. $92\frac{1}{2}$ qt. |
| 10. 96 pt. | 12. 85 pt. | 14. 475 pt. | 16. $75\frac{1}{2}$ pt. |

Reduce to pints :

- | | | | |
|-------------|-------------|--------------------------|--------------------------|
| 17. 396 gi. | 18. 892 gi. | 19. $126\frac{1}{2}$ qt. | 20. $79\frac{1}{2}$ gal. |
|-------------|-------------|--------------------------|--------------------------|

Reduce to quarts :

21. 792 gi. 22. 298 pt. 23. $926\frac{3}{4}$ gal. 24. 72 gal. 3 qt.

The following examples review liquid measures and other measures already studied.

Reduce to the next lower units :

25. 64 lb. 27. $9\frac{3}{4}$ lb. 29. 96 T. 31. $48\frac{3}{4}$ T.
26. $5\frac{1}{2}$ cu. ft. 28. $7\frac{1}{2}$ sq. ft. 30. $6\frac{1}{2}$ sq. rd. 32. $2\frac{1}{2}$ A.

Reduce to higher units :

33. 528 oz. 35. 375 oz. 37. 35,000 oz. 39. 12,500 lb.
34. 624 oz. 36. 450 oz. 38. 32,176 oz. 40. 75,550 lb.

Reduce to ounces :

41. $\frac{3}{4}$ lb. 43. $72\frac{3}{8}$ lb. 45. $96\frac{7}{8}$ lb. 47. 1 T. 200 lb.
42. $\frac{7}{32}$ lb. 44. $29\frac{5}{32}$ lb. 46. 29.75 lb. 48. 9 lb. 13 oz.

Reduce to pounds :

49. $42\frac{3}{4}$ T. 51. 3.6 T. 53. 432 oz. 55. 2 T. 800 lb.
50. $65\frac{4}{5}$ T. 52. $3.3\frac{1}{3}$ T. 54. 4832 oz. 56. 5 T. 650 lb.

57. How many pounds in $26\frac{1}{2}$ long tons ?

58. How many pounds in 1750 hundredweight ?

59. How many more pounds in 86 long hundredweight than in 86 common hundredweight ?

60. Allowing $7\frac{1}{2}$ gal. to a cubic foot, and taking the weight of a cubic foot of water as $62\frac{1}{2}$ lb., what is the weight of the water needed to fill a tank containing 75,000 gal. ?

61. Taking the weight of a cubic foot of water as $62\frac{1}{2}$ lb., and ice as being 0.92 as heavy as water, should we buy a piece of ice of 50 lb., or one piece of 50 lb. and another of 25 lb., in order to fill approximately an ice chamber that is 12 in. by 12 in. by 18 in. ?

62. Taking the weight of 1 gal. of water as 8.3 lb., what is the weight of 25 gal. of water? of 48 gal.?

63. How much does $\frac{1}{2}$ gal. of water weigh? 1 pt. of water? 3 qt. of water? 2.5 gal. of water?

64. If a 3-gallon pail weighs 3 lb., how much will it weigh when full of water? when half full? when 0.75 full?

65. How much does 125 gal. of water weigh? 355 gal.? 45.8 gal.? 1500 gal.?

66. How much does 1 bbl. (of $31\frac{1}{2}$ gal.) of water weigh?

67. Milk being 0.03 heavier than water, how much does it weigh to the gallon? the quart? the pint?

68. Alcohol being 0.79 as heavy as water, how much does it weigh per gallon? per barrel of $31\frac{1}{2}$ gal.?

69. Petroleum being 0.7 as heavy as water, how much does it weigh per gallon? per barrel of $31\frac{1}{2}$ gal.?

70. Copper being 8.95 times as heavy as water, and a cubic foot of water weighing $62\frac{1}{2}$ lb., how much does 1 cu. ft. of copper weigh? also 231 cu. in.?

71. Turpentine being 0.01 lighter than water, how much does it weigh per gallon? per barrel of $31\frac{1}{2}$ gal.?

72. A tank 8 ft. long, 6 ft. wide, and 4 ft. deep contains how many cubic feet? how many cubic inches? how many gallons?

73. A tank 8 ft. long, 5.5 ft. wide, and 3 ft. deep contains how many cubic feet? how many cubic inches? how many gallons?

74. A tank 7 ft. long by $5\frac{1}{2}$ ft. wide is filled with water to the depth of 3 ft. How many gallons of water in it?

75. The water of the Dead Sea is so salty that it is $1\frac{6}{5}$ times as heavy as fresh water. How much does 1 gal. of Dead Sea water weigh?

129. Equivalent Measures. In cooking, the following equivalents are often used:

4 teaspoons	= 1 tablespoon, often written	$4\ t = 1\ T$;
16 tablespoons	= 1 cup,	often written $16\ T = 1\ C$;
2 cups	= 1 pint,	often written $2\ C = 1\ pt.$

EXERCISE 70

1. Given the number of cups of anything, how do you find the equivalent number of pints?
2. Given a certain fraction of a cup of anything, how do you find the equivalent number of tablespoons?
3. Given the number of pints of anything, how do you find the equivalent number of cups?
4. Given the number of tablespoons of anything, how do you find the equivalent number of cups?
5. Given the number of teaspoons of anything, how do you find the equivalent number of tablespoons?
6. Given a certain fraction of a tablespoon, how do you find the equivalent number of teaspoons?
7. Given a recipe, how do you make a certain fractional part of it?
8. Given a recipe, how do you double it or make any given number of times the amount?
9. How many cups will you allow to 2 pt.? to 4 pt.?
10. How many cups will you allow to $1\frac{1}{2}$ pt.? to 1 qt.?
to $1\frac{1}{2}$ qt.? to $1\frac{1}{4}$ qt.?
11. How many tablespoons will you allow to $\frac{3}{4}$ of a cup?
to $1\frac{1}{2}$ cups? to $1\frac{1}{4}$ cups? to $1\frac{3}{4}$ cups?
12. How many teaspoons will you allow to $\frac{1}{8}$ of a cup?
to $\frac{3}{4}$ of a cup? to 4 tablespoons? to 8 tablespoons? to $\frac{1}{2}$
of a tablespoon?

130. English Money. The following is the table of English money :

- 4 farthings = 1 penny (d.)
 12 pence = 1 shilling (s. or /)
 20 shillings = 1 pound or sovereign (£)

The *florin* is 2 shillings and the *crown* is 5 shillings. The half crown is therefore 2 shillings 6 pence, commonly called "two and six."

The value of the pound in our money is \$4.866 + ; of the shilling, 24.3¢ + ; of the penny, about 2¢. We commonly think of the pound as about \$5, and the shilling as about 25¢.

We write 2 pounds 4 shillings 3 pence thus : £2 4s. 3d. It is customary to write 2 shillings 6 pence thus : 2/6.

EXERCISE 71

Express as pence :

- | | | |
|--------|------------|-----------------|
| 1. £1. | 4. £1 2s. | 7. £1 2s. 6d. |
| 2. £2. | 5. £1 9s. | 8. £3 9s. 5d. |
| 3. 5s. | 6. £2 16s. | 9. £4 12s. 4½d. |

Express as shillings :

- | | | |
|---------|-------------|-----------|
| 10. £2. | 13. £1 10s. | 16. 48d. |
| 11. £5. | 14. £3 15s. | 17. 96d. |
| 12. £7 | 15. £6 19s. | 18. 240d. |

Express as pounds :

- | | | |
|-----------|------------|------------|
| 19. 60s. | 22. 240d. | 25. 700s. |
| 20. 140s. | 23. 480d. | 26. 720d. |
| 21. 480s. | 24. 1200d. | 27. 1500s. |

Taking \$4.87 to the pound, express as dollars :

- | | | |
|----------|----------|-----------|
| 28. £48. | 29. £75. | 30. £475. |
|----------|----------|-----------|

Taking \$4.87 to the pound, express as pounds :

- | | | |
|--------------|--------------|---------------|
| 31. \$24.35. | 32. \$34.09. | 33. \$535.70. |
|--------------|--------------|---------------|

131. French Money. The French table of money is as follows:

$$100 \text{ centimes (c.)} = 1 \text{ franc (fr.)} = \$0.193$$

We usually think of the franc as about 20¢, and the 5-franc piece as about \$1. This system is used in several other countries besides France. In Italy, for example, 100 centesimi = 1 lira, the lira having the same value as the franc.

132. German Money. The German table of money is as follows:

$$100 \text{ pfennigs (pf.)} = 1 \text{ mark (M.)} = \$0.238$$

We usually think of 4 pf. as 1¢, and 1 M. as about 25¢.

EXERCISE 72

Express as francs and decimals:

1. 850 c. 2. 375 c. 3. 480 c. 4. 1250 c.

Express as marks and decimals:

5. 400 pf. 6. 175 pf. 7. 1600 pf. 8. 9275 pf.

9. What part of a franc is 75 centimes?

10. What part of a mark is 75 pfennigs?

11. Taking the franc as 19.3¢, this is what part of our quarter?

12. At the same rate, the 5-franc piece is what part of our dollar?

13. Taking the mark as 23.8¢, this is what part of our quarter?

14. At the same rate, 4 marks is what part of our dollar?

15. Find $\frac{1}{4}$ of 75 francs 40 centimes. (This may be written 75.40 francs.)

16. Find 0.06 of 25 marks 50 pfennigs. (This may be written 25.50 marks.)

133. Time Measure. The table of time is as follows:

- ✓ **60 seconds (sec.) = 1 minute (min.)**
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days = 1 common year (yr.)
366 days = 1 leap year
100 years = 1 century

There are 12 months (mo.) or about 52 weeks in a year. The names of the months, called calendar months, are:

	da.		da.
1. January (Jan.)	31	7. July	31
2. February (Feb.)	28 or 29	8. August (Aug.)	31
3. March (Mar.)	31	9. September (Sept.) . . .	30
4. April (Apr.)	30	10. October (Oct.)	31
5. May	31	11. November (Nov.) . . .	30
6. June	30	12. December (Dec.) . . .	31

The number of days in each month may be easily remembered by committing to memory the following lines:

"Thirty days hath September,
 April, June, and November;
 All the rest have thirty-one,
 Except the second month alone,
 Which has but twenty-eight, in fine,
 Till leap year gives it twenty-nine."

We usually write 15 minutes after 10 in the forenoon thus: 10 hr. 15 min. A.M., 10.15 A.M., or 10 15 A.M. We usually write 20 minutes before 3 in the afternoon thus: 2 hr. 40 min. P.M., 2.40 P.M., or 2 40 P.M.

A *year* is the time in which the earth performs one revolution in its orbit around the sun, and consists of 365 da. 5 hr. 48 min. 46 sec.; that is, nearly $365\frac{1}{4}$ days. As there are 365 days in a common year, a common year lacks nearly $\frac{1}{4}$ of a day of being a solar year, and this defect is made up by reckoning for some years (leap years) 366 days.

The leap years are the years whose dates are exactly divisible by 4; except in the case of complete hundreds, and these are leap years if exactly divisible by 400. Thus, 1600 and 1904 were leap years; 1800, 1900, and 1911 were not; 1928 and 1940 will be.

EXERCISE 73

Reduce to the next lower units :

- | | | | |
|-------------|------------------------|------------------------|------------|
| 1. 360 min. | 4. 16 da. | 7. 52 wk. | 10. 8 yr. |
| 2. 18 hr. | 5. 36 da. | 8. 26 wk. | 11. 16 yr. |
| 3. 36 hr. | 6. $48\frac{1}{2}$ da. | 9. $46\frac{3}{7}$ wk. | 12. 32 yr. |

Reduce to higher units :

- | | | | |
|--------------|-------------|--------------------------|-------------|
| 13. 360 sec. | 15. 504 hr. | 17. 132 mo. | 19. 25 yr. |
| 14. 720 min. | 16. 168 hr. | 18. $182\frac{5}{8}$ da. | 20. 365 da. |
21. How many days from March 5 to June 5 ?
22. How many days from June 10 to November 10 ?
23. Allowing $365\frac{1}{4}$ da. to the year, how many days in half a century ?
24. Allowing $365\frac{1}{4}$ da. to the year, how many years in 182,625 da. ?
25. Name the leap years in the following: 1925, 1926, 1927, 1928, 1929, 1930, 1940, 1950, 1960, 1972, 1982, 2000.
26. January 1, 1914, being Thursday, on what day of the week does January 1, 1916, fall ?
27. Reduce 15 hr. 32 min. 17 sec. to seconds.
28. Reduce 2 da. 7 hr. 29 min. to minutes.
29. Reduce 8 wk. 5 da. 2 hr. 3 min. to minutes.
30. How many days will the month of February, 1948, contain ? How many hours ?
31. A lunar month (the time from one new moon to the next one) contains 2,551,443 seconds. Express a lunar month in units of higher denominations.
32. A man received \$2.75 a day every week day during March, in a year when that month had five Sundays. How much did he receive during the month ?

33. If it takes 45 min. to bake potatoes and $\frac{2}{3}$ as long to boil them, how many minutes does it take to boil them?

34. If 9 min. for each pound is allowed for roasting beef, how long will it take to roast a piece weighing $7\frac{1}{4}$ lb.?

35. If 12 min. for each pound is allowed for roasting mutton, how long will it take for a piece weighing $6\frac{3}{4}$ lb.?

36. If 20 min. for each pound is allowed for roasting veal, how long will it take to roast a piece weighing $8\frac{3}{4}$ lb.?

37. If bread rises from 7 P.M. to 7 A.M. at the first rising, and 1 hr. at the second rising, how long does it take for both risings?

38. If it takes bread 55 min. to bake and it takes rolls 25 min. to bake, rolls will bake in what fraction of the time required for bread?

39. If quick-process bread is set for its first rising at 8 A.M. and rises $3\frac{3}{4}$ hr., and if 15 min. is allowed for getting it ready for the second rising, and it then rises 1 hr. 5 min. and is baked for 50 min., at what time will it be finished?

40. When should a chicken, which needs 1 hr. 30 min. for roasting, be put into the oven to be ready to serve at 6 P.M.?

41. At what time should peas begin to boil, if they should boil 20 min. and be served at 5.50 P.M.?

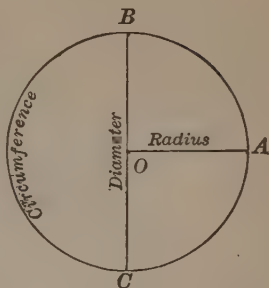
42. If we allow 15 min. for each pound in roasting a chicken, how much time should we allow for a chicken weighing $5\frac{3}{4}$ lb.?

43. If a class in cooking meets from 1.30 to 2.40 P.M., and is to make a cake that must bake 40 min., when must the cake be ready for the oven in order to be done 10 min. before the close of the class period?

44. How many minutes from 9.45 A.M. to 11.25 A.M.?
to noon? to 1.15 P.M.?

134. Measuring Arcs. The circumference of every circle is divided into 360 equal parts called *degrees*.

Corresponding to every arc, like AB in the figure, is an angle at the center, AOB . Therefore to every degree on the circumference (*arc degree*) there is a degree at the center (*angle degree*), and an arc has as many degrees as its angle.



135. Right Angle and Perpendicular. The angle at the center of a circle corresponding to one fourth of a circumference is called a *right angle*, and the two lines that form a right angle are said to be *perpendicular* to each other.

Thus angle AOB is a right angle, and OB is perpendicular to OA . A right angle, therefore, contains $\frac{1}{4}$ of 360 degrees, or 90 degrees.

An angle less than a right angle is called an *acute angle*, and one greater than a right angle is called an *obtuse angle*.

136. Angle Measure and Arc Measure. The following are the tables of circular measure :

Angle Measure

$$\begin{aligned} 60 \text{ seconds (")} &= 1 \text{ minute (')} \\ 60 \text{ minutes} &= 1 \text{ degree (}^\circ\text{)} \\ 360 \text{ degrees} &= 4 \text{ right angles} \end{aligned}$$

Arc Measure

$$\begin{aligned} 60 \text{ seconds (")} &= 1 \text{ minute (')} \\ 60 \text{ minutes} &= 1 \text{ degree (}^\circ\text{)} \\ 360 \text{ degrees} &= 1 \text{ circumference} \end{aligned}$$

That is, angle measures and arc measures are practically the same.

A degree of the circumference of the earth at the equator contains 60 geographical miles, or 69.16 statute miles.

The teacher should explain the relation of this table to the study of the latitude and longitude of places represented on the map.

EXERCISE 74

Express as seconds :

1. $17'$. 4. 1° . 7. $3^\circ 5'$. 10. $9^\circ 6' 2''$. 13. 90° .
 2. $32'$. 5. 3° . 8. $4^\circ 8'$. 11. $15^\circ 10' 30''$. 14. 45° .
 3. $48'$. 6. 27° . 9. $8^\circ 5' 8''$. 12. $18^\circ 20' 35''$. 15. $30^\circ 30' 30''$.

Express as minutes :

16. $480''$. 19. $900''$. 22. 27° . 25. $9^\circ 9'$. 28. $27^\circ 56'$.
 17. $560''$. 20. $360''$. 23. 35° . 26. $10^\circ 10'$. 29. $48^\circ 50'$.
 18. $420''$. 21. $240''$. 24. 45° . 27. $35^\circ 30'$. 30. $59^\circ 20'$.

Express as degrees :

31. $4800''$. 33. $1800''$. 35. $2700'$. 37. $50,400''$.
 32. $3600''$. 34. $9600''$. 36. $5400'$. 38. $108,000''$.

39. How many degrees in $\frac{1}{6}$ of a circumference? Draw a circumference and divide it approximately into sixths.

40. How many degrees in $\frac{1}{2}$ of a circumference? in $\frac{1}{4}$ of a circumference? Draw the figures.

41. There are 360° in the equator. How many minutes?

42. There are 90° from the north pole to the equator. How many minutes are there? How many seconds?

43. The circumference of a sailor's compass is divided into thirty-two equal parts. How many degrees are there in each part?

44. How many degrees in $\frac{1}{8}$ of the circumference of the earth? in $\frac{1}{16}$ of the circumference?

45. There are $23\frac{1}{2}^\circ$ from the north pole to the arctic circle. How many minutes?

46. There are $23\frac{1}{2}^\circ$ from the equator to the Tropic of Cancer. How many degrees from the arctic circle to the Tropic of Cancer?

137. Miscellaneous Measures. The following tables are used:

<i>Numbers</i>	<i>Paper</i>
12 units = 1 dozen (doz.)	24 sheets = 1 quire
12 dozen = 1 gross (gro.)	20 quires = 1 ream
12 gross = 1 great gross	2 reams = 1 bundle
20 units = 1 score	5 bundles = 1 bale

The great gross and score are becoming obsolete. For convenience in counting, 500 sheets are now commonly called a ream.

EXERCISE 75

- How many sheets in 24 quires?
- How many pens in 18 doz.? in 9 gro.? in 35 gro.?
- How many dozen in $\frac{1}{2}$ gro.? in $2\frac{2}{3}$ gro.? in $5\frac{3}{4}$ gro.?
- What will a gross of padlocks cost at \$5.25 a dozen?
- How many years old is a man who is 4 score? one who is 3 score? one who is 4 score years and 10?
- How many sheets of fine writing paper in 2 quires? in 35 quires? in 75 quires?
- If paper costs 15¢ a quire, how much will 14 quires cost? How many sheets will there be?
- At 20¢ a quire, how much will 7 quires of paper, with \$1 worth of envelopes, cost?
- What will 9 gro. of tablets cost at 25¢ a dozen?
- At \$7.20 a gross, what will 12 lamp chimneys cost?
- In 7 doz. reams of 500 sheets each, how many more sheets than in 7 doz. reams of 480 sheets each?
- At $12\frac{1}{2}$ ¢ a quire, how much will $9\frac{1}{2}$ quires of paper cost? $6\frac{1}{2}$ quires? a dozen quires?
- At 24¢ a quire for paper, and 1¢ each for envelopes, find the cost for paper, envelopes, and postage for sending 8 letters of one sheet each to some place in this country.

138. Wood Measure. Wood is sold by the cord. The standard cord is a pile 8 ft. long, 4 ft. wide, and 4 ft. high, and therefore contains 128 cu. ft.

The cord varies, often being considered to be a pile 8 ft. long and 4 ft. high, the price then varying with the length of the stick.

For example, how many cords of wood in a pile 24 ft. long, 4 ft. wide, and 6 ft. high ?

Here we have
$$\frac{\overset{3}{24} \times \overset{3}{4} \times \overset{3}{6}}{\underset{2}{8} \times 4 \times 4} \text{ cd.} = 4\frac{1}{2} \text{ cd.}$$

EXERCISE 76

Find the number of cords of wood in piles 4 ft. wide, whose length and height are as follows :

- | | | |
|------------------|------------------|------------------------|
| 1. 10 ft., 4 ft. | 4. 16 ft., 5 ft. | 7. 24 ft., 4 ft. |
| 2. 12 ft., 6 ft. | 5. 18 ft., 4 ft. | 8. 26 ft., 5½ ft. |
| 3. 15 ft., 5 ft. | 6. 20 ft., 6 ft. | 9. 32 ft., 5 ft. 8 in. |

10. How much are 7½ cd. of wood worth at \$2.50 a cord ?
11. How much are 16¾ cd. of wood worth at \$2.25 a cord ?
12. If wood is worth \$2.40 a cord, how many cords can be bought for \$42.60 ?

13. What is the cost of a pile of wood 22 ft. long and 5 ft. high, when wood of that length is selling for \$3.50 a cord ? (Take the cord as a pile 8 ft. long and 4 ft. high.)

14. What is the cost of a pile of wood 36 ft. long and 6 ft. high, when wood of that length is selling for \$3.75 a cord ?

15. A pile of wood 32 ft. long and 6 ft. high sells for \$24. What is the price of wood of that length per cord ?

16. When wood of a certain length is worth \$3.80 a cord, a pile 4 ft. high sells for \$11.40. How long is the pile ?



17. These children are standing by a cord of wood. It is 8 ft. long and the wood is cut in 4-foot lengths. How high is the pile?

18. How many cubic feet are there in $3\frac{3}{4}$ cords?

19. A pile of 4-foot wood is 4 rd. long and 6 ft. high. How many cords does it contain?

20. A pile of 4-foot wood is 75 ft. long and 7 ft. high. How many cords does it contain?

21. How much is a pile of 4-foot wood 60 ft. long and 6 ft. high worth at \$3.75 a cord?

22. How much is a pile of 4-foot wood 75 ft. long and 7 ft. high worth at \$4 a cord?

23. A pile of wood 8 ft. long and 4 ft. high is usually called a cord, the price varying with the length of the wood. How many cords in a pile 28 ft. long and 6 ft. high?

24. A pile of wood is 50 ft. long and 5 ft. high. How many cords does it contain?

25. How much is a pile of wood 30 ft. long and 5 ft. high worth at \$3.50 a cord?

139. Addition of Compound Numbers. The general principles for adding compound numbers are the same as those for adding integers.

The only difference arises from the fact that in integers ten units of each order always make one unit of the next higher, while in compound numbers the number of units of a lower denomination required to make one unit of the next higher varies with the denomination we are using.

Required to add 2 hr. 38 min. 42 sec., 3 hr. 25 min. 36 sec., and 5 hr. 17 min. 58 sec.

The sum of the seconds is 136 sec., or 2 min. 16 sec. We write the 16 sec. under the seconds, and add the 2 min. to the minutes. The sum of the minutes is 82 min., or 1 hr. 22 min. Writing the 22 min., we add the 1 hr. to the hours. The sum of the hours is 11 hr. Hence the result is 11 hr. 22 min. 16 sec.

In practical life to-day we rarely have to add numbers of more than two denominations, and hence the examples given in this book are in general of not more than two denominations.

EXERCISE 77

1. A girl made two aprons. For one she used 1 yd. 9 in. of cloth and for the other 27 in. How much did she use in all?

Add the following :

2.		3.		4.		5.	
ft.	in.	lb.	oz.	rd.	ft.	rd.	ft.
13	2	36	9	10	4	17	6½
24	8	15	3	25	11	18	10

6. If a boy weighs 73 lb. 12 oz. and his sister weighs 69 lb. 12 oz., how much do they weigh together?

7. The class in cooking had one piece of meat weighing 1 lb. 14 oz. and another weighing 2 lb. 9 oz. What was the total weight?

Add the following :

8.

lb.	oz.
25	12
24	4

9.

rd.	ft.
73	$4\frac{1}{2}$
26	12

10.

mi.	rd.
26	300
23	20

11.

mi.	rd.
37	280
42	50

12.

yd.	in.
41	25
41	11

13.

ft.	in.
60	9
10	4

14.

lb.	oz.
30	10
20	8

15.

lb.	oz.
41	8
16	8

16.

rd.	ft.
30	$6\frac{1}{2}$
40	10

17.

mi.	rd.
70	300
80	25

18.

yd.	in.
30	30
20	8

19.

yd.	in.
27	27
36	27

20.

da.	hr.
3	23
11	3
9	10

21.

wk.	da.
2	6
3	5
2	4

22.

bu.	pk.
44	2
17	3
16	2

23.

bu.	pk.
62	$3\frac{1}{2}$
18	$3\frac{1}{2}$
12	2

24.

ft.	in.
3	9
7	10
12	8

25.

sq. ft.	sq. in.
96	36
72	87
81	92

26.

cu. ft.	cu. in.
87	921
94	436
76	997

27.

lb.	oz.
147	7
962	12
321	8

28.

ft.	in.
29	11
16	8
37	6
16	4

29.

sq. ft.	sq. in.
39	47
46	83
92	54
33	2

30.

cu. ft.	cu. in.
37	692
49	875
89	981
75	100

31.

lb.	oz.
482	15
621	12
537	8
215	8

32.	33.	34.	35.
gal. qt.	gal. pt.	ft. in.	hr. min. sec.
29 3	47 5	17 8	2 17 36
37 2	36 4	26 7	3 39 48
16 1	42 6	32 5	5 17 50
19 3	71 1	41 7	3 56 49
<u>36 2</u>	<u>43 5</u>	<u>20 9</u>	<u>5 48 35</u>

36.	37.	38.	39.
gal. qt.	bu. pk.	ft. in.	° ' "
36 3	29 3	47 8	5 37 49
48 1	36 2	69 7	9 26
29 2	48 3	33 9	2 52 46
43 2	73 1	83 6	48 32
<u>51 3</u>	<u>49 2</u>	<u>50</u>	<u>6 15 9</u>

40.	41.	42.	43.
qt. pt.	pk. qt.	wk. da.	hr. min. sec.
17 1	19 3	12 4	17 23 32
36 1	17 6	32 5	23 15 48
49 1	23 4	16 6	4 7 15
27	19 7	10 5	16 19 49
<u>35 1</u>	<u>13 5</u>	<u>13 3</u>	<u>43 6 33</u>

44. Add $16^{\circ} 27' 32''$, $37^{\circ} 48' 16''$, $48^{\circ} 29' 36''$, $28^{\circ} 13' 23''$.

45. Add 2 A. 40 sq. rd., 3 A. 80 sq. rd., 4 A. 30 sq. rd., 10 sq. rd.

46. Add 5 sq. rd. 6 sq. yd., 7 sq. rd. $20\frac{1}{4}$ sq. yd., 6 sq. rd. 4 sq. yd.

47. Add 9 cu. yd. 18 cu. ft., 7 cu. yd. 12 cu. ft., 24 cu. ft. 920 cu. in., 808 cu. in.

48. Add $2\frac{1}{2}$ lb., 4 lb. 4 oz., $3\frac{3}{4}$ lb., 5 lb. 8 oz., 6 lb. 9 oz., $5\frac{1}{4}$ lb., $2\frac{1}{2}$ lb., 8 oz.

140. Subtraction of Compound Numbers. The general principles for subtracting compound numbers are the same as those for subtracting integers, except as explained in addition.

For example, from 22 hr. 16 min. 27 sec. required to subtract 14 hr. 25 min. 40 sec.

Since we cannot subtract 40 sec. from 27 sec., we take 1 min. (60 sec.) of the 16 min. Then 60 sec. + 27 sec. = 87 sec., and 87 sec. - 40 sec. = 47 sec. Since we cannot subtract 25 min. from 16 min., we take 1 hr. (60 min.) of the 22 hr. Then 60 min. + 16 min. = 76 min., and 76 min. - 25 min. = 51 min. Then 21 hr. - 14 hr. = 7 hr. The difference is 7 hr. 51 min. 47 sec.

EXERCISE 78

1. The class made a lamp screen. Each wing was 1 ft. $6\frac{3}{4}$ in. high and $9\frac{3}{4}$ in. wide. It was how much higher than wide?

2. A girl tried some bluing in laundry work, using 2 gal. 1 qt. in one experiment and 2 qt. in another. How much more did she use in the first experiment than in the second?

3. If one boy is 5 ft. 1 in. in height and another is 4 ft. 10 in. in height, how much taller is the first boy?

Subtract:

4.		5.		6.		7.	
ft.	in.	lb.	oz.	rd.	ft.	mi.	rd.
58	10	25	4	35	3	27	140
9	6	17	6	26	7	16	300

8. A lady buys $7\frac{1}{2}$ yd. of cloth and uses 27 in. How much has she left?

9. A field is 42 rd. 8 ft. long and 24 rd. 12 ft. wide. The length of the field is how much more than the width?

10. A man starts out to travel $3\frac{1}{2}$ mi. After he has gone 200 rd. how much farther has he to go?

Subtract :

11.

rd.	ft.
6	$6\frac{1}{2}$
4	

12.

rd.	ft.
6	$6\frac{1}{2}$
13	

13.

rd.	ft.
26	$8\frac{1}{2}$
18	$9\frac{3}{4}$

14.

bu.	pk.
42	1
26	2

15.

mi.	rd.
10	60
3	20

16.

mi.	rd.
10	0
	20

17.

mi.	rd.
76	$7\frac{3}{4}$
29	8

18.

gal.	qt.
39	0
17	2

19.

lb.	oz.
37	$6\frac{1}{8}$
28	3

20.

lb.	oz.
37	$6\frac{1}{8}$
28	$7\frac{2}{3}$

21.

yd.	in.
53	$28\frac{1}{2}$
47	$32\frac{3}{4}$

22.

qt.	pt.
56	0
29	1

23.

ft.	in.
29	7
16	10

24.

yd.	ft.
12	2
11	1

25.

lb.	oz.
173	0
69	10

26.

gal.	qt.
42	2
29	3

27.

ft.	in.
82	$5\frac{1}{2}$
64	7.25

28.

bu.	qt.
28	24
19	17

29.

lb.	oz.
251	4
176	12

30.

qt.	pt.
63	0
28	1

31.

sq.ft.	sq.in.
78	92
29	106

32.

hr.	min.
19	40
12	25

33.

cu.ft.	cu.in.
250	67
175	998

34.

mi.	rd.
27	4
9	56

35.

sq.ft.	sq.in.
69	75.25
49	$26\frac{1}{2}$

36.

A.	sq.rd.
58	82
42	120

37.

cu.ft.	cu.in.
126	750
78	1025

38.

min.	sec.
106	27
49	49

Subtract :

39.		40.		41.		42.		
ft.	in.	lb.	oz.	rd.	ft.	yd.	in.	
93	7	47	8	32	4	27	4	
<u>61</u>	<u>8</u>	<u>26</u>	<u>9</u>	<u>26</u>	<u>5</u>	<u>19</u>	<u>16</u>	
43.		44.		45.		46.		
mi.	rd.	yd.	in.	ft.	in.	mi.	rd.	
81	7	87	4	65½	0	246	30	
<u>65</u>	<u>98</u>	<u>69</u>	<u>8</u>	<u>48</u>	<u>9</u>	<u>129</u>	<u>75</u>	
47.		48.		49.		50.		
lb.	oz.	rd.	ft.	mi.	rd.	rd.	ft.	
85	6	67	8½	89	80	127	3	
<u>36½</u>	<u>0</u>	<u>39½</u>	<u>0</u>	<u>69½</u>	<u>0</u>	<u>99</u>	<u>12½</u>	
51.			52.			53.		
yr.	mo.	da.	yr.	mo.	da.	yr.	mo.	da.
1916	6	8	1919	7	3	1920	3	0
<u>1914</u>	<u>7</u>	<u>9</u>	<u>1915</u>	<u>2</u>	<u>15</u>	<u>1917</u>	<u>9</u>	<u>16</u>

In Exs. 51-53 reckon 1 mo. as 30 da.

54. The latitude of Albany is $42^{\circ} 39' 13''$ N. (that is, north of the equator). How much does this lack of being 45° ?

55. The latitude of Clinton, N. Y., is $43^{\circ} 3' 17''$ N. This is how much more than that of Albany, as given in Ex. 54?

56. The latitude of New York City is $40^{\circ} 45' 23''$ N. This is how much less than that of Albany?

57. The latitude of West Point is $41^{\circ} 23' 22''$ N. How many degrees, minutes, and seconds is this from the north pole?

58. The longitude of Albany is $73^{\circ} 44' 48''$ W. (that is, west of the prime meridian which passes through Greenwich, England). The longitude of New York City is $73^{\circ} 58' 25.5''$ W. What is the difference in longitude?

141. Multiplication of Compound Numbers. Required to multiply 3 ft. 9 in. by 5.

ft.	in.	
3	9	Since 5×9 in. = 45 in., or 3 ft. 9 in., we write 9 in. under the inches, and reserve the 3 ft. to add to the product of
	5	5×3 ft. Then 5×3 ft. = 15 ft., and 15 ft. + 3 ft. = 18 ft.
18	9	Therefore the product is 18 ft. 9 in.

EXERCISE 79

Multiply:

- | | |
|--|----------------------------------|
| 1. 3 ft. 4 in. by 2. | 23. 18 lb. 6 oz. by 6. |
| 2. 7 ft. 6 in. by 4. | 24. 7 rd. 9 ft. by 8. |
| 3. 8 ft. 2 in. by 9. | 25. 11 lb. 7 oz. by 4. |
| 4. 9 yd. 6 in. by 3. | 26. 9 gal. 3 qt. by 3. |
| 5. 8 yd. 5 in. by 8. | 27. 12 gal. 3 qt. by 2. |
| 6. 6 yd. 6 in. by 7. | 28. 10 mi. 1200 ft. by 5. |
| 7. 47 ft. 6 in. by 25. | 29. 27 lb. 3 oz. by 84. |
| 8. 49 yd. 2 in. by 96. | 30. 44 mi. 3 ft. by 89. |
| 9. 10 ft. 3 in. by 47. | 31. 273 ft. 2 in. by 35. |
| 10. 12 lb. 4 oz. by 92. | 32. 309 lb. 5 oz. by 67. |
| 11. 40 yd. 3 in. by 68. | 33. 370 yd. 1 in. by 83. |
| 12. 37 ft. 6 in. by 7. | 34. 27 sq. ft. 60 sq. in. by 7. |
| 13. 49 ft. 11 in. by 15. | 35. 6 cu. ft. 700 cu. in. by 9. |
| 14. 64 ft. 10 in. by $16\frac{1}{2}$. | 36. 27 cu. ft. 200 cu. in. by 8. |
| 15. 35 qt. 1 pt. by 17. | 37. 28 lb. 4 oz. by 35. |
| 16. 62 gal. 3 qt. by 25. | 38. 48 min. 25 sec. by 16. |
| 17. 48 bu. 2 pk. by 62. | 39. 55 min. 30 sec. by 20. |
| 18. 15 hr. 27 min. by 6. | 40. 18 bu. 3 pk. by 8. |
| 19. 621 mi. 20 ft. by 97. | 41. 9 T. 1650 lb. by 4. |
| 20. 32 min. 48 sec. by 12. | 42. 404 mi. 70 ft. by 73. |
| 21. 45 min. 15 sec. by 16. | 43. 210 mi. 10 yd. by 50. |
| 22. 22 min. 30 sec. by 20. | 44. 175 mi. 10 rd. by 60. |

45. If the average weight of the 35 members of your class is 57 lb. 4 oz., what is the total weight of the class?

46. If the average length of your steps is 2 ft. 2 in., how far will you go in 425 steps?

47. If a wheel is 9 ft. 2 in. in circumference, how far will it travel in making 125 revolutions?

48. If one side of a square is 8 ft. 9 in., how far is it around the square; that is, what is the perimeter?

49. One side of a square is 7 ft. 9 in. What is the perimeter?

50. If a barrel will hold 31 gal. 2 qt., how much water will 15 barrels hold?

51. If one can of fruit weighs 1 lb. 12 oz., what will a dozen such cans weigh?

52. If a school has 6 recitations a day, each of 45 min., how many hours and minutes of recitations a day?

53. A year being 365 da. 5 hr. 48 min. 46 sec., how many days, hours, minutes, and seconds in 6 yr.?

54. If a cubic foot of water weighs 62 lb. 8 oz., what is the weight of the water in a tank that contains 48 cu. ft.?

55. What is the weight of the water in a tank that is 6 ft. long, 5 ft. wide, and 13 ft. deep?

56. What is the weight of the water in a tank that is 6 ft. 6 in. long, 5 ft. wide, and 12 ft. deep?

57. What is the weight of the water in a tank that is 6 ft. 6 in. long, 4 ft. 6 in. wide, and 8 ft. deep?

58. If the height of each story above the first or ground story in a city office building is 11 ft. 9 in., and that of the first story is 15 ft. 8 in., and there are nine stories in all, what is the height of the building?

142. Division of a Compound Number by an Abstract Number.

Required to divide 7 hr. 5 min. 24 sec. by 2.

Arranging as in ordinary division, we have

hr. min. sec.	7 hr. $\div 2 = 3$ hr., with remainder 1 hr., or 60 min.
2) 7 5 24	Then 60 min. + 5 min. = 65 min., and 65 min.
3 32 42	$\div 2 = 32$ min., with remainder 1 min., or 60 sec.
	Then 60 sec. + 24 sec. = 84 sec., and 84 sec. $\div 2$
	= 42 sec. Therefore the quotient is 3 hr. 32 min. 42 sec.

In most cases in practical business the dividend has only two denominations. Indeed, it is so much more common to write $2\frac{1}{2}$ bu., $7\frac{1}{2}$ lb., 28.25 A., instead of 2 bu. 2 pk., 7 lb. 8 oz., 28 A. 40 sq. rd., that compound numbers are not nearly as important as formerly.

143. Division of One Compound Number by Another. Required to divide 41 ft. 3 in. by 2 ft. 9 in.

Here it is advisable to reduce both numbers to the same denomination, and it makes no difference in principle whether we reduce to feet or to inches.

Reducing to inches, we have 41 ft. 3 in. = 495 in. and 2 ft. 9 in. = 33 in. Then $495 \text{ in.} \div 33 \text{ in.} = 15$.

Or, reducing to feet, we have

$$41\frac{1}{4} \text{ ft.} \div 2\frac{3}{4} \text{ ft.} = \frac{4}{11} \text{ of } \frac{165}{4} = 15.$$

We aim to choose the denomination that makes the work the shorter.

EXERCISE 80

Divide :

- | | |
|--------------------------------------|--------------------------------|
| 1. 9 ft. 8 in. by 4. | 5. 47 mi. 288 ft. by 12. |
| 2. 175 ft. 4 in. by 8. | 6. 4 hr. 3 min. 30 sec. by 10. |
| 3. 34 lb. 6 oz. by 11. | 7. 25 ft. 1 in. by 3 ft. 7 in. |
| 4. 29 yd. 18 in. by 6. | 8. 60 ft. 9 in. by 6 ft. 9 in. |
| 9. 8 hr. 3 min. by 2 hr. 45 sec. | |
| 10. 59 yd. 21 in. by 5 yd. 15 in. | |
| 11. 240 lb. 12 oz. by 26 lb. 12 oz. | |
| 12. 21 mi. 1620 ft. by 1 mi. 345 ft. | |

13. How many planks, each 9 ft. 6 in. long, will it take to reach 76 ft. ?

14. How many cans, each containing $\frac{7}{8}$ qt., can be filled from 7 gal. of sirup ?

15. If a train travels 20 mi. in 24 min. 20 sec., how long does it take to go a mile ?

16. How many rods are there in 44 yd. ? How many yards are there in 44 rd. ?

17. From 9 gal. of sirup 40 cans are filled. How many pints does each can hold ?

18. If a train moves 73 ft. 4 in. in a second, how many seconds will it take to go a mile ?

19. How many lots, each containing 2 A. 20 sq. rd., can be cut from a lot containing 17 A. ?

20. If a train is traveling at the rate of a mile in 72 sec., how many miles will it travel in 42 min. ?

21. If a man walks at the rate of a mile in 19 min., how many miles can he walk in 1 hr. 54 min. ?

22. If a man can build 30 ft. 8 in. of fence a day, how many days will it take him to build 214 ft. 8 in. ?

23. A steamship changed its longitude $52^{\circ} 30'$ in 7 days. How many degrees of longitude did it sail per day ?

24. If each tile has a surface area of 1 sq. ft. 12 sq. in., how many tiles will be required to cover a space containing 156 sq. ft. ?

25. A real estate dealer has $3\frac{1}{2}$ A. of land that he purposes to cut into five equal lots. How many square rods will there be in each lot ?

26. A man is walking at the rate of a mile in 21 min., and a train is going at the rate of a mile in 72 sec. The train is traveling how many times as fast as the man ?

EXERCISE 81

State the units used in measuring the following :

1. The length of dress goods ; the width.
2. The quantity of a liquid.
3. The quantity of dry substances, like grain.
4. Sugar, in buying it ; in cooking.
5. Rice, in buying it ; in cooking.
6. Tea, in buying it ; in making it for the table.
7. Cocoa, in buying it ; in making it for the table.
8. Salt, in buying it ; in cooking.
9. Berries, in buying them ; in cooking.
10. Oranges, in large quantities ; in small quantities.
11. Kerosene, in large quantities ; in small quantities.
12. Water, in large quantities ; in cooking.
13. The length of carpeting ; the width.
14. The dimensions of a book cover ; of a floor ; of a field.
15. The area of a sheet of paper ; of a floor ; of a field.
16. The capacity of an aquarium ; the contents of a grain elevator ; the capacity of an oil tank.
17. Milk, when sold to a creamery ; when sold to a dealer ; when sold to a family.
18. Coffee, when sold in large quantities ; when sold to families ; when used for cooking.
19. Potatoes, when sold in large quantities ; when sold in small quantities at a city grocery ; when used in cooking.
20. Butter, when sold in large quantities ; when sold in small quantities ; when used in cooking.
21. Flour, when sold in large quantities ; when sold in small quantities ; when used in cooking.

EXERCISE 82

REVIEW QUESTIONS

1. What are denominate numbers? compound numbers?
2. What is meant by reduction descending? Illustrate.
3. What is meant by reduction ascending? Illustrate.
4. Write the common table of length. Make up and solve an example in addition of three compound numbers.
5. Write the common table of square measure. Make up and solve an example in subtraction in this table.
6. Write the common table of cubic measure. Make up and solve an example in multiplication in this table.
7. Write the common table of weight. Make up and solve an example in division in this table.
8. Write the common table of liquid measure. Make up and solve an example in division in this table.
9. Write the common table of dry measure. Make up an example involving this table, and solve it.
10. Why is it necessary to have some years of 366 days, and how often do they occur? What is the exception?
11. Write the tables of angle and arc measure. Make up an example involving each table, and solve it.
12. How does the addition of compound numbers differ from that of integers? Illustrate.
13. How does the subtraction of compound numbers differ from that of integers? Illustrate by the case of 42 — 39, and 4 ft. 2 in. — 3 ft. 9 in.
14. How does the multiplication of a compound number differ from that of an integer? Illustrate.
15. Illustrate the two kinds of division of compound numbers.

144. Solving Problems. We have now learned the operations of addition, subtraction, multiplication, and division with integers, common fractions, decimals, and compound numbers. We are therefore ready to consider more fully the methods of solving ordinary business problems involving denominate numbers.

145. Three Necessary Things. In solving a problem there is one thing that the teacher will ask you to do, and two things that may be required. These three are: (1) to work rapidly and accurately; that is, to take the shortest road to the answer, and to be certain the answer is correct; (2) to put neatly on paper not merely the operation but a brief explanation; (3) to give a brief analysis or oral explanation, showing that you understand the work.

For example, if 5 yd. of cloth cost \$2.10, how much will 12 yd. cost?

(1) <i>The number work.</i>	\$0.42
$12 \times \$2.10$	$\begin{array}{r} 12 \\ 84 \\ 42 \\ \hline \end{array}$
$\frac{5}{}$	$\begin{array}{r} 42 \\ \hline \$5.04 \end{array}$

(2) *The written explanation:*

5 yd. cost \$2.10.

1 yd. cost $\frac{1}{5}$ of \$2.10.

12 yd. cost $12 \times \frac{1}{5}$ of \$2.10, or \$5.04.

(3) *The oral analysis:*

Since 5 yd. cost \$2.10, 1 yd. will cost $\frac{1}{5}$ of \$2.10, and 12 yd. will cost $12 \times \frac{1}{5}$ of \$2.10, or \$5.04.

Teachers will not need to call for all this work with every example. Sometimes it will be necessary to emphasize the written explanation and sometimes the oral analysis, but the important thing is that the number work should be quickly and neatly, and, above all, *accurately* done.

146. Estimating the Answer. One of the best ways to avoid absurd answers is to *estimate the result* in advance.

For example, if 5 yd. cost \$2.10, we know 12 yd. will cost nearly $2\frac{1}{2}$ times as much, or somewhere near \$5. When we solve, if we find such a result as \$50.40, we see at once that there is a mistake, probably in the position of the decimal point. The correct result is \$5.04.

Always estimate the answer in advance, write down the estimate, and compare it with the result. If there is a great difference, the work is probably wrong.

It is almost always possible to estimate an answer within a reasonable amount. The more one practices, the easier this becomes.

147. Accuracy. In any single operation the work is *right* or it is *wrong*. A business man will not excuse a bookkeeper who writes \$9250.75 instead of \$90,250.75. Only a zero is missing, but it means a difference of over \$80,000.

A pupil may be excused if he does not understand the wording or the meaning of a problem, but for inaccurate numerical work he must be judged as he will be judged in business, — his work is right or else it is wrong. Accuracy is far more important than speed.

148. Checking the Work. *Always check the number work in some way. The first consideration in arithmetic is accuracy.*

One of the simplest checks is estimating the answer in advance, or seeing if it is reasonable after the work is performed.

149. Businesslike Work. Work as business men work, putting the additions and subtractions in columns except when a written analysis is required.

Much time is wasted over unbusinesslike analyses of problems that require nothing more than a rapid, accurate solution.

150. Neatness. Always have the work neatly done. Slovenly work leads to inaccuracy. A business man does not wish to employ any one whose letters, accounts, or computations have a slovenly appearance.

EXERCISE 83

1. A schoolroom is 32 ft. 8 in. long. Express the length in inches.

2. A room is 282 in. wide. Express the width in feet and inches; in feet and a fraction.

3. How much will a strip of carpet cost at 50¢ a yard, the strip being 16 ft. 6 in. long?

4. A wheel is 11 ft. in circumference. How many times will it turn in going $2\frac{1}{2}$ mi.?

5. If a lot is 197 ft. 9 in. long and $82\frac{1}{4}$ ft. wide, the length is how many rods greater than the width?

6. If a man is walking 88 yd. a minute, he is walking at the rate of how many miles per hour?

7. If a cubic inch of cast iron weighs $4\frac{5}{8}$ oz., how many pounds does a cubic foot weigh?

8. If a cubic foot of ice weighs 57 lb. 8 oz., how many cubic feet of ice will it take to weigh a ton?

9. The radius of the earth at the equator is estimated to be 20,926,202 ft. Express this in miles and feet.

10. If a pile of wood contains 2760 cu. ft., how many cords does it contain?

11. Express 2 ft. 6 in. as inches; as feet and a common fraction; as feet and a decimal; as a fraction of a yard; as a fraction of a rod.

12. A horse is 14 hands high. Express this in inches; in feet and inches; in feet and a common fraction; in feet and a decimal fraction.

13. A cubic foot of water weighs nearly 1000 oz. How many pounds is this? Express the result in pounds and ounces; in pounds and a common fraction of a pound; in pounds and a decimal fraction of a pound.

14. How many pounds does 5 cu. yd. 18 cu. ft. of earth weigh, if a cubic yard weighs 1 T. 825 lb.?

15. If the average weight of a certain kind of water pipe is 15 lb. 8 oz. per foot, how many feet of the pipe will it take to weigh a ton?

16. If the water in a certain harbor is 14 fathoms deep, a fathom being 6 ft., what is the depth in feet? If the depth is 165 ft., what is the depth in fathoms?

17. If a man takes 100 steps of 0.8 yd. each in a minute, how long will it take him to walk a distance of 16 mi.?

18. When sound travels 1116 ft. a second, how far distant is the lightning flash (which is seen practically instantaneously) when a man hears the thunder 9 sec. after he sees the flash?

19. If a cubic foot of ice weighs 57 lb. 8 oz., what will be the weight in tons of the mass of ice 1 ft. thick that covers a pond whose surface is 100,000 sq. yd.?

20. A train leaves Binghamton at 5 min. past 8, and reaches a place 59.6 mi. distant at 5 min. before 10. What is the average time per mile? Answer to two decimal places.

21. A train leaves Albany at 8.45 A.M., and reaches a place 284.74 mi. distant at 3.30 P.M. What is the average time per mile? Answer to two decimal places.

22. On a trip of three miles an automobile required 2 min. 34 sec. for the first mile, 2 min. 17 sec. for the second, and 1 min. 56 sec. for the third. How long did it take for the three miles?

23. When a man was three miles from his railroad station he had 20 min. 30 sec. before his train was due to leave. He then drove one mile in 6 min. 48 sec., and a second mile in 6 min. 54 sec. What is the longest time in which he can drive the last mile and still catch his train?



24. This farmer is pointing to a field that is 80 rd. long and 40 rd. wide. How many square rods does it contain? How many acres?

25. If a field is 80 rd. square, how many acres does it contain? How many acres will it contain if it is 160 rd. square?

26. A man thinks of replacing the fence around a field by a 4-strand barbed wire fence. The field is a rectangle 120 rd. by 30 rd. How many pounds of barbed wire will he need, allowing 1 lb. of barbed wire to the rod?

27. If an artesian well produces 3.265 cu. ft. of water per minute, and a cubic foot of water weighs 62.5 lb., what is the weight of the water produced in an hour?

28. If a man allows 350 cu. ft. of solidly stacked hay to the ton, how many tons of hay are there in a mow that contains 4375 cu. ft.?

29. If corn fodder loses $\frac{3}{4}$ of its weight in curing, how much will 28 T. of green fodder weigh after it is cured? How much will 45 T. weigh?

30. Taking the annual rainfall of New York as 37.52 in., what will be the weight of water that will fall on 1 sq. ft. of land per year? on 1 sq. rd.? on 1 A.?

31. If a farmer has 64 A. of wheat yielding $24\frac{1}{4}$ bu. per acre, what will it cost him for threshing at 4¢ a bushel?

32. What is the total cost of $15\frac{1}{2}$ bu. of wheat at 80¢, 17 bu. of potatoes at 28¢, 13 bu. of corn at 45¢, and 21 bu. of oats at 41¢?

33. If a man and team can seed 8 A. a day, how long will it take to seed a field of 96 A., and what will it cost at \$6.12 $\frac{1}{2}$ a day?

34. Supposing they are able to plow 2 A. a day, how much will it cost to plow a 76-acre field at \$6.12 $\frac{1}{2}$ a day?

35. If three fourths of the weight of potatoes is water, and if a bushel of potatoes weighs 60 lb., what is the weight of water in 275 bu. of potatoes?

36. If two twenty-fifths of the weight of barley is water, and if a bushel of barley weighs 48 lb., what is the weight of water in 275 bu. of barley?

37. When wheat is worth 92¢ a bushel, what is the value of the wheat grown on 75 A., if the average yield per acre is 22 bu.?

38. If wheat weighs 60 lb. per bushel, and the average yield is $24\frac{1}{2}$ bu. per acre, what will be the weight of the wheat grown on a field of $42\frac{1}{2}$ A.?

39. An experiment has shown that it requires a pull of 92 lb. to draw a ton over a good gravel road, and 2.35 times as much to draw the same load over a common dirt road. What pull is required for the dirt road?

40. It is estimated that a pine pillar of a certain length, and 4 in. square at the base, will support 12.1 T. of the weight of a barn floor. At this rate, how much will such a pillar 4 in. thick and 8 in. wide support? a pillar 4 in. thick and 10 in. wide? a pillar 4 in. thick and 12 in. wide?

151. Advantage of Cancellation. As we have already found, there is often an advantage in stating a problem in a single step, endeavoring then to simplify the work by cancellation.

For example, a man in rapid walking takes 120 steps a minute, averaging 26 in. each. How long will it take him to walk a mile?

Since he walks 26 in. in taking 1 step and takes 120 steps a minute, in 1 min. he will walk 120×26 in. Since in 1 mi. there are 5280 ft., or 5280×12 in., it will take him as many minutes to walk a mile as 5280×12 in. divided by 120×26 in. Therefore we have

$$\frac{5280 \times 12 \text{ in.}}{120 \times 26 \text{ in.}} = \frac{\overset{264}{\cancel{528}} \times \overset{12}{\cancel{12}}}{\underset{10}{\cancel{120}} \times \underset{13}{\cancel{26}}} = 20\frac{4}{13}.$$

It will therefore take the man $20\frac{4}{13}$ minutes to walk 1 mile.

EXERCISE 84

1. A manufacturer ships 2500 lb. of brass bedsteads from New York to Chicago. Such freight takes the first-class rate, 75¢ per hundred pounds. The manufacturer agrees to pay two thirds of the freight. How much does he pay?

2. A manufacturer ships 1800 lb. of iron bedsteads, this kind of freight taking the second-class rate, 65¢ per hundred pounds. He agrees to pay one third of this freight. How much does he pay?

3. A manufacturer ships 650 lb. of furniture casters in boxes, this kind of freight taking the third-class rate, 50¢ per hundred pounds. If he pays three fourths of this freight, how much does he pay?

4. A manufacturer ships 900 lb. of crated bamboo bedsteads, this kind of freight taking double the first-class rate. If the first-class rate is 75¢ per hundred pounds, how much is the freight?

5. He ships 1200 lb. of crated bamboo cabinets, the rate being three times the first-class rate. If the first-class rate is 75¢ per hundred pounds, how much is the freight?

6. A dealer ships some bundles of green hides weighing 1900 lb., these taking the fourth-class rate, which is $\frac{7}{13}$ of the first-class rate. If the first-class rate is 75¢ per hundred pounds, how much is the freight?

7. A dealer ships 2200 lb. of dry hides, the rate being $1\frac{1}{2}$ times the first-class rate. If the first-class rate is 90¢ per hundred pounds, how much is the freight?

8. Car loads of green hides take the fifth-class rate. If the first-class rate is 80¢ per hundred pounds, and the fifth-class two fifths as much, and a dealer ships a car load of 36,000 lb., how much is the freight?

9. Wheat in car-load lots takes the sixth-class rate. If the first-class rate is 90¢ per hundred pounds, and the sixth-class one third as much, and a dealer ships a car-load lot of 40,000 lb., how much is the freight?

10. If a dealer sends by express 250 lb. of goods, the rate being \$3.50 per hundred pounds, and he pays half of the transportation charges, how much does he pay?

11. A dealer sent an express package weighing 150 lb. and agreed to pay $\frac{2}{3}$ of the transportation charges. If the rate was \$2.50 per hundred pounds, how much did he pay?

12. A canal boat carries 211.75 T. of freight, the rate being 20¢ per hundredweight. If $\frac{3}{7}$ of the freight is owned by one man, $\frac{2}{7}$ by another, and the rest by a third, what is the charge to be paid by each?

13. If a drayman in a city averages 3 loads of freight a day, and receives on an average \$2.10 a load, how much will he earn in a month of 26 working days?

152. Unitary Analysis. A common form of analysis is illustrated in § 145, where we went from the cost of 5 yd. to the cost of 1 yd., and then to the cost of 12 yd.

An analysis that proceeds first to the unit and then to the required number of units is called *unitary analysis*.

In all such problems the words "at the same rate" are understood.

For example, if 50 bbl. of flour cost \$235, how many barrels can be bought for \$126.90?

$$\begin{array}{l} \text{If \$235 buys 50 bbl., \$1 will buy } \frac{50 \text{ bbl.}}{235}, \text{ and \$126.90 will buy} \\ 126.90 \times \frac{50 \text{ bbl.}}{235} = \frac{1269 \text{ bbl.}}{47} = 27 \text{ bbl.} \end{array}$$

Sometimes the problem requires us to proceed to two or more units, as in some of the examples already given.

For example, if 16 horses eat 42 bu. of oats in 2 weeks, how many bushels will 32 horses eat in 3 weeks?

1. In 2 weeks 16 horses eat 42 bu.
2. In 1 week 16 horses eat $\frac{1}{2}$ of 42 bu.
3. In 1 week 1 horse eats $\frac{1}{16}$ of $\frac{1}{2}$ of 42 bu.
4. In 3 weeks 1 horse eats $3 \times \frac{1}{16}$ of $\frac{1}{2}$ of 42 bu.
5. In 3 weeks 32 horses eat $32 \times 3 \times \frac{1}{16}$ of $\frac{1}{2}$ of 42 bu.

Canceling, we find the result to be 126 bu.

EXERCISE 85

1. If 100 laborers can do a piece of work in 12 days, how many will be needed to do it in 8 days?
2. How many men will it take to do a piece of work in 168 days that 108 men can do in 266 days?
3. If a stable has on hand enough oats to last 25 horses 105 days, how long will the oats last 15 horses?
4. How long will it take 16 men to do a piece of work that 18 men can do in 24 days?

5. If a clock gains 1 min. 15 sec. in 24 hours, how much will it gain in 42 hours at the same rate?

6. If 80 ft. of water pipe weigh 1240 lb., how much will 4 rd. of the pipe weigh?

7. If a candle $5\frac{3}{4}$ in. long burns 3 hr. 24 min., how many hours will a candle 6.3 in. long burn?

8. If a certain weight of sheet iron $\frac{3}{8}$ in. thick covers 15 sq. ft., how many square feet would the same weight of sheet iron $\frac{1}{4}$ in. thick cover?

9. If a wheel 12 ft. 1 in. in circumference makes 3432 revolutions in going a certain distance, how many revolutions will a wheel 11 ft. 11 in. in circumference make?

10. If a wheel 10 ft. 2 in. in circumference makes 3036 revolutions in going a certain distance, what is the circumference of a wheel that makes 3660 revolutions in going the same distance?

11. At the rate of $2\frac{3}{4}$ mi. an hour, I can walk a certain distance in 2 hr. 30 min. How long will it take me to walk the same distance at the rate of $3\frac{1}{8}$ mi. an hour?

12. At the rate of $2\frac{7}{8}$ mi. an hour, I can walk a certain distance in 2 hr. 20 min. What is my rate per hour if I walk the same distance in 2 hr. 33 min. 20 sec.?

13. Using 24 electric lights, a man's electricity bill is \$9 for 3 months. How much will it be for 4 months, if he uses half a dozen more lights, all lights running equally?

14. If 144 reams of paper are used in printing 7600 books, 84 sheets to a book, how many reams will be required to print 4900 books, containing 57 sheets of the same size?

15. If 27 sacks, each containing $3\frac{1}{3}$ bu., will hold a given quantity of grain, what must be the capacity of each sack that 18 sacks may hold the same quantity?

EXERCISE 86

PROBLEMS IN SHIPPING OUR FOOD

✓ 1. A dealer ships 75 bu. of blueberries to market, using cases of 24 qt. each. How many cases does he need?

1 2. A gardener ships 1272 bunches of asparagus in boxes of 2 dozen bunches each. How many boxes does he need?

3. A market gardener ships 960 bu. of early potatoes in 3-peck boxes. How many boxes does he need?

4. A Monroe County fruit grower ships 150 baskets of peaches to New York, each basket containing $\frac{1}{2}$ bu. If the dealers sell these by the quart, how many quarts will they sell?

5. A man ships 150 boxes of damson plums to a city dealer. The boxes hold $\frac{1}{2}$ bu. each. The plums are sold in the city by the quart. How many quarts are there?

6. A dealer shipped 1250 lambs to New York. The average weight was 81 lb., and he paid \$4.75 per 100 lb. How much did he pay for the lot?

✓ 7. A Florida grower sends to the northern market 250 crates of strawberries, each containing 32 quart boxes. The berries retail at 40¢ a box. What is the total retail price?

8. A sheep raiser shipped a lot of lambs to a packing house, receiving \$342 for the lot. The total weight was 7200 lb. At this rate, how much should he receive for another shipment of 3200 lb.?

9. A grape grower ships a ton of grapes to market. Of these $\frac{1}{10}$ were put into 9-pound baskets, and the rest into 10-pound baskets. How many baskets did he ship?

10. A gardener shipped 260 lb. of horse-radish to a city market, where it was sold for \$9.85. If the transportation charges were 75¢, what was the net price per pound?

EXERCISE 87

PROBLEMS IN MANUFACTURING

1. If the output of rope in a single year in this country was 200,824,974 lb., how many tons were manufactured?
2. Our silk mills consumed 17,471,000 lb. of raw silk in a single year. How many tons were consumed?
3. If there were manufactured in the United States in a certain year 35,553,731 gross of buttons, how many buttons were manufactured?
4. If there were manufactured in a certain year in this country 136,887,782 gross of pins, how many pins were manufactured?
5. A tannery paid \$390 for tanning materials, which was 0.13 of the total cost of the materials it used. What did it pay for all the materials?
6. If our cotton mills use 4,485,000 bales of cotton in one year, weighing 2,242,500,000 lb., what is the average weight per bale?
7. If the capital of a certain establishment is \$75,500, and its annual business is 3.2 times its capital, what is its annual business?
8. If we have 1400 establishments engaged in manufacturing boots and shoes, and all but one twentieth are in the New England, North Central, and Middle Atlantic States, how many are in the other states?
9. A manufacturing establishment makes a certain article for which it pays 65¢ for material, 25¢ for labor, 24¢ for expenses and advertising, and 8¢ for putting the article on the market. It sells this article for \$1.30. How much is its profit on 1728 such articles? How much is its profit on 60 doz. such articles?

EXERCISE 88

PROBLEMS IN HOME PURCHASES

1. If 16 lb. of a certain kind of biscuit can be bought for \$1.44, how much will 10 lb. cost at the same rate ?

2. A woman can buy pastry flour in 10-pound bags at 44¢, or in $24\frac{1}{2}$ -pound bags at 98¢. How much does she save a pound by buying the flour in the larger bags ?

3. A dealer sells Ceylon tea at 68¢ a pound, and 5-pound tins at \$3.14. If two families want $2\frac{1}{2}$ lb. each, how much will each save if they buy a tin and divide it ?

4. A dealer sells coffee in half-pound bags at 17¢, and in 5-pound cans at \$1.64. If a woman wishes 5 lb., how much does she save by purchasing it by the can ?

5. A woman can buy Dutch cocoa in $\frac{1}{4}$ -pound boxes at 22¢, or in 5-pound cans at \$2.78. If she wishes 5 lb., how much does she save by purchasing it by the can ?

6. A woman can buy maple sirup at 44¢ a quart, or in gallon cans at \$1.38. If she wishes 4 qt., how much does she save by purchasing it by the can ?

7. A dealer sells large glass jars of comb honey at \$5.06 a dozen, or 44¢ each. How much more will he receive by selling a gross by the jar than by the dozen ?

8. A woman buys a pound each of mustard seed, allspice, and cayenne pepper, paying 60¢ for the three. The mustard seed costs 12¢ a pound, and the allspice costs the same as the pepper. How much does the allspice cost ?

9. A woman bought a can each of cinnamon, nutmeg, and pepper, paying 39¢ for the three. The pepper cost 9¢, and the nutmeg cost twice as much as the cinnamon. How much did the cinnamon cost ? How much did the cinnamon and nutmeg together cost ?

EXERCISE 89

MISCELLANEOUS PROBLEMS

1. If a man walks at the rate of a mile in 18 min. 20 sec., how many miles can he walk in 1 hr. 31 min. 40 sec.?

2. When sailors measure the depth of the water they use as a measure a fathom of 6 ft. If the water is 120 fathoms deep, how many feet is this?

3. When we speak of a distance at sea being 480 mi., we mean nautical miles, or knots. A knot being nearly $1\frac{1}{8}$ mi., how many miles does a ship sail in going 480 knots?

4. The more exact length of a knot being 6080.27 ft., how many miles in 3000 knots?

5. A sheet of paper 10 in. wide is to be cut into 4 leaves, each 8 in. long and 5 in. wide. What is the length of the sheet? Draw a plan.

6. A sheet of paper 10 in. wide is to be cut into 6 leaves, each 8 in. long and 5 in. wide. What is the length of the sheet? Draw a plan.

7. A ceiling is 24 ft. long and 18 ft. wide. It is to be calcimined at 9¢ a square yard. What will be the cost of calcimining it?

8. A field is 32 rd. long and 25 rd. wide. What is its area in acres?

9. A rectangular city lot is 106 ft. 7 in. long and 28 ft. 8 in. wide. Find the number of square feet in the lot.

10. A rectangular field containing 4 A. is 32 rd. long. How wide is it?

11. A rectangular field containing $4\frac{1}{2}$ A. is 36 rd. long. How wide is it?

12. How many square rods in a rectangular field $27\frac{1}{2}$ rd. long and 24 rd. wide? How many acres?

CHAPTER VIII

BILLS AND ACCOUNTS

153. Bill. A statement of the amount owed by one person to another is called a *bill*.

The person who owes the money is called the *debtor*.

The person to whom the money is owed is called the *creditor*.

The debtor or creditor may be a company of persons, instead of one person.

To *make the extensions in a bill* means to write the amount after each item. To *foot a bill* means to add the amounts of the several items and find the total amount.

To *receipt a bill* means to stamp or write the words "Paid" or "Received payment" at the foot, followed by the name of the creditor or his authorized agent.

154. Specimen of a Bill. The following is a receipted bill:

Ithaca, N. Y., Feb. 1, 1915

Mr. David Brownson, Newfield, N. Y.

Bought of Charles Dunham

1272 State St., Ithaca, N. Y.

Jan.	5	6 cans soup	@ 20¢	\$1	20		
	20	10 lb. pearl barley	@ 9¢		90	\$2	10

Received payment, Feb. 8, 1915

Chas. Dunham

Items quoted by the dozen or fraction of a dozen have prices given by the dozen. Thus, $\frac{1}{2}$ doz. oranges @ 50¢ means that the price is 50¢ a dozen.

EXERCISE 90

Write and receipt bills for the following, inserting the place, date, and names, and completing each item :

1. 14 yd. flannel @ 45¢, 32 yd. sheeting @ 30¢, 4½ yd. velvet @ \$1.60, 2½ yd. ribbon @ 40¢, 2 pr. gloves @ \$1.75.

2. 6 lb. ham @ 19¢, 8 lb. bacon @ 22¢, 4 lb. sausage @ 24¢, 2 tins sliced smoked beef @ 25¢, 3 cans corned beef @ 13¢, 4 cans tomato soup @ 22¢, 3 jars extract of beef @ 40¢, 2 lb. tapioca @ 8¢.

3. 3 qt. Lima beans @ 15¢, 4 cans tomatoes @ 14¢, 6 cans French peas @ 22¢, 5 cans California asparagus @ 42¢, 2 cans imported asparagus @ 85¢, 4 cans sardines @ 18¢, 2 packages oatmeal @ 12¢.

4. 5 boxes Smyrna figs @ 60¢, 4 boxes stuffed dates @ 40¢, 6 lb. California prunes @ 20¢, 3 lb. Malaga raisins @ 22¢, 4 tins crystallized ginger @ 40¢, 6 jars marmalade @ 28¢, 6 cans peas @ 18¢.

5. 6 heads lettuce @ 6¢, 9½ lb. butter @ 32¢, 7½ gal. oil @ 18¢, 12 lb. raisins @ 12½¢, 6 lb. coffee @ 32¢, 4½ doz. eggs @ 36¢, ½ doz. oranges @ 60¢, 6 lb. powdered sugar @ 7½¢, 2 qt. lima beans @ 15¢.

6. 4 lb. pearl barley @ 9¢, 6 lb. cornstarch @ 8¢, 4 packages hominy @ 20¢, 6 packages oatmeal @ 12¢, 6 lb. rice @ 11¢, 5 lb. sago @ 10¢, 3 lb. tapioca @ 8¢.

7. 5 gal. molasses @ 45¢, 10 lb. coffee @ 18¢, 15 lb. sugar @ 7¢, 6 boxes crackers @ 8¢, 6½ doz. eggs @ 34¢, 9 lb. butter @ 33¢, 6½ lb. cheese @ 12¢.

8. 7 gro. toothbrushes @ \$9.65, 8 doz. brushes @ \$17.75, 2½ gro. nailbrushes @ \$26.80, ½ doz. clothes brushes @ \$2.92, 6 doz. boxes tooth powder @ \$2.04, 4½ doz. atomizers @ \$19.60.

9. 6 bedroom sets @ \$48.75, 15 rockers @ \$4.20, 9 tables @ \$7.50, 16 chairs @ \$2.10.

10. 75 gro. bone buttons @ 22¢, 675 yd. cashmere @ 85¢, 840 yd. cambric @ 17½¢, 560 yd. chambray @ 22½¢, 620 yd. flannel @ 37½¢, 950 yd. sheeting @ 24½¢.

11. 75 bbl. flour @ \$5.23, 63 gal. molasses @ 39¢, 75 gal. fancy sirup @ 58¢, 3 doz. packages pancake flour @ \$1.20, 260 lb. coffee @ 30¢.

12. 8 doz. hatchets @ \$10.75, 6 doz. pr. hinges @ \$4.35, 5 doz. carpenter's squares at \$34.50, ½ gro. locks @ \$42.50, 8 doz. files @ \$6.25.

13. 575 yd. cotton flannel @ 38¢, 684 yd. cotton @ 7¢, 364 yd. buckram @ 9¢, 648 yd. sheeting @ 26¢, 327 yd. Java canvas @ 62¢.

14. 84 gro. bone buttons @ 18¢, 72 doz. pearl buttons @ 9¢, 694 yd. cambric @ 17¢, 364 yd. cashmere @ 82¢, 408 yd. chambray @ 21¢.

15. 4 gro. toothbrushes @ \$9.35, 3 doz. atomizers @ \$19.25, 8 doz. combs @ \$1.95, 4 doz. brushes @ \$18.37, ½ gro. nailbrushes @ \$27.50.

16. 480 yd. matting @ 18¢, 375 yd. matting @ 19¢, 284 yd. carpet @ 48¢, 8 rugs @ \$7.33, 4 doz. door mats @ \$4.75, 240 yd. border @ \$1.12½.

17. 9 dining sets @ \$62.50, 16 rockers @ \$5.35, 8 sideboards at \$32.50, 6 card tables @ \$8.75, 4 china closets @ \$17.50, 6 dining tables @ \$9.25.

18. 325 yd. carpet @ 48¢, 520 yd. matting @ 22¢, 16 rugs at \$6.40, 4 rugs at \$12.50, 3 doz. door mats @ \$5.30, 328 yd. calico @ 6¢, 825 yd. sheeting @ 24¢.

19. 36 yd. carpet @ \$1.12½, 15 yd. border @ \$1.25, 15½ yd. cotton @ 38¢, 22 yd. cloth @ 37½¢, 7½ yd. silk @ \$1.40, 2½ doz. buttons @ 48¢, 12½ yd. calico @ 8¢.

155. Accounts. A statement of the amount owed by one person to another, with any payments that may have been made, is called an *account*.

The words "bill" and "account" are often used for either a bill or an account.

156. Specimen of an Account. The following is an account:

Elmira, N. Y., March 30, 1916

Mr. Robert J. Spencer, Oswego, N. Y.

To Henderson Bros., Dr.

Wholesale Stationers, 4056 Water St.

		Dr.		Cr.	
Mar.	7	50 gro. pens @ 78¢	\$39		
		15 doz. notebooks @ \$1. ⁷⁰	25 50		
	9	Cash			\$40
	12	4 doz. rulers @ 1. ³²	5 28		
	13	Cash			10
	20	30 gro. lead pencils @ 3. ³⁵	100 50		
	25	Cash to balance			120 28
			\$170 28	\$170 28	

The difference between the sum of the items of indebtedness and those representing payments is the *balance*.

EXERCISE 91

Make out accounts for the following, inserting the place, date, and names, and completing each item:

1. Feb. 4, 12 lb. ham @ 21¢, 6½ lb. butter @ 33¢, 8 doz. eggs @ 45¢; Feb. 7, 8 lb. sugar @ 6¾¢, 15 lb. raisins @ 15¢, 8 lb. prunes @ 19¢, 6 lb. tapioca @ 9¢. Payments: Feb. 5, \$3; Feb. 9, \$2; balance on Feb. 13.

2. March 2, 10 doz. brooms @ \$3.72, 8 doz. mops @ \$7.90, 12 doz. mop handles @ \$2.45, 9 doz. laundry boards @ \$5.25; March 7, 15 doz. clotheslines @ \$5.50, 6 doz. cedar pails @ \$2.70; March 12, 10 doz. feather dusters @ \$3.20. Payments: March 3, \$75; March 8, \$50; balance on March 15.

3. April 3, 6 doz. wool dusters @ \$4.80, 5 doz. dustpans @ \$3.15, 2 doz. rattan carpet beaters @ \$2.65, 5 waxing brushes @ \$2.32, 3 doz. shoe brushes @ \$6.35; April 8, 5 doz. scrub brushes @ \$1.60, 9 floor brushes @ \$1.44; April 12, 10 doz. whisk brooms @ \$2.88. Payments: April 5, \$25; April 10, \$15; balance on April 15.

4. May 1, 8 doz. quarts ammonia @ \$2.56, 75 lb. borax @ 12¢, 8 boxes sal soda @ \$1.08; May 5, 15 doz. cans brick dust @ 67¢, 25 doz. cans emery powder @ \$1.67, 12 boxes scouring soap @ \$3.07; May 12, 35 boxes laundry soap @ \$3.88, 8 doz. packages soap powder @ \$1.88. Payments: May 3, \$20; May 6, \$25; balance on May 14.

5. June 7, 8 doz. bottles mixed pickles @ \$2.67, 9 doz. bottles gherkins @ \$3.07, 35 doz. packages gelatin @ \$1.48; June 12, 9 doz. pints chowchow @ \$3.12, 6 doz. bottles horse-radish @ \$0.98, 4 doz. bottles mustard sauce @ \$1.95; June 16, 8 doz. bottles salad dressing @ \$2.58, 75 doz. cans tomatoes @ \$2.48, 6 doz. bottles catsup @ \$2.44, 5 doz. bottles chutney sauce @ \$1.02. Payments: June 8, \$50; June 14, \$30; balance on June 20.

6. July 9, 6 gro. toothbrushes @ \$9.68, 4 gro. nailbrushes @ \$24.35, 16 doz. combs @ \$1.88; July 15, 7 doz. whisk brooms @ \$2.94, 5 doz. hairbrushes @ \$9.60, 4 doz. bath brushes @ \$6.48; July 20, 4 doz. hand mirrors @ \$7.20, 7 doz. tubes cold cream @ \$1.98. Payments: July 12, \$50; July 16, \$20; balance on July 21.

157. Cash Accounts. In keeping a cash account it is customary to write each item of the receipts on the left side (called the *debit* side), and each item of the expenditures on the right side (called the *credit* side), thus:

	1915					1915				
<i>a</i>	Mar.	4	Cash on hand	\$125	50	Mar.	5	Groceries	\$17	22 <i>f</i>
<i>b</i>		6	P. F. Roberts	30			6	Meat bill	6	90 <i>g</i>
<i>c</i>		7	R. S. Ball	72	25		7	Balance	203	63 <i>h</i>
<i>d</i>				227	75				227	75 <i>i</i>
<i>e</i>	Mar.	7	Balance	203	63					

Here on the left side *a* shows the balance on hand when this page of the account is begun; *b* and *c* are receipts from Messrs. Roberts and Ball; *d* is the sum of these three items on the day that the account is added, or balanced. On the right side *f* and *g* are bills paid. To find how much is now on hand we subtract the sum of \$17.22 and \$6.90, or \$24.12, from \$227.75, and we find that the remainder (in cash accounts called the *balance*) is \$203.63, which is written on the right at *h*, and again on the left at *e*. To check the work, *f*, *g*, and *h* are added, and the sums at *d* and *i* must agree.

EXERCISE 92

Make out cash accounts and balance them as above:

1. Receipts: May 1, cash on hand, \$36.50; May 2, R. S. Jacobs, \$20; May 4, M. R. Randall, \$32.60. Expenses: May 3, groceries, \$7.86; meat, \$3.50. Balanced on May 5.

2. Receipts: April 3, cash on hand, \$25.30; April 4, J. B. Fuller, \$30; April 7, R. H. Bates, \$4.30. Expenses: April 4, groceries, \$9.32. Balanced on April 8.

3. Receipts: Feb. 6, cash on hand, \$135.60; Feb. 6, R. S. Roberts, \$75.50; Feb. 7, M. L. Richards, \$87.90; Feb. 8, R. P. Jasper, \$42.25. Expenses: Feb. 6, rent, \$75; Feb. 7, salaries, \$60; Feb. 8, coal, \$15.30; Feb. 8, account book, \$2.25; Feb. 8, gas bill, \$3.75. Balanced on Feb. 9.

Insert dates and items as in Exs. 1-3, and balance the following accounts :

4. Receipts : \$125, \$32.60, \$28.90, \$50; expenses : \$35, \$15.30, \$28.40.

5. Receipts : \$16.30, \$15.50, \$10.10, \$3.50; expenses : \$4.80, \$3.45, \$20, \$4.75.

6. Receipts : \$10.30, \$5.25, \$6.32, \$7.20; expenses : \$5.30, \$2.40, \$3.80, \$1.50.

7. Receipts : \$34.30, \$15.30, \$10.75, \$6.32; expenses : \$32.50, \$8.20, \$4.30, \$3.

8. Receipts : \$42.60, \$18.50, \$13.60, \$2.80, \$4; expenses : \$15.40, \$15, \$32.75, \$4.60.

9. Receipts : \$75.50, \$62.30, \$4.72, \$16, \$25.30; expenses : \$3.25, \$80, \$14.20, \$15, \$3.25.

10. Receipts : \$62.30, \$15.40, \$14.75, \$21.32, \$5; expenses : \$35.75, \$10.42, \$29.60, \$28.40.

11. Receipts : \$1250.50, \$620, \$300, \$175, \$16.25, \$120.50, \$32.75, \$68.50; expenses : \$600, \$360, \$166.67, \$33.33, \$240.

12. Receipts : \$625.75, \$130.50, \$200, \$75, \$50, \$15.20, \$14.60, \$33.80; expenses : \$430, \$250, \$100, \$32.75, \$28.50.

13. Receipts : \$128.50, \$32.75, \$68.75, \$43.50, \$28.90; expenses : \$62.30, \$28.50, \$75.50, \$38.40, \$25.25, \$20.30.

14. Receipts : \$875.80, \$260.50, \$350, \$400, \$80, \$25, \$380, \$125; expenses : \$250, \$375, \$480.70, \$200, \$75.

15. Receipts : \$1025.30, \$535.40, \$287.50, \$126.75, \$25, \$324.80, \$125.45, \$135.42, \$12.87; expenses : \$10, \$225, \$430.75, \$223.50, \$121.60, \$123.75, \$2.35, \$6.27, \$4.68.

16. Receipts : \$287.60, \$821.75, \$327.42, \$196.80, \$427, \$236.90, \$481.73, \$87.96, \$52.80; expenses : \$327.50, \$427.39, \$179.86, \$321.42, \$83.96, \$75, \$1.24, \$3.50.

EXERCISE 93

PROBLEMS WITHOUT NUMBERS

1. If you know the cost per dozen of certain articles, how do you find the cost of a given number of dozen of each article?

2. How do you make the extensions in a bill? foot the bill? receipt the bill?

3. If you know the items representing purchases, and those representing payments on an account, how do you balance the account?

4. If you know the amount of indebtedness on an account, and the balance, how will you find the sum of the payments?

5. How do you keep a cash account? How do you balance it?

6. If you know the cost of each of several things, how do you find the cost of all of them?

7. If you know the sum of two numbers and one of them, how do you find the other?

8. If you know the cost of one thing, how do you find the cost of several things of the same kind?

9. If you know the cost of several things of the same kind, how do you find the cost of one?

10. Name some things that are sold by the dry quart; by the peck; by the bushel.

11. Name some things that are weighed by the ounce; by the pound; by the ton.

12. If you know the cost of an ounce of anything, how do you find the cost per pound?

13. If you know the cost of a pound of anything, how do you find the cost per ounce?

Sixth Grade

CHAPTER IX

FACTORS, MEASURES, MULTIPLES

158. Factors. Integral numbers whose product is a given number are called *factors* of that number.

Thus, since $2 \times 3 \times 5 \times 7 = 210$, some of the factors of 210 are 2, 3, 5, and 7. We also see that 210 has other factors, like 6, 10, 14, and so on. Although every number has as factors itself and 1, we do not usually speak of these as factors.

159. Prime Number. A number that has no factors is called a *prime number*.

As stated in § 158, this means that the number itself and 1 are excluded. Thus 2, 3, 5, and 7 are prime numbers.

160. Prime Factor. A factor that is a prime number is called a *prime factor*.

Thus 2, 3, and 5 are the prime factors of 30. Although 6 is a factor of 30, it is not a prime factor.

161. Composite Number. A number that is not a prime number is called a *composite number*.

Thus 4, 15, 100, and 250 are composite numbers.

162. Exact Divisor. A number that divides another without a remainder is called an *exact divisor* of that number.

163. Exact Division. When we have an exact divisor, the case is said to be one of *exact division*.

Thus 15 is *exactly divisible* by 5. Usually we speak of an exact divisor simply as a *divisor* of a number. Then again we often speak of an exact divisor as a *measure* of a number, because with it we may exactly measure the number, just as with a yardstick we may measure the length of a carpet that is 10 yd. long.

164. Divisible Numbers. When we speak of one number as being *divisible* by another we mean exactly divisible, that is, divisible without a remainder.

165. Even Number. A number that is divisible by 2 is called an *even number*.

166. Odd Number. A number that is not divisible by 2 is called an *odd number*.

167. Divisibility by 2. *A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8.*

Thus 36 must be divisible by 2 if 6 is; because 30, or any other number of tens, is divisible by 2, ten being 2×5 .

Thus we may tell without dividing that 1,023,578 is divisible by 2.

168. Divisibility by 3. *A number is divisible by 3 if the sum of the digits is divisible by 3.*

We often use the word *digits* to mean the *value* of the symbols 0, 1, 2, and so on to 9, as well as the characters from 1 to 9.

Thus 54 is divisible by 3 since $5 + 4$ is divisible by 3.

Likewise, 534 is divisible by 3 because $5 + 3 + 4$, or 12, is divisible by 3.

169. Divisibility by 5. *A number is divisible by 5 if it ends in 0 or 5.*

For if a number ends in 0, it is tens, and 10 is divisible by 5. If it ends in 5, it is tens plus 5, and both are divisible by 5.

There is no easy test for divisibility by 7.

170. Finding Prime Factors. Find the prime factors of 420.

By § 167, 2 is a factor of 420 and of 210; by § 168, 3 is a factor of 105; by § 169, 5 is a factor of 35, the other factor being 7. Hence the prime factors of 420 are 2, 2, 3, 5, 7.

$$\begin{array}{r} 2 \overline{)420} \\ 2 \overline{)210} \\ 3 \overline{)105} \\ 5 \overline{)35} \\ 7 \end{array}$$

That is, *divide as often as possible by the smallest prime factor, and then by the next larger, and so on until a prime quotient is reached. The several divisors and the last quotient are the prime factors.*

171. Proving a Number Prime. Required to find if 239 is a prime number.

The last digit is not 0, 2, 4, 6, or 8, so 239 cannot contain the factor 2, and therefore cannot contain the factor 4, 6, 8, or any other multiple of 2. The sum of the digits does not contain the factor 3, and therefore 239 cannot contain the factor 3 or any multiple of 3. The last digit is not 0 or 5, and therefore 239 cannot contain the factor 5 or any multiple of 5. Therefore 2, 3, 4, 5, 6, 8, 9, 10, 12, 14, 15, 16, cannot be factors of 239. By trial we find that 7, 11, 13, 17, are not factors. We need not try any prime number higher than 17, as the quotient when 17 is tried is less than 17, and we have tried all prime factors below 17. Hence 239 is prime.

Try the various prime numbers, beginning with 2, to see if they are factors. If no prime factor is found before the quotient becomes less than the divisor, the number is prime.

EXERCISE 94

1. Select the even numbers : 432, 640, 287, 1024, 3036, 8292, 4427, 8329, 4878.

2. Select the numbers divisible by 3 : 424, 636, 292, 444, 1002, 3027, 1104, 8822, 6777.

3. Select the numbers divisible by 5 : 830, 272, 637, 4225, 3010, 7200, 6305, 9270, 4005.

4. Select the prime numbers : 53, 61, 97, 111, 121, 143, 187, 1005, 1001, 1203, 2112, 1331, 1337.

Find the prime factors of the following :

5. 120.	/ 10. 175.	15. 1001.	/ 20. 1728.
6. 210.	11. 225.	16. 1245.	21. 5280.
7. 150.	12. 303.	17. 1331.	22. 6006.
8. 220.	13. 750.	18. 3645.	23. 3737.
9. 170.	14. 666.	19. 3432.	24. 9768.

172. Common Measure. A number that exactly divides each of two or more numbers is called a *common measure* of those numbers.

Thus 7 is a common measure of 28 and 42, since it exactly divides each of them, as a foot length exactly divides a 10-foot length and a 12-foot length that we are measuring. A common measure is also called a *common divisor*, or a *common factor*.

173. Greatest Common Measure. The greatest number that exactly divides each of two or more numbers is called the *greatest common measure* of those numbers.

Thus, while 7 is a common measure of 28 and 42, 14 is the *greatest* common measure. It is also called the *greatest common divisor*. The letters G.C.M. stand for the words Greatest Common Measure, and G.C.D. for Greatest Common Divisor.

174. Numbers Prime to Each Other. If two integers have no common measure, always excepting 1, they are said to be *prime to each other*.

Thus, 25 and 33 are prime to each other, although each has factors.

175. Finding the G.C.M. The greatest common measure of such numbers as we shall use in studying fractions is found easily by factoring.

For example, required the G.C.M. of 24, 30, and 36.

Finding the prime factors of each, we have the following:

$$\begin{array}{r} 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \overline{)30} \\ 3 \overline{)15} \\ 5 \end{array}$$

$$\begin{array}{r} 2 \overline{)36} \\ 2 \overline{)18} \\ 3 \overline{)9} \\ 3 \end{array}$$

The factor 2 occurs once in each of the numbers. The factor 3 occurs once in each of the numbers. No other factor occurs in *each* of the numbers. Therefore the G.C.M. is 2×3 , or 6.

This solution may be expressed as follows:

$$24 = 2 \times 2 \times 2 \times 3; \quad 30 = 2 \times 3 \times 5; \quad 36 = 2 \times 2 \times 3 \times 3.$$

Therefore the G.C.M. is 2×3 , or 6.

Required the G.C.M. of 54, 81, and 108.

$$\begin{aligned}\text{Now} \quad & 54 = 2 \times 3 \times 3 \times 3; \\ & 81 = 3 \times 3 \times 3 \times 3; \\ & 108 = 2 \times 2 \times 3 \times 3 \times 3.\end{aligned}$$

Here 3 occurs three times as a factor in each number, and there is no other common factor. Hence the G.C.M. is $3 \times 3 \times 3$, or 27.

Hence, to find the G.C.M. of two or more numbers,

Separate the numbers into prime factors and find the product of the prime factors common to the numbers.

If one of the given numbers is a multiple of one of the others, it may be canceled, since it must contain all the factors of that other. Thus the G.C.M. of 48, 54, and 108 is the same as that of 48 and 54.

EXERCISE 95

Find the G.C.M. of the following:

- | | | |
|--------------|-----------------|-------------------|
| 1. 2, 8. | 16. 6, 8, 42. | 31. 63, 84, 126. |
| 2. 4, 16. | 17. 9, 12, 33. | 32. 33, 77, 121. |
| 3. 9, 33. | 18. 8, 28, 84. | 33. 30, 60, 250. |
| 4. 12, 78. | 19. 9, 33, 81. | 34. 32, 48, 128. |
| 5. 14, 49. | 20. 10, 35, 85. | 35. 51, 63, 132. |
| 6. 15, 75. | 21. 12, 48, 68. | 36. 36, 84, 132. |
| 7. 16, 56. | 22. 14, 21, 77. | 37. 60, 144, 576. |
| 8. 17, 51. | 23. 16, 72, 96. | 38. 38, 190, 760. |
| 9. 18, 81. | 24. 18, 72, 81. | 39. 32, 160, 808. |
| 10. 19, 171. | 25. 20, 80, 95. | 40. 33, 132, 143. |
| 11. 20, 175. | 26. 24, 60, 96. | 41. 50, 125, 250. |
| 12. 21, 105. | 27. 30, 90, 96. | 42. 64, 128, 256. |
| 13. 22, 121. | 28. 32, 48, 96. | 43. 64, 160, 640. |
| 14. 23, 207. | 29. 36, 54, 90. | 44. 63, 126, 333. |
| 15. 24, 246. | 30. 39, 52, 91. | 45. 75, 125, 375. |

176. Finding the G.C.M. of Large Numbers. It is not often necessary to find the G.C.M. of large numbers that cannot be separated readily into factors. An example will illustrate a process often called the long-division method.

This method depends upon the principle that a common factor of two numbers is also a factor of their sum and of their difference.

Required the G.C.M. of 323 and 357.

323)357(1

323

34)323(9

306

17)34(2

34

Here we cannot easily factor the two numbers. We divide the larger number by the smaller to see if the smaller is the G.C.M. It is not because there is a remainder 34. We then divide the smaller number by this remainder and find there is a remainder 17. Hence 34 is not the G.C.M. Then we divide

the last divisor 34 by the last remainder 17 and find that 17 is an exact divisor of 34. Then 17 is the G.C.M. required.

EXERCISE 96

Find the G.C.M. of the following :

1. 195 and 260.

9. 2813 and 3973.

2. 294 and 350.

10. 2002 and 3010.

3. 333 and 592.

11. 7099 and 9389.

4. 410 and 697.

12. 2220 and 3293.

5. 720 and 1728.

13. 2650 and 8003.

6. 1875 and 2500.

14. 1140 and 1767.

7. 1220 and 2013.

15. 3621 and 7100.

8. 1010 and 2121.

16. 3650 and 3796.

17. What is the largest number that will exactly divide 2378 and 3915 ?

18. What is the largest number that will exactly divide 551, 667, and 841 ?

177. Multiple. The product of two or more integers is called a *multiple* of each of them.

Thus 20 is a multiple of 2, 4, 5, and 10, because $2 \times 2 \times 5 = 4 \times 5 = 2 \times 10 = 20$. This has already been defined (§ 36). We speak of 75, for example, as a multiple of 25, and of 51 as a multiple of 17, or we speak of each as a multiple of any other of its factors.

178. Common Multiple. A multiple of two or more numbers is called a *common multiple* of the numbers.

Thus 60 is a common multiple of 6 and 10, and so are 30, 90, 120, and so on, common multiples of 6 and 10.

179. Least Common Multiple. The smallest common multiple of two or more numbers is called their *least common multiple*. It is the least number that is exactly divisible by each of them.

Thus 60 is a common multiple of 6 and 10, but the *least common multiple* is 30.

The letters L.C.M. are used for the words Least Common Multiple.

Since the L.C.M. must contain each of the numbers, it must contain all of the prime factors of each, and this is the principle upon which the finding of the L.C.M. rests.

180. Finding the L.C.M. Required the L.C.M. of 24, 30, 36.

Resolving each number into its prime factors, we have

$$\begin{array}{r} 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \overline{)30} \\ 3 \overline{)15} \\ 5 \end{array}$$

$$\begin{array}{r} 2 \overline{)36} \\ 2 \overline{)18} \\ 3 \overline{)9} \\ 3 \end{array}$$

The L.C.M. must contain the factors of 24, which are 2, 2, 2, and 3. These are therefore taken as part of its factors.

It must contain the factors of 30, which are 2, 3, and 5; but we have already taken 2 and 3, so we need take only 5.

It must contain the factors of 36, which are 2, 2, 3, and 3; but we have already taken 2, 2, and 3, so we need take only one more 3.

Hence the factors of the L.C.M. are 2, 2, 2, 3, 5, 3. Their product is 360, and 360 is the L.C.M.

It is often convenient to adopt another arrangement of the work, as in finding the L.C.M. of 12, 18, 30, 45.

Here we arrange the numbers in a horizontal line, and divide by the smallest prime factor common to *two* or more of them.

$$\begin{array}{r} 2 \overline{) 12 \quad 18 \quad 30 \quad 45} \\ 3 \overline{) 6 \quad 9 \quad 15 \quad 45} \\ \quad 2 \qquad \qquad 15 \end{array}$$

We first divide by 2, and write the quotients and the *undivided numbers* as well in a line below. In the first line of quotients we cancel the 9 and the 15 because each is a factor of 45, and therefore 45 contains all the factors of 9 and of 15. We next divide by 3, and in the last line of quotients we have the numbers 2 and 15, which are prime to each other. Hence the L.C.M. is $2 \times 3 \times 2 \times 15$, or 180.

Required the L.C.M. of 5, 15, 30, 60.

$$\begin{array}{r} 5 \quad 15 \quad 30 \quad 60 \end{array}$$

Here we cancel the 5 because it is a factor of 15. We cancel the 15 because it is a factor of 30, and cancel the 30 because it is a factor of 60. Therefore 60 contains all the factors of the other numbers and is itself the L.C.M.

181. General Directions for Finding the L.C.M. From the above examples the following directions may be observed:

Separate each number into its prime factors in any convenient manner. Find the product of these factors, using each factor the greatest number of times it occurs in any one of the given numbers.

EXERCISE 97

Find the L.C.M. of the following:

- | | | |
|-----------------------|------------------|--------------------------------|
| 1. 22, 33. | 7. 3, 21, 35. | 13. 6, 9, 12, 15. |
| 2. 17, 51. | 8. 4, 12, 38. | ~~14.~~ 7, 14, 21, 35. |
| 3. 16, 24. | 9. 6, 15, 39. | 15. 8, 28, 36, 52. |
| 4. 15, 35. | 10. 8, 20, 48. | 16. 9, 21, 39, 93. |
| 5. 27, 39. | 11. 9, 42, 49. | ~~17.~~ 10, 15, 25, 50. |
| ~~6.~~ 33, 55. | 12. 12, 24, 144. | 18. 16, 24, 64, 96. |

- | | |
|---------------------|--------------------------|
| 19. 6, 9, 12, 33. | 29. 4, 8, 12, 16, 32. |
| 20. 9, 27, 36, 63. | 30. 5, 10, 20, 35, 45. |
| 21. 20, 24, 30, 40. | 31. 6, 18, 30, 42, 66. |
| 22. 24, 36, 48, 60. | 32. 7, 21, 63, 70, 84. |
| 23. 11, 22, 33, 66. | 33. 8, 32, 64, 72, 96. |
| 24. 14, 28, 77, 91. | 34. 9, 27, 54, 63, 90. |
| 25. 20, 28, 56, 80. | 35. 10, 11, 12, 22, 33. |
| 26. 30, 40, 50, 60. | 36. 11, 15, 33, 45, 55. |
| 27. 32, 40, 48, 80. | 37. 12, 16, 24, 36, 72. |
| 28. 45, 63, 72, 99. | 38. 13, 26, 52, 78, 104. |

39. Find the least number of marbles that can be divided equally among 15, 20, or 25 boys.

40. Find the shortest distance that can be exactly measured by a 3-foot rule, or a 5-foot pole, or a 10-foot pole.

41. What is the smallest quantity of liquid that can be measured exactly by 2-quart, 4-quart, 6-quart, or 8-quart measures?

42. What is the smallest sum of money that can be counted out in 5-cent pieces, 10-cent pieces, 25-cent pieces, or 50-cent pieces?

43. What is the capacity of the smallest water tank that can be filled in an integral number of minutes by either of two pipes, if one discharges 32 gallons a minute and the other 48 gallons a minute?

44. Find the least number of oranges that, when arranged in groups of 6, 7, 8, or 9, has just 5 over in each case. (Find the L.C.M. and add 5. Test the result.)

45. Find the least number of marbles that, when arranged in groups of 8, 12, 15, 60, or 120, has just 7 over in each case.

182. Cancellation. As in § 97, we may apply our knowledge of divisibility of numbers, and shorten the work by canceling equal factors from both dividend and divisor.

For example, divide $11 \times 27 \times 30$ by $3 \times 9 \times 15$.

The common factor 3 is canceled from the 3 in the divisor and the 27 in the dividend. The common factor 9 is canceled from the 9 in the divisor and the 9 that remains uncanceled in the 27 of the dividend.

$$\frac{11 \times \overset{9}{\cancel{27}} \times \overset{2}{\cancel{30}}}{\cancel{3} \times \cancel{9} \times \cancel{15}} = \frac{11 \times 1 \times 2}{1 \times 1 \times 1} = 22.$$

The common factor 15 is canceled from the divisor and the 30 in the dividend. There remains for a dividend $11 \times 1 \times 2$; and for a divisor

$1 \times 1 \times 1$. Hence the quotient is 22. This is much shorter than finding two products, 8910 and 405, and then the quotient 22.

EXERCISE 98

Find the quotient of:

1. $\frac{6 \times 8 \times 99}{3 \times 4 \times 11}$.

9. $\frac{4 \times 5 \times 6 \times 8}{2 \times 3 \times 4 \times 5}$.

2. $\frac{8 \times 12 \times 51}{4 \times 9 \times 17}$.

10. $\frac{8 \times 16 \times 28 \times 92}{2 \times 32 \times 23 \times 14}$.

3. $\frac{9 \times 30 \times 38}{6 \times 10 \times 19}$.

11. $\frac{36 \times 19 \times 29 \times 39}{3 \times 38 \times 58 \times 13}$.

4. $\frac{12 \times 21 \times 57}{9 \times 14 \times 19}$.

12. $\frac{27 \times 35 \times 42 \times 72}{5 \times 9 \times 14 \times 18}$.

5. $\frac{13 \times 20 \times 91}{13 \times 13 \times 14}$.

13. $\frac{34 \times 35 \times 36 \times 37}{5 \times 17 \times 9 \times 74}$.

6. $\frac{12 \times 32 \times 78}{9 \times 16 \times 26}$.

14. $\frac{24 \times 34 \times 44 \times 54}{3 \times 8 \times 17 \times 33}$.

7. $\frac{15 \times 25 \times 64}{10 \times 80 \times 10}$.

15. $\frac{15 \times 18 \times 48 \times 75}{25 \times 30 \times 36 \times 24}$.

8. $\frac{18 \times 27 \times 96}{12 \times 36 \times 54}$.

16. $\frac{24 \times 63 \times 82 \times 91}{41 \times 13 \times 21 \times 12}$.

17. What is the quotient of 35 times the product of 48 and 119, divided by 14 times the product of 21 and 17?

18. If a dividend is the product of 68, 91, and 95, and the divisor is the product of all the prime numbers between 12 and 20, what is the quotient?

19. If we divide the product of 16, twice 16, and three times 16, by the product of the square of 2 taken three times as a factor, what is the quotient?

20. How many times is the product of all the prime numbers between 20 and 30 contained in the product of 203 and 230?

21. A man worked 4 weeks, 6 days a week, for a market gardenèr, and received for his pay 24 crates of berries valued at \$2 a crate. How much did he earn a day?

22. How many dozen quart bottles of imported pickles at \$6 a dozen must be given in exchange for 3 cases of olives, each containing 2 dozen bottles at \$3 a dozen?

23. A grocer exchanged 16 boxes of soap, each containing 60 cakes at 4 cents a cake, for a certain number of cases of preserved fruit, 2 dozen jars to a case at 40 cents a jar. How many cases did he get?

24. A man exchanged 9 firkins of butter weighing 56 pounds each at 30 cents a pound for several rolls of matting, 40 yards to the roll worth 42 cents a yard. How many rolls did he get?

25. If 24 men, working 9 hours a day, can do a piece of work in 12 days, how many days will it take 18 men, working 8 hours a day, to do the work?

26. If 15 men can do a piece of work in 16 days, working 9 hours a day, how many days will it take 20 men working 12 hours a day to do the work?

27. A clerk working 48 weeks in a store, at \$12 a week, receives the same amount that another clerk receives in 36 weeks. What is the weekly wages of the other clerk ?

28. A dealer sells a shipment of 36 horses for \$2700. He sells another shipment of 25 horses for the same amount. What is the average price per horse in each of the two shipments ?

29. We find the volume of a rectangular box by taking the product of its three dimensions expressed in the same unit of measure. The dimensions of one box, expressed as abstract numbers, are 4, 5, and 6, and two dimensions of another box of the same volume are 3 and 10. What is the other dimension of the second box ?

30. Expressed as abstract numbers, the dimensions of one box are 8, 12, and 15, and two dimensions of another box of the same volume are 6 and 30. What is the other dimension of the second box ?

31. Expressing the dimensions as abstract numbers, a cellar is 20 long, 14 wide, and 6 deep. How deep must another cellar be that is 16 long and 15 wide, in order that the capacity of both cellars may be the same ?

32. How many times as much earth must be removed from a cellar excavation 175 feet long, 80 feet wide, and 15 feet deep, as from a cellar excavation 150 feet long, 40 feet wide, and 5 feet deep ?

33. How many times as much earth must be removed from a cellar excavation 160 feet long, 40 feet wide, and 9 feet deep, as from a cellar excavation 60 feet long, 40 feet wide, and 6 feet deep ?

34. How many times as much will a box contain whose dimensions are 8, 27, and 35, as one whose dimensions are 7, 12, and 18 ?

CHAPTER X

COMMON FRACTIONS

183. Fraction. A number that shows what part or what number of parts of a unit is taken is called a *fraction*.

Fractions have already been studied in the Fourth Year. They are now reviewed and are studied more carefully.

184. Common Fraction. A fraction expressed by two numbers, one under the other with a line between them, is called a *common fraction*.

185. Terms of a Fraction. In a common fraction the number above the line is called the *numerator*; the number below the line is called the *denominator*; the two together are called the *terms* of the fraction.

In the fraction $\frac{3}{4}$ the numerator is 3 and the denominator is 4. The denominator (namer) shows into how many equal parts the unit has been divided, and the numerator (numberer) shows how many of these equal parts have been taken.

186. Unit Fraction. A fraction whose numerator is one is called a *unit fraction*.

Thus $\frac{1}{4}$ is a unit fraction. It is also called a *fractional unit*.

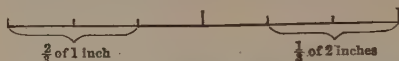
187. Proper Fraction. A fraction whose numerator is less than the denominator is called a *proper fraction*.

For example, $\frac{3}{4}$ is a proper fraction.

188. Improper Fraction. A fraction whose numerator is not less than the denominator is called an *improper fraction*.

For example, $\frac{4}{4}$ and $\frac{5}{4}$ are improper fractions.

189. Fraction as an Expressed Division. In this figure we see that $\frac{2}{3}$ of 1 inch equals $\frac{1}{3}$ of 2 inches, or the result of



dividing 2 inches by 3; that is, $\frac{2}{3}$ is the same as $\frac{1}{3}$ of 2, or 2 divided by 3. Therefore,

A fraction is an expressed division, the numerator being the dividend and the denominator the divisor.

190. Reduction of a Fraction to Lowest Terms. Since we may divide both terms of a fraction by the same number without changing the value of the fraction, we may continue this division until the terms are prime to each other.

If the terms of a fraction are prime to each other, the fraction is expressed in *lowest terms*.

For example, $\frac{15}{17}$ is expressed in lowest terms, but $\frac{3}{12}$ is not.

Therefore, *to reduce a fraction to lowest terms, cancel all the factors common to both numerator and denominator.*

For example, reduce $\frac{24}{30}$ to lowest terms.

Canceling 2, we have $\frac{12}{15}$; canceling 3, we have $\frac{4}{5}$.

EXERCISE 99

Reduce to lowest terms:

- | | | | |
|----------------------|-----------------------|------------------------|-------------------------|
| 1. $\frac{24}{36}$. | 9. $\frac{32}{48}$. | 17. $\frac{36}{144}$. | 25. $\frac{26}{144}$. |
| 2. $\frac{36}{84}$. | 10. $\frac{16}{80}$. | 18. $\frac{48}{144}$. | 26. $\frac{72}{256}$. |
| 3. $\frac{72}{88}$. | 11. $\frac{17}{51}$. | 19. $\frac{64}{156}$. | 27. $\frac{64}{448}$. |
| 4. $\frac{48}{84}$. | 12. $\frac{19}{57}$. | 20. $\frac{72}{156}$. | 28. $\frac{78}{273}$. |
| 5. $\frac{54}{99}$. | 13. $\frac{23}{92}$. | 21. $\frac{84}{168}$. | 29. $\frac{111}{120}$. |
| 6. $\frac{23}{89}$. | 14. $\frac{42}{82}$. | 22. $\frac{64}{192}$. | 30. $\frac{121}{132}$. |
| 7. $\frac{21}{63}$. | 15. $\frac{35}{63}$. | 23. $\frac{64}{160}$. | 31. $\frac{132}{144}$. |
| 8. $\frac{18}{38}$. | 16. $\frac{48}{64}$. | 24. $\frac{33}{144}$. | 32. $\frac{144}{156}$. |

191. Reduction of a Fraction to Higher Terms. By multiplying both terms of a fraction by the same number we may reduce a fraction to *higher terms*.

For example, reduce $\frac{1}{2}$ to sixtieths.

Here $60 \div 12 = 5$, so that the denominator must be multiplied by 5 to make 60. Therefore we multiply both terms by 5, and $\frac{1}{2} = \frac{5}{10}$.

Therefore, *to reduce a fraction to a fraction having a higher denominator, divide the required denominator by the given denominator and multiply both terms of the fraction by the quotient.*

192. Reduction of an Integer to an Improper Fraction. An integer may be expressed as an improper fraction with any required denominator.

Reduce 3 to an improper fraction with denominator 7.

We may simply think of 3 as equal to $\frac{3}{1}$. We have, then, the same case as in § 191. Multiplying both terms by 7, we have $\frac{3}{1} = \frac{21}{7}$.

Therefore, *to reduce an integer to a fraction with a given denominator, multiply it by the given denominator and under this product write the denominator.*

EXERCISE 100

Reduce as indicated:

To 60ths:	1. $\frac{1}{2}$.	2. $\frac{8}{15}$.	3. $\frac{9}{20}$.	4. $\frac{11}{30}$.
To 80ths:	5. $\frac{1}{2}$.	6. $\frac{6}{15}$.	7. $\frac{7}{20}$.	8. $\frac{9}{20}$.
To 144ths:	9. $\frac{1}{2}$.	10. $\frac{1}{12}$.	11. $\frac{5}{24}$.	12. $\frac{7}{72}$.
To 156ths:	13. $\frac{1}{2}$.	14. $\frac{2}{39}$.	15. $\frac{5}{8}$.	16. $\frac{7}{8}$.
To 256ths:	17. $\frac{1}{2}$.	18. $\frac{3}{16}$.	19. $\frac{3}{32}$.	20. $\frac{7}{4}$.
To 288ths:	21. $\frac{1}{2}$.	22. $\frac{1}{12}$.	23. $\frac{5}{24}$.	24. $\frac{7}{8}$.
To 320ths:	25. $\frac{1}{2}$.	26. $\frac{3}{16}$.	27. $\frac{5}{32}$.	28. $\frac{9}{64}$.
To 100ths:	29. $\frac{1}{2}$.	30. $\frac{3}{10}$.	31. $\frac{4}{25}$.	32. $\frac{9}{50}$.

193. Mixed Number. A number composed of an integer and a fraction taken together is called a *mixed number*.

Thus $2\frac{3}{4}$ is a mixed number. It is read "two and three fourths."

194. Reduction of an Improper Fraction. An improper fraction may always be reduced to an integer or a mixed number.

For example, required to reduce $7\frac{2}{3}$ to an integer or a mixed number.

Dividing both terms by 12, we have $\frac{6}{1}$, which equals 6.

Required to reduce $3\frac{6}{8}$ to an integer or a mixed number.

Since $3\frac{6}{8}$ is an expressed division, we divide 36 by 16 and the quotient is $2\frac{4}{16}$, or $2\frac{1}{4}$.

Therefore, *to reduce an improper fraction to an integer or a mixed number, divide the numerator by the denominator.*

EXERCISE 101

Reduce to integers or mixed numbers :

1. $\frac{36}{12}$.	10. $\frac{96}{16}$.	19. $\frac{84}{48}$.	28. $\frac{80}{64}$.	37. $\frac{198}{96}$.
2. $\frac{38}{12}$.	11. $\frac{36}{24}$.	20. $\frac{240}{48}$.	29. $\frac{88}{64}$.	38. $\frac{288}{96}$.
3. $\frac{40}{12}$.	12. $\frac{72}{24}$.	21. $\frac{60}{50}$.	30. $\frac{448}{64}$.	39. $\frac{480}{96}$.
4. $\frac{60}{12}$.	13. $\frac{96}{24}$.	22. $\frac{75}{30}$.	31. $\frac{80}{72}$.	40. $\frac{576}{96}$.
5. $\frac{72}{12}$.	14. $\frac{72}{18}$.	23. $\frac{90}{50}$.	32. $\frac{84}{72}$.	41. $\frac{250}{100}$.
6. $\frac{36}{16}$.	15. $\frac{96}{32}$.	24. $\frac{95}{50}$.	33. $\frac{96}{72}$.	42. $\frac{375}{100}$.
7. $\frac{40}{16}$.	16. $\frac{96}{48}$.	25. $\frac{350}{50}$.	34. $\frac{144}{72}$.	43. $\frac{425}{100}$.
8. $\frac{72}{16}$.	17. $\frac{72}{38}$.	26. $\frac{64}{64}$.	35. $\frac{288}{72}$.	44. $\frac{550}{100}$.
9. $\frac{80}{16}$.	18. $\frac{56}{48}$.	27. $\frac{72}{64}$.	36. $\frac{192}{96}$.	45. $\frac{750}{100}$.

46. If the numerator of a fraction is equal to the denominator, to what integer is the fraction equal ?

47. If the numerator of a fraction is $2\frac{1}{2}$ times the denominator, to what mixed number is the fraction equal ? To what integer is it equal if the denominator is $\frac{1}{3}$ of the numerator ?

195. Reduction of a Mixed Number. A mixed number may always be reduced to an improper fraction.

For example, required to reduce $2\frac{3}{4}$ to an improper fraction.

Since 1 equals 4 fourths ($\frac{4}{4}$), 2 equals 2×4 fourths, or 8 fourths ($\frac{8}{4}$); and 8 fourths and 3 fourths are 11 fourths ($\frac{11}{4}$).

Therefore, *to reduce a mixed number to an improper fraction, multiply the denominator by the integral part of the mixed number, and to the product add the numerator; under this sum write the denominator.*

EXERCISE 102

Reduce to improper fractions:

- | | | | |
|-----------------------|-----------------------|-----------------------|------------------------|
| 1. $3\frac{7}{8}$. | 11. $2\frac{5}{24}$. | 21. $8\frac{5}{36}$. | 31. $1\frac{7}{84}$. |
| 2. $4\frac{7}{8}$. | 12. $5\frac{7}{24}$. | 22. $6\frac{7}{36}$. | 32. $5\frac{1}{64}$. |
| 3. $7\frac{5}{8}$. | 13. $8\frac{1}{24}$. | 23. $4\frac{1}{36}$. | 33. $7\frac{1}{84}$. |
| 4. $6\frac{7}{8}$. | 14. $2\frac{1}{24}$. | 24. $7\frac{1}{36}$. | 34. $3\frac{2}{64}$. |
| 5. $9\frac{5}{8}$. | 15. $6\frac{1}{24}$. | 25. $2\frac{1}{36}$. | 35. $6\frac{3}{64}$. |
| 6. $7\frac{3}{16}$. | 16. $7\frac{3}{32}$. | 26. $5\frac{5}{8}$. | 36. $1\frac{5}{72}$. |
| 7. $5\frac{5}{16}$. | 17. $9\frac{5}{32}$. | 27. $8\frac{7}{8}$. | 37. $6\frac{7}{72}$. |
| 8. $8\frac{9}{16}$. | 18. $3\frac{7}{32}$. | 28. $2\frac{1}{48}$. | 38. $5\frac{3}{72}$. |
| 9. $9\frac{7}{16}$. | 19. $6\frac{9}{32}$. | 29. $6\frac{1}{48}$. | 39. $35\frac{5}{72}$. |
| 10. $6\frac{1}{16}$. | 20. $2\frac{1}{32}$. | 30. $1\frac{3}{48}$. | 40. $72\frac{1}{2}$. |

41. How many eighths ($\frac{1}{8}$) are there in $\frac{7}{8}$? in 3? in $3\frac{7}{8}$?

42. How many sixteenths ($\frac{1}{16}$) are there in $\frac{5}{16}$? in 5? in $5\frac{5}{16}$?

43. How many thirty-seconds ($\frac{1}{32}$) are there in $\frac{7}{32}$? in 3? in $3\frac{7}{32}$?

44. How many tins, $\frac{1}{8}$ of a pound each, can be filled from $\frac{7}{8}$ of a pound of pepper? from 2 pounds? from $2\frac{7}{8}$ pounds?

196. Common Denominator. If two or more fractions have the same denominator they are said to have a *common denominator*.

Thus $\frac{3}{8}$ and $\frac{5}{8}$ have the common denominator 8. The letters L.C.D. stand for the words Least Common Denominator.

197. Similar Fractions. Fractions that have a common denominator are called *similar fractions*. It is always possible to reduce fractions to similar fractions.

Forexample, required to reduce $\frac{2}{3}$ and $\frac{3}{4}$ to similar fractions.

The L.C.M. of the denominators is 12. Reducing $\frac{2}{3}$ to twelfths (§ 191) we have $\frac{8}{12}$, and reducing $\frac{3}{4}$ to twelfths we have $\frac{9}{12}$.

The L.C.D. of several fractions is the L.C.M. of the given denominators.

Hence, *to reduce fractions to similar fractions, reduce them to fractions whose common denominator is the L.C.M. of the given denominators.*

EXERCISE 103

Reduce to similar fractions, using the L.C.D. :

- | | | |
|------------------------------------|---|---|
| 1. $\frac{3}{8}, \frac{5}{12}$ | 13. $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}$ | 25. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}$ |
| 2. $\frac{5}{6}, \frac{7}{8}$ | 14. $\frac{1}{3}, \frac{1}{4}, \frac{5}{8}$ | 26. $\frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \frac{5}{6}$ |
| 3. $\frac{5}{8}, \frac{3}{12}$ | 15. $\frac{2}{3}, \frac{1}{4}, \frac{3}{8}$ | 27. $\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, \frac{1}{8}$ |
| 4. $\frac{7}{8}, \frac{4}{6}$ | 16. $\frac{2}{3}, \frac{3}{4}, \frac{5}{8}$ | 28. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{3}{8}$ |
| 5. $\frac{5}{12}, \frac{7}{18}$ | 17. $\frac{1}{6}, \frac{1}{8}, \frac{1}{12}$ | 29. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{7}{8}$ |
| 6. $\frac{7}{12}, \frac{9}{16}$ | 18. $\frac{2}{5}, \frac{3}{8}, \frac{5}{12}$ | 30. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{1}{16}$ |
| 7. $\frac{11}{12}, \frac{15}{16}$ | 19. $\frac{3}{5}, \frac{7}{8}, \frac{7}{12}$ | 31. $\frac{3}{4}, \frac{7}{8}, \frac{5}{16}, \frac{3}{32}$ |
| 8. $\frac{15}{16}, \frac{19}{24}$ | 20. $\frac{1}{8}, \frac{1}{10}, \frac{1}{12}$ | 32. $\frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{24}$ |
| 9. $\frac{13}{16}, \frac{29}{32}$ | 21. $\frac{3}{8}, \frac{7}{10}, \frac{7}{12}$ | 33. $\frac{3}{4}, \frac{3}{8}, \frac{1}{16}, \frac{5}{24}$ |
| 10. $\frac{13}{24}, \frac{5}{12}$ | 22. $\frac{3}{8}, \frac{1}{10}, \frac{5}{12}$ | 34. $\frac{5}{8}, \frac{1}{16}, \frac{3}{32}, \frac{1}{64}$ |
| 11. $\frac{7}{32}, \frac{11}{48}$ | 23. $\frac{5}{8}, \frac{5}{16}, \frac{7}{24}$ | 35. $\frac{7}{8}, \frac{5}{16}, \frac{5}{32}, \frac{7}{64}$ |
| 12. $\frac{15}{32}, \frac{29}{48}$ | 24. $\frac{7}{8}, \frac{1}{16}, \frac{7}{24}$ | 36. $\frac{7}{8}, \frac{1}{24}, \frac{1}{32}, \frac{5}{64}$ |

198. Addition of Fractions. If we add 3 marbles and 5 pencils, we have as a sum neither marbles nor pencils, but just 8 things, of two different kinds. So if we add $\frac{3}{4}$ and $\frac{5}{6}$ we shall have as a sum neither fourths nor sixths, but just 8 parts of a unit, of two different sizes.

Therefore, to add fractions that are not similar and obtain a definite sum, we must reduce them to similar fractions.

Required the sum of $\frac{3}{4}$ and $\frac{5}{6}$.

Here the L.C.M. of 4 and 6 is 12. Therefore the L.C.D. is 12.

Then $\frac{3}{4} = \frac{9}{12}$,

and $\frac{5}{6} = \frac{10}{12}$.

Now just as $\$9 + \$10 = \$19$, 9 twelfths + 10 twelfths = 19 twelfths.

That is, $\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12} = 1\frac{7}{12}$.

Therefore, to add fractions, reduce them to similar fractions having the least common denominator, if they are not similar. Write the sum of the numerators over the least common denominator, and reduce the result to lowest terms.

EXERCISE 104

Find the sum of:

- | | | | |
|----------------------------------|------------------------------------|-------------------------------------|--------------------------------------|
| 1. $\frac{1}{2}, \frac{1}{4}$. | 11. $\frac{3}{4}, \frac{5}{8}$. | 21. $\frac{1}{4}, \frac{5}{24}$. | 31. $\frac{3}{4}, \frac{15}{8}$. |
| 2. $\frac{1}{2}, \frac{3}{4}$. | 12. $\frac{3}{4}, \frac{7}{8}$. | 22. $\frac{3}{4}, \frac{11}{24}$. | 32. $\frac{3}{8}, \frac{13}{48}$. |
| 3. $\frac{1}{2}, \frac{1}{8}$. | 13. $\frac{1}{4}, \frac{1}{16}$. | 23. $\frac{3}{8}, \frac{7}{24}$. | 33. $\frac{5}{8}, \frac{17}{48}$. |
| 4. $\frac{1}{2}, \frac{5}{8}$. | 14. $\frac{3}{4}, \frac{7}{16}$. | 24. $\frac{5}{8}, \frac{11}{24}$. | 34. $\frac{7}{8}, \frac{23}{48}$. |
| 5. $\frac{1}{2}, \frac{7}{8}$. | 15. $\frac{3}{8}, \frac{5}{16}$. | 25. $\frac{3}{16}, \frac{1}{24}$. | 35. $\frac{1}{16}, \frac{17}{48}$. |
| 6. $\frac{1}{4}, \frac{1}{8}$. | 16. $\frac{7}{8}, \frac{11}{16}$. | 26. $\frac{5}{16}, \frac{5}{24}$. | 36. $\frac{7}{16}, \frac{31}{48}$. |
| 7. $\frac{1}{4}, \frac{3}{8}$. | 17. $\frac{1}{2}, \frac{3}{32}$. | 27. $\frac{7}{16}, \frac{11}{24}$. | 37. $\frac{1}{8}, \frac{5}{64}$. |
| 8. $\frac{1}{4}, \frac{5}{8}$. | 18. $\frac{3}{4}, \frac{3}{32}$. | 28. $\frac{9}{16}, \frac{13}{24}$. | 38. $\frac{5}{12}, \frac{1}{64}$. |
| 9. $\frac{1}{4}, \frac{7}{8}$. | 19. $\frac{5}{8}, \frac{7}{32}$. | 29. $\frac{1}{2}, \frac{1}{48}$. | 39. $\frac{7}{12}, \frac{9}{64}$. |
| 10. $\frac{3}{4}, \frac{3}{8}$. | 20. $\frac{7}{8}, \frac{13}{32}$. | 30. $\frac{3}{4}, \frac{1}{48}$. | 40. $\frac{15}{16}, \frac{63}{64}$. |

199. Addition of Mixed Numbers. In adding mixed numbers we may proceed as in the following example:

Required the sum of $3\frac{4}{5}$, $5\frac{7}{8}$, and $16\frac{3}{4}$.

The L.C.D. is $2 \times 2 \times 2 \times 5$, or 40. Reducing the fractions to similar fractions with denominator 40,

we have

$$3\frac{4}{5} = 3\frac{16}{20},$$

$$5\frac{7}{8} = 5\frac{35}{40},$$

and

$$16\frac{3}{4} = 16\frac{30}{40}.$$

Therefore

$$\text{the sum} = 24\frac{81}{40} = 24 + 2\frac{1}{40} = 26\frac{1}{40}.$$

Therefore, to add mixed numbers, add separately the integers and the fractions and find the sum of the results.

EXERCISE 105

Find the sum of:

- | | | |
|--------------------------------------|--|---|
| 1. $2\frac{1}{2}$, $3\frac{5}{8}$. | 7. $27\frac{3}{4}$, $42\frac{3}{8}$. | 13. $1\frac{5}{8}$, $3\frac{1}{3}$, $4\frac{3}{4}$. |
| 2. $2\frac{3}{4}$, $7\frac{7}{8}$. | 8. $26\frac{4}{5}$, $37\frac{1}{3}$. | 14. $1\frac{2}{3}$, $3\frac{4}{5}$, $5\frac{2}{15}$. |
| 3. $4\frac{1}{4}$, $6\frac{5}{8}$. | 9. $29\frac{3}{5}$, $32\frac{2}{3}$. | 15. $2\frac{1}{5}$, $4\frac{1}{8}$, $6\frac{3}{16}$. |
| 4. $3\frac{1}{2}$, $5\frac{1}{3}$. | 10. $46\frac{1}{8}$, $35\frac{3}{16}$. | 16. $3\frac{1}{8}$, $6\frac{3}{4}$, $5\frac{3}{16}$. |
| 5. $4\frac{1}{2}$, $6\frac{2}{3}$. | 11. $27\frac{2}{3}$, $62\frac{1}{8}$. | 17. $4\frac{2}{3}$, $3\frac{1}{8}$, $7\frac{1}{2}$. |
| 6. $5\frac{1}{4}$, $7\frac{1}{3}$. | 12. $37\frac{7}{8}$, $71\frac{5}{16}$. | 18. $3\frac{7}{8}$, $4\frac{3}{4}$, $5\frac{5}{16}$. |

19. In a pile of four books the first is $\frac{7}{8}$ of an inch thick, the second $1\frac{1}{8}$ in., the third $\frac{3}{4}$ of an inch, and the fourth $2\frac{1}{3}$ in. What is the height of the pile?

20. A merchant had four pieces of cloth of the same kind. The first piece contained $84\frac{3}{8}$ yd., the second $112\frac{5}{8}$ yd., the third $132\frac{3}{4}$ yd., and the fourth $148\frac{1}{2}$ yd. How many yards of cloth had he in all?

21. Five cans of sirup were measured very carefully and the first was found to contain $2\frac{1}{4}$ qt., the second $2\frac{3}{8}$ qt., the third $2\frac{7}{16}$ qt., the fourth $2\frac{5}{2}$ qt., and the fifth $2\frac{3}{8}$ qt. How much sirup was there in all?

200. Subtraction of Fractions. In subtraction we reduce the fractions to similar fractions with the L.C.D. as in addition.

For example, required to take $\frac{3}{8}$ from $\frac{2}{3}$.

The L.C.D. is 24. Reducing to twenty-fourths,

we have $\frac{2}{3} = \frac{16}{24}$,

and $\frac{3}{8} = \frac{9}{24}$.

Therefore the difference is $\frac{16}{24} - \frac{9}{24}$, or $\frac{7}{24}$.

Therefore, to subtract one fraction from another, reduce them to similar fractions having the least common denominator, if they are not similar. Write the difference of the numerators over the least common denominator, and reduce the result to lowest terms.

EXERCISE 106

Subtract as indicated :

- | | | |
|-------------------------------------|--------------------------------------|---|
| 1. $\frac{1}{2} - \frac{3}{8}$. | 16. $\frac{1}{2} - \frac{1}{8}$. | 31. $\frac{1}{2} - \frac{5}{72}$. |
| 2. $\frac{3}{4} - \frac{3}{8}$. | 17. $\frac{1}{2} - \frac{15}{64}$. | 32. $\frac{1}{2} - \frac{11}{72}$. |
| 3. $\frac{7}{8} - \frac{3}{4}$. | 18. $\frac{1}{2} - \frac{23}{64}$. | 33. $\frac{1}{4} - \frac{7}{72}$. |
| 4. $\frac{7}{8} - \frac{1}{2}$. | 19. $\frac{1}{4} - \frac{3}{64}$. | 34. $\frac{3}{4} - \frac{35}{72}$. |
| 5. $\frac{5}{8} - \frac{1}{4}$. | 20. $\frac{3}{4} - \frac{31}{64}$. | 35. $\frac{1}{8} - \frac{1}{72}$. |
| 6. $\frac{1}{2} - \frac{5}{16}$. | 21. $\frac{3}{4} - \frac{35}{64}$. | 36. $\frac{3}{8} - \frac{23}{72}$. |
| 7. $\frac{1}{2} - \frac{3}{16}$. | 22. $\frac{57}{64} - \frac{3}{4}$. | 37. $\frac{5}{8} - \frac{41}{72}$. |
| 8. $\frac{1}{4} - \frac{1}{16}$. | 23. $\frac{1}{8} - \frac{3}{64}$. | 38. $\frac{7}{8} - \frac{61}{72}$. |
| 9. $\frac{3}{4} - \frac{5}{16}$. | 24. $\frac{3}{8} - \frac{5}{64}$. | 39. $\frac{1}{6} - \frac{1}{72}$. |
| 10. $\frac{5}{8} - \frac{5}{16}$. | 25. $\frac{5}{8} - \frac{21}{64}$. | 40. $\frac{3}{16} - \frac{5}{72}$. |
| 11. $\frac{7}{8} - \frac{9}{16}$. | 26. $\frac{7}{8} - \frac{51}{64}$. | 41. $\frac{5}{16} - \frac{7}{72}$. |
| 12. $\frac{1}{2} - \frac{3}{32}$. | 27. $\frac{3}{16} - \frac{3}{64}$. | 42. $\frac{7}{72} - \frac{11}{144}$. |
| 13. $\frac{1}{4} - \frac{3}{32}$. | 28. $\frac{5}{16} - \frac{7}{64}$. | 43. $\frac{11}{12} - \frac{37}{144}$. |
| 14. $\frac{29}{32} - \frac{3}{4}$. | 29. $\frac{7}{16} - \frac{13}{48}$. | 44. $\frac{7}{10} - \frac{37}{100}$. |
| 15. $\frac{29}{32} - \frac{7}{8}$. | 30. $\frac{9}{16} - \frac{17}{48}$. | 45. $\frac{9}{10} - \frac{113}{1000}$. |

201. Subtraction of Mixed Numbers. In subtracting mixed numbers we proceed as in the following examples :

(1) Required to subtract $4\frac{3}{8}$ from $7\frac{7}{8}$.

The L.C.D. is 3×8 , or 24. Reducing to twenty-fourths,

we have $7\frac{7}{8} = 7\frac{21}{24}$,

and $4\frac{3}{8} = 4\frac{9}{24}$.

Therefore the difference is $7\frac{21}{24} - 4\frac{9}{24}$, or $3\frac{12}{24}$.

(2) Required to subtract $5\frac{3}{4}$ from $8\frac{1}{8}$.

The L.C.D. is $2 \times 2 \times 3$, or 12. Reducing to twelfths,

we have $8\frac{1}{8} = 8\frac{2}{12}$,

and $5\frac{3}{4} = 5\frac{9}{12}$.

Since we cannot take 9 twelfths from 2 twelfths, we change $8\frac{2}{12}$ to $7 + 1\frac{2}{12}$, or $7\frac{14}{12}$. Then $7\frac{14}{12} - 5\frac{9}{12} = 2\frac{5}{12}$.

Therefore, *to subtract one mixed number from another, reduce the fractions to similar fractions; then subtract the fractions and integers separately.*

EXERCISE 107

Subtract as indicated :

1. $7\frac{7}{8} - 2\frac{1}{4}$.

6. $11\frac{1}{8} - 2\frac{1}{4}$.

11. $12\frac{1}{3} - 8\frac{1}{4}$.

2. $9\frac{7}{8} - 3\frac{3}{4}$.

7. $10\frac{3}{8} - 3\frac{3}{4}$.

12. $15\frac{1}{4} - 7\frac{1}{3}$.

3. $6\frac{5}{8} - 2\frac{1}{2}$.

8. $15\frac{5}{8} - 7\frac{3}{4}$.

13. $16\frac{1}{3} - 2\frac{3}{8}$.

4. $8\frac{5}{8} - 2\frac{1}{4}$.

9. $12\frac{3}{4} - 6\frac{7}{8}$.

14. $19\frac{2}{3} - 3\frac{4}{5}$.

5. $7\frac{3}{16} - 5\frac{1}{8}$.

10. $14\frac{1}{4} - 9\frac{3}{8}$.

15. $31\frac{5}{8} - 7\frac{7}{12}$.

16. From a piece of silk containing $126\frac{1}{8}$ yd. there is cut $23\frac{3}{4}$ yd., $36\frac{5}{8}$ yd., and $24\frac{1}{2}$ yd. How much is left?

17. A man had a farm of $275\frac{1}{8}$ acres. He sold $45\frac{3}{4}$ acres to one man, $27\frac{7}{8}$ acres to another, and $16\frac{1}{8}$ acres to a third. How much had he left?

18. A train that reached Rochester at $9\frac{1}{4}$ o'clock A.M. left New York $7\frac{1}{2}$ hours earlier. When did it leave New York?

202. Multiplication of a Fraction by an Integer. Just as

2 times 2 feet are 4 feet,

so

2 times 2 fifths ($\frac{2}{5}$) are 4 fifths ($\frac{4}{5}$).In the same way, $3 \times \frac{1}{8} = \frac{3}{8}$, $3 \times \frac{2}{7} = \frac{6}{7}$, $5 \times \frac{3}{8} = \frac{15}{8} = 1\frac{7}{8}$.If we have to find $6 \times \frac{3}{4}$, it is better to indicate the multiplication first and shorten the actual work by cancellation, thus :

$$6 \times \frac{3}{4} = \frac{\overset{3}{\cancel{6}} \times 3}{\underset{2}{\cancel{4}}} = \frac{9}{2} = 4\frac{1}{2}.$$

Therefore, to multiply a fraction by an integer, multiply the numerator by the integer and write the product over the denominator.

Before actually multiplying, indicate the multiplication and cancel if possible. Reduce the result to lowest terms.

EXERCISE 108*Multiply :*

- | | | |
|--------------------------------|----------------------------------|----------------------------------|
| 1. $6 \times \frac{3}{8}$. | 11. $120 \times \frac{9}{16}$. | 21. $126 \times \frac{5}{12}$. |
| 2. $8 \times \frac{3}{4}$. | 12. $128 \times \frac{11}{16}$. | 22. $140 \times \frac{7}{12}$. |
| 3. $15 \times \frac{2}{3}$. | 13. $160 \times \frac{3}{32}$. | 23. $150 \times \frac{11}{12}$. |
| 4. $18 \times \frac{3}{4}$. | 14. $176 \times \frac{5}{32}$. | 24. $216 \times \frac{5}{24}$. |
| 5. $22 \times \frac{3}{8}$. | 15. $192 \times \frac{7}{32}$. | 25. $228 \times \frac{7}{24}$. |
| 6. $26 \times \frac{5}{8}$. | 16. $208 \times \frac{9}{32}$. | 26. $252 \times \frac{11}{24}$. |
| 7. $32 \times \frac{7}{8}$. | 17. $224 \times \frac{11}{32}$. | 27. $264 \times \frac{13}{24}$. |
| 8. $35 \times \frac{3}{16}$. | 18. $240 \times \frac{15}{32}$. | 28. $325 \times \frac{9}{16}$. |
| 9. $42 \times \frac{5}{16}$. | 19. $256 \times \frac{7}{32}$. | 29. $432 \times \frac{7}{8}$. |
| 10. $56 \times \frac{7}{16}$. | 20. $272 \times \frac{1}{32}$. | 30. $500 \times \frac{1}{8}$. |

31. If a book costs $\$ \frac{3}{4}$, how much will 17 books cost ?32. If a train travels $\frac{5}{6}$ of a mile in a minute, how far will it travel in 132 minutes ?

203. To find a Fractional Part of an Integer. One of the most important operations with fractions is to find a fractional part of a number. To take a fractional part of a number is called *multiplication by a fraction*.

That is, to take $\frac{1}{3}$ of 6 is called the multiplication of 6 by $\frac{1}{3}$. We indicate the operation by the expression: $\frac{1}{3}$ of 6 = 2, or $\frac{1}{3} \times 6 = 2$, the symbol $\times$ being read *of* in the latter case. Special emphasis should be given to this important case.

Therefore, *to multiply an integer by a fraction, multiply the integer by the numerator of the fraction and write the product over the denominator.*

It is advisable to cancel before the actual multiplication, if possible. Thus, in taking $\frac{4}{15}$ of 12, the work should be arranged as follows:

$$\frac{4}{15} \text{ of } 12 = \frac{4 \times \overset{4}{\cancel{12}}}{\underset{5}{\cancel{15}}} = \frac{16}{5} = 3\frac{1}{5}.$$

EXERCISE 109

Multiply:

1. $\frac{2}{3}$ of 18.

8. $\frac{5}{12}$ of 72.

15. $\frac{5}{24}$ of 72.

2. $\frac{3}{4}$ of 24.

9. $\frac{7}{12}$ of 80.

16. $\frac{7}{24}$ of 80.

3. $\frac{2}{5}$ of 35.

10. $1\frac{1}{2}$ of 96.

17. $1\frac{1}{4}$ of 84.

4. $\frac{3}{5}$ of 48.

11. $\frac{3}{16}$ of 72.

18. $1\frac{3}{4}$ of 96.

5. $\frac{4}{5}$ of 75.

12. $\frac{5}{16}$ of 80.

19. $1\frac{7}{4}$ of 100.

6. $\frac{5}{6}$ of 93.

13. $\frac{7}{16}$ of 96.

20. $3\frac{5}{2}$ of 100.

7. $\frac{3}{8}$ of 24.

14. $\frac{9}{16}$ of 100.

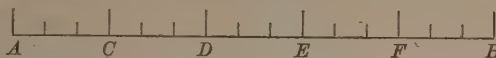
21. $3\frac{7}{2}$ of 200.

22. At 72¢ a yard, how much will $\frac{5}{8}$ of a yard of cloth cost? $\frac{3}{4}$ of a yard?

23. At 96¢ a yard, how much will $\frac{7}{8}$ of a yard of cloth cost? $\frac{5}{8}$ of a yard?

24. At 88¢ a yard, how much will $\frac{3}{4}$ of a yard of cloth cost? $\frac{3}{8}$ of a yard? $\frac{5}{8}$ of a yard?

204. To find a Fractional Part of a Fraction. Suppose it is required to find $\frac{2}{3}$ of $\frac{4}{5}$.



In this figure let the line AB be divided into five equal parts at the points C , D , E , and F . Then AF is $\frac{4}{5}$ of AB .

Let each of the 5 parts be divided into 3 equal parts. Then each of these parts evidently is $\frac{1}{15}$ of AB .

That is, $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$ of the whole.

Then $\frac{1}{3}$ of $\frac{4}{5}$ must be 4 times as much, or $\frac{4}{15}$.

Then $\frac{2}{3}$ of $\frac{4}{5}$ must be twice $\frac{4}{15}$, or $\frac{8}{15}$.

Therefore, to multiply a fraction by a fraction, find the product of the numerators for the required numerator and the product of the denominators for the required denominator.

This case also demands special emphasis.

Always cancel as far as possible. For example :

$$\frac{11}{12} \text{ of } \frac{15}{22} = \frac{11 \times 15}{12 \times 22} = \frac{5}{8}.$$

$\begin{array}{cc} 5 & \\ 4 & 2 \end{array}$

EXERCISE 110

Multiply :

- | | | |
|---------------------------------------|--|---|
| 1. $\frac{3}{4}$ of $\frac{8}{9}$. | 7. $\frac{1}{8}$ of $\frac{4}{5}$. | 13. $\frac{1}{32}$ of $\frac{4}{5}$. |
| 2. $\frac{2}{3}$ of $\frac{9}{10}$. | 8. $\frac{3}{8}$ of $\frac{7}{8}$. | 14. $\frac{1}{48}$ of $\frac{24}{5}$. |
| 3. $\frac{2}{3}$ of $\frac{15}{16}$. | 9. $\frac{7}{8}$ of $\frac{4}{7}$. | 15. $\frac{7}{48}$ of $\frac{16}{21}$. |
| 4. $\frac{2}{3}$ of $\frac{11}{24}$. | 10. $\frac{5}{8}$ of $\frac{3}{16}$. | 16. $\frac{3}{8}$ of $\frac{5}{48}$. |
| 5. $\frac{2}{3}$ of $\frac{35}{32}$. | 11. $\frac{3}{16}$ of $\frac{8}{9}$. | 17. $\frac{4}{5}$ of $\frac{25}{48}$. |
| 6. $\frac{4}{5}$ of $\frac{5}{24}$. | 12. $\frac{7}{16}$ of $\frac{8}{21}$. | 18. $\frac{5}{9}$ of $\frac{81}{100}$. |
19. How much is half of three quarters of a yard ?
20. How much is $\frac{2}{3}$ of $\frac{5}{8}$ of an acre ?
21. How much is a quarter of $\frac{7}{8}$ of a mile ?
22. How much is an eighth of $\frac{5}{16}$ of an inch ?

Find the products of the following :

23. $\frac{5}{8} \times \frac{2}{3}\frac{1}{2}$.

29. $\frac{1}{8}\frac{3}{4} \times \frac{2}{3}$.

35. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$.

24. $\frac{3}{8} \times \frac{1}{2}\frac{1}{4}$.

30. $\frac{1}{8}\frac{5}{6} \times \frac{2}{5}$.

36. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$.

25. $\frac{7}{8} \times \frac{3}{5}\frac{2}{5}$.

31. $\frac{7}{12} \times \frac{4}{9}$.

37. $\frac{2}{3} \times \frac{2}{3} \times \frac{3}{4}$.

26. $\frac{9}{16} \times \frac{4}{9}$.

32. $\frac{5}{12} \times \frac{3}{5}$.

38. $\frac{3}{5} \times \frac{3}{4} \times \frac{1}{8}$.

27. $\frac{7}{16} \times \frac{5}{7}$.

33. $\frac{1}{2}\frac{1}{2} \times \frac{4}{5}$.

39. $\frac{3}{4} \times \frac{3}{8} \times \frac{4}{5}$.

28. $\frac{5}{16} \times \frac{3}{4}\frac{2}{5}$.

34. $\frac{1}{8}\frac{1}{4} \times \frac{4}{11}$.

40. $\frac{5}{8} \times \frac{7}{8} \times \frac{3}{8}$.

41. The perimeter of (distance around) a square being $\frac{7}{8}$ of an inch, what is the length of each side?

42. The perimeter of a triangle being $\frac{7}{8}$ of an inch and the sides being equal, what is the length of each side?

43. If the distance to a certain house is $\frac{3}{8}$ of a mile, what fraction of a mile has a boy gone when he has covered $\frac{5}{16}$ of the distance?

44. If it takes a train $\frac{3}{4}$ of an hour to reach a certain station, what fraction of an hour will it take to cover $\frac{3}{5}$ of the distance?

45. A remnant of lace cost \$ $\frac{5}{8}$. How much did $\frac{4}{5}$ of the remnant cost? How much did $\frac{2}{3}$ of it cost?

46. If $\frac{3}{4}$ of a yard of lace is needed to trim two neck bands, how much lace will be needed to trim one neck band?

47. If an automobile travels a mile in $\frac{5}{6}\frac{2}{3}$ of a minute in a certain race, how long would it take to travel $\frac{3}{4}$ of a mile at the same rate?

48. If a woman pays \$ $\frac{3}{4}$ for a yard of ribbon, how much would she pay for $\frac{2}{3}$ of a yard? for $\frac{1}{3}$ of a yard? for $\frac{1}{2}$ of a yard?

49. A milliner used $\frac{7}{8}$ of a yard of velvet for the upper brim and the crown of a hat, and $\frac{2}{5}$ as much for the lower brim. How much velvet did she use for the hat?

✓ 50. A piece of goods $\frac{7}{8}$ of a yard long is $\frac{2}{3}$ as long after tucking. How long is it after tucking?

✓ 51. A piece of cloth $\frac{5}{8}$ of a yard long loses $\frac{2}{5}$ of its length in tucking. How long is it then?

52. A woman has cut some goods $\frac{1}{4}$ of a yard wide for a ruffle. She uses $\frac{1}{4}$ of this width for a hem and $\frac{2}{3}$ of the rest for tucking. How wide is the ruffle when it is finished?

53. If I pay \$2 for $\frac{3}{4}$ of a yard of a certain kind of velvet, what part of a yard will I buy with \$1? What part with \$1.50, which is $\frac{3}{4}$ of \$2?

54. A girl in a shop makes $\frac{5}{6}$ of a yard of a certain kind of chiffon appliqué in an hour. How much does she make in 20 minutes? in 45 minutes?

✓ 55. If it takes $\frac{3}{4}$ of an hour to solve a page of examples, how long would it take to solve $\frac{2}{3}$ of them at the same rate? How many minutes?

56. A man who owned $\frac{5}{8}$ of the stock in a certain company sold $\frac{2}{3}$ of what he owned. What part of the stock of the company did he then own?

57. In making the plans for a house an architect wishes to draw a line that shall be $\frac{2}{3}$ as long as one that is $\frac{1}{5}$ of an inch long. How long shall he draw it?

58. In 40 minutes the minute hand of a clock makes $\frac{2}{3}$ of a revolution. What part of a revolution will it make in 32 minutes? (32 minutes = $\frac{3}{4}$ of 40 minutes.)

59. In 55 minutes the minute hand of a clock makes $1\frac{1}{2}$ of a revolution. What part of a revolution will it make in 33 minutes?

60. A boy finds that when the front wheel of his cart has made one revolution the cart has moved $\frac{5}{6}$ of a yard. How far has it gone when the wheel has made $\frac{3}{4}$ of a revolution?

205. Multiplication of a Mixed Number by an Integer. Since $3 \times 5 = 15$, and $3 \times \frac{5}{8} = \frac{5}{2} = 2\frac{1}{2}$, therefore $3 \times 5\frac{5}{8}$ equals the sum of these products, that is, $15 + 2\frac{1}{2}$, or $17\frac{1}{2}$.

Therefore, *to multiply a mixed number by an integer, multiply the whole part and the fractional part separately by the integer and add the products.*

EXERCISE 111

Multiply:

- | | | |
|---------------------------------|---------------------------------|-----------------------------------|
| 1. $2 \times 17\frac{3}{8}$. | 11. $2 \times 16\frac{3}{16}$. | 21. $4 \times \$32\frac{3}{4}$. |
| 2. $3 \times 26\frac{1}{4}$. | 12. $3 \times 17\frac{5}{16}$. | 22. $7 \times \$21\frac{1}{4}$. |
| 3. $3 \times 18\frac{3}{4}$. | 13. $4 \times 25\frac{7}{16}$. | 23. $6 \times \$35\frac{1}{2}$. |
| 4. $4 \times 35\frac{3}{4}$. | 14. $8 \times 39\frac{1}{16}$. | 24. $8 \times \$27\frac{3}{4}$. |
| 5. $2 \times 43\frac{3}{8}$. | 15. $8 \times 58\frac{1}{8}$. | 25. $12 \times 117\frac{1}{2}$. |
| 6. $3 \times 25\frac{5}{8}$. | 16. $2 \times 45\frac{3}{32}$. | 26. $15 \times 222\frac{1}{5}$. |
| 7. $6 \times 56\frac{7}{8}$. | 17. $4 \times 74\frac{5}{32}$. | 27. $12 \times 305\frac{1}{8}$. |
| 8. $4 \times 33\frac{5}{12}$. | 18. $9 \times 86\frac{7}{32}$. | 28. $16 \times 211\frac{7}{8}$. |
| 9. $3 \times 25\frac{7}{12}$. | 19. $8 \times 69\frac{9}{32}$. | 29. $25 \times 107\frac{1}{4}$. |
| 10. $5 \times 48\frac{1}{12}$. | 20. $6 \times 97\frac{1}{32}$. | 30. $35 \times 121\frac{3}{10}$. |
31. At $\$2\frac{1}{4}$ a yard, how much will 9 yards of cloth cost?
32. How many feet in the perimeter of a square $26\frac{3}{8}$ feet on a side?
33. A dealer sells 16 notebooks at $3\frac{3}{4}$ cents each. How much does he receive?
34. A rod being $16\frac{1}{2}$ feet long, how many feet in the perimeter of a lot that is 26 rods long and 16 rods wide?
35. In a motor race one of the cars averaged $1\frac{3}{8}$ miles a minute. How many miles did it travel in 7 minutes?
36. If a boy lives $1\frac{3}{4}$ miles from this school, and has a cousin living 25 times as far from here, how far from here does the cousin live?

206. Multiplication of an Integer by a Mixed Number. Since $2 \times 9 = 18$, and $\frac{3}{4}$ of $9 = \frac{27}{4} = 6\frac{3}{4}$, therefore $2\frac{3}{4} \times 9$ equals the sum of these products, or $18 + 6\frac{3}{4}$, or $24\frac{3}{4}$.

The symbol $\times$ after a mixed number is read *times*, as with integers.

Therefore, *to multiply an integer by a mixed number, multiply the integer by the whole part and the fractional part separately and add the products.*

In practice, to take the product of an integer and a mixed number we use as the multiplier whichever is the more convenient. For example, it is shorter to multiply $249\frac{3}{4}$ by 2 than to multiply 2 by $249\frac{3}{4}$. But if it is required to multiply $4\frac{7}{8}$ by 648, it is easier to change the order and multiply 648 by $4\frac{7}{8}$. These two examples may be arranged thus:

$$\begin{array}{r} 249\frac{3}{4} \\ \times 2 \\ \hline 498\frac{3}{2} = 499\frac{1}{2} \end{array}$$

$$\begin{array}{r} 648 \\ \times 4\frac{7}{8} \\ \hline 5184 \\ 4536 \\ \hline 26520 \end{array}$$

(1 of 648)
(7 of 648)
(4 of 648)

EXERCISE 112

Multiply:

- | | | |
|--------------------------------|---------------------------------|-----------------------------------|
| 1. $3\frac{1}{2} \times 75$. | 11. $3\frac{7}{8} \times 96$. | 21. $3\frac{3}{2} \times 160$. |
| 2. $4\frac{2}{3} \times 69$. | 12. $5\frac{7}{8} \times 91$. | 22. $4\frac{5}{2} \times 640$. |
| 3. $5\frac{1}{4} \times 72$. | 13. $2\frac{1}{16} \times 32$. | 23. $7\frac{7}{2} \times 176$. |
| 4. $7\frac{3}{4} \times 84$. | 14. $3\frac{3}{16} \times 64$. | 24. $8\frac{9}{2} \times 656$. |
| 5. $6\frac{1}{4} \times 31$. | 15. $5\frac{5}{16} \times 80$. | 25. $9\frac{1}{2} \times 352$. |
| 6. $8\frac{3}{4} \times 39$. | 16. $9\frac{7}{16} \times 96$. | 26. $12\frac{3}{2} \times 288$. |
| 7. $7\frac{3}{8} \times 48$. | 17. $5\frac{3}{16} \times 38$. | 27. $15\frac{5}{2} \times 256$. |
| 8. $9\frac{3}{8} \times 50$. | 18. $4\frac{5}{16} \times 60$. | 28. $20\frac{5}{8} \times 240$. |
| 9. $5\frac{5}{8} \times 64$. | 19. $9\frac{7}{16} \times 88$. | 29. $300\frac{7}{8} \times 528$. |
| 10. $7\frac{5}{8} \times 70$. | 20. $7\frac{9}{16} \times 62$. | 30. $500\frac{9}{4} \times 384$. |

207. Multiplication of Two Mixed Numbers. We may multiply one mixed number by another by the method explained in § 206 or by that in § 204.

For example, we may multiply $9\frac{3}{4}$ by $2\frac{1}{3}$ as here shown. But it is usually easier to reduce both to improper fractions and multiply as in § 204. Thus in the case of $6\frac{2}{3} \times 4\frac{1}{6}$ we have $6\frac{2}{3} = \frac{20}{3}$, and $4\frac{1}{6} = \frac{25}{6}$. Then

$$\begin{array}{r} 9\frac{3}{4} \\ 2\frac{1}{3} \\ \hline 3\frac{1}{4} (\frac{1}{3} \text{ of } 9\frac{3}{4}) \\ 19\frac{2}{4} (2 \times 9\frac{3}{4}) \\ \hline 22\frac{3}{4} \end{array}$$

For example, we may multiply $9\frac{3}{4}$ by $2\frac{1}{3}$ as here shown. But it is usually easier to reduce both to improper fractions and multiply as in § 204. Thus in the case of $6\frac{2}{3} \times 4\frac{1}{6}$ we have $6\frac{2}{3} = \frac{20}{3}$, and $4\frac{1}{6} = \frac{25}{6}$. Then

$$6\frac{2}{3} \times 4\frac{1}{6} = \frac{2}{3} \times \frac{15}{6} = \frac{32 \times 75}{5 \times 18} = 30.$$

Therefore, *to multiply a mixed number by a mixed number, reduce each to an improper fraction and multiply.*

In the case of a fraction and a mixed number, reduce the mixed number to an improper fraction.

EXERCISE 113

Multiply:

- | | | |
|---|---|---|
| 1. $2\frac{1}{2} \times 3\frac{3}{8}$. | 6. $2\frac{1}{2} \times 31\frac{1}{2}$. | 11. $33\frac{1}{3} \times 66\frac{2}{3}$. |
| 2. $3\frac{1}{3} \times 4\frac{3}{4}$. | 7. $3\frac{1}{2} \times 25\frac{1}{3}$. | 12. $33\frac{1}{3} \times 120\frac{2}{3}$. |
| 3. $2\frac{2}{3} \times 3\frac{1}{4}$. | 8. $7\frac{1}{2} \times 33\frac{1}{3}$. | 13. $16\frac{2}{3} \times 132\frac{2}{3}$. |
| 4. $5\frac{3}{5} \times 7\frac{3}{4}$. | 9. $3\frac{3}{4} \times 66\frac{2}{3}$. | 14. $16\frac{2}{3} \times 336\frac{2}{3}$. |
| 5. $4\frac{1}{3} \times 6\frac{3}{8}$. | 10. $5\frac{1}{3} \times 29\frac{2}{3}$. | 15. $21\frac{3}{4} \times 441\frac{1}{3}$. |

16. At the rate of \$66 $\frac{2}{3}$ a month, how much rent will a man pay for a city apartment in 6 months?

17. If a train travels at the rate of 48 $\frac{3}{4}$ miles per hour, how far will it travel in 2 $\frac{1}{2}$ hours?

18. Some blotting paper is $\frac{3}{64}$ of an inch thick. What will be the height of a pile of 2 $\frac{3}{4}$ dozen sheets?

19. If an arithmetic is 1 $\frac{1}{8}$ inches thick and a reader is $\frac{1}{8}$ of an inch thick, what is the height of a pile of 7 arithmetics and 9 readers?

208. Division of a Fraction by an Integer. There are two cases to be considered:

(1) When the numerator of the fraction is divisible by the divisor.

In this case, just as $8 \text{ ft.} \div 2 = 4 \text{ ft.}$, and $\$8 \div 2 = \$4$, so $8 \text{ ninths} (\frac{8}{9}) \div 2 = 4 \text{ ninths} (\frac{4}{9})$. In the same way, $\frac{9}{10} \div 3 = \frac{3}{10}$, and $\frac{15}{16} \div 5 = \frac{3}{16}$.

(2) When the numerator of the fraction is not divisible by the divisor.

Take the case of $\frac{3}{8} \div 2$. This is the same as $\frac{1}{2}$ of $\frac{3}{8}$, which equals $\frac{3}{16}$. In the same way, $\frac{5}{8} \div 3 = \frac{1}{3}$ of $\frac{5}{8} = \frac{5}{24}$.

Therefore, *to divide a fraction by an integer, divide the numerator by the integer if possible, and write the quotient over the denominator.*

If it is impossible to divide the numerator by the integer, multiply the denominator by the integer and write the product under the numerator.

It is easier to indicate this multiplication and then cancel if possible. Thus

$$\frac{15}{16} \div 6 = \frac{\overset{5}{15}}{\underset{2}{6} \times 16} = \frac{5}{32}.$$

EXERCISE 114

Find the quotient of:

- | | | |
|-----------------------------|-------------------------------|----------------------------------|
| 1. $\frac{36}{48} \div 7.$ | 8. $\frac{15}{32} \div 20.$ | 15. $\frac{21}{24} \div 126.$ |
| 2. $\frac{27}{32} \div 9.$ | 9. $\frac{12}{24} \div 57.$ | 16. $\frac{23}{24} \div 161.$ |
| 3. $\frac{51}{64} \div 17.$ | 10. $\frac{17}{48} \div 51.$ | 17. $\frac{27}{100} \div 170.$ |
| 4. $\frac{3}{16} \div 6.$ | 11. $\frac{34}{100} \div 51.$ | 18. $\frac{37}{100} \div 333.$ |
| 5. $\frac{15}{16} \div 10.$ | 12. $\frac{38}{100} \div 57.$ | 19. $\frac{33}{1000} \div 77.$ |
| 6. $\frac{25}{32} \div 15.$ | 13. $\frac{5}{32} \div 100.$ | 20. $\frac{51}{1000} \div 170.$ |
| 7. $\frac{15}{64} \div 25.$ | 14. $\frac{25}{64} \div 125.$ | 21. $\frac{143}{1000} \div 130.$ |

209. Division of a Mixed Number by an Integer. A mixed number may be divided by an integer by first reducing it to an improper fraction and then proceeding as in § 208.

For example, $3\frac{3}{8} \div 7 = \frac{27}{8} \div 7 = \frac{27}{56}$.

In case the mixed number is long, however, it is easier to divide as with integers, continuing the division until a remainder is found that is smaller than the divisor.

For example, $177\frac{1}{2} \div 5 = 35$ with remainder $2\frac{1}{2}$. Dividing the remainder, we have $2\frac{1}{2} \div 5 = \frac{5}{2} \div 5 = \frac{1}{2}$. Hence the entire quotient is $35\frac{1}{2}$.

Therefore, *to divide a mixed number by an integer, divide as with integers until a remainder is found that is less than the divisor; then divide this remainder by the divisor.*

We may reduce to a decimal fraction before dividing.

EXERCISE 115

Find the quotient of:

- | | | |
|-----------------------------|-------------------------------|---------------------------------|
| 1. $3\frac{3}{4} \div 9$. | 7. $23\frac{3}{4} \div 8$. | 13. $142\frac{1}{2} \div 5$. |
| 2. $4\frac{3}{8} \div 7$. | 8. $28\frac{3}{4} \div 7$. | 14. $166\frac{2}{3} \div 10$. |
| 3. $5\frac{5}{8} \div 6$. | 9. $32\frac{5}{8} \div 9$. | 15. $133\frac{1}{3} \div 20$. |
| 4. $6\frac{7}{8} \div 5$. | 10. $41\frac{5}{8} \div 6$. | 16. $275\frac{3}{4} \div 12$. |
| 5. $8\frac{3}{8} \div 4$. | 11. $34\frac{1}{16} \div 7$. | 17. $266\frac{7}{8} \div 15$. |
| 6. $7\frac{3}{8} \div 59$. | 12. $78\frac{3}{16} \div 9$. | 18. $345\frac{5}{16} \div 25$. |

19. If 9 books cost $\$6\frac{3}{4}$, or $\$6.75$, what is the average cost?

20. If 7 yards of cloth cost $\$8\frac{3}{4}$, or $\$8.75$, what does it cost per yard?

21. If 19 yards of basket-weave burlap cost $\$21\frac{3}{8}$, or $\$21.37\frac{1}{2}$, what does it cost per yard?

22. If 15 dozen pairs of buttonhole scissors cost $\$67\frac{1}{2}$, or $\$67.50$, what do they cost per dozen?

210. Dividing by a Unit Fraction. Since there are 4 fourths of an inch in 1 inch, therefore $1 \text{ inch} \div \frac{1}{4} \text{ of an inch} = 4$, and $2 \text{ inches} \div \frac{1}{4} \text{ of an inch} = 8$. In the same way, $5 \div \frac{1}{4} = 20$, $7 \div \frac{1}{4} = 28$, and so on.

And if $5 \div \frac{1}{4} = 20$, then $\frac{5}{8} \div \frac{1}{4}$ must equal $\frac{1}{8}$ as much, or $\frac{20}{8}$, or $2\frac{1}{2}$.

Therefore, *to divide by a unit fraction, multiply by the denominator.*

That is,

$$15 \div \frac{1}{8} = 8 \times 15 = 120,$$

$$27 \div \frac{1}{4} = 4 \times 27 = 108,$$

$$\frac{3}{8} \div \frac{1}{16} = 16 \times \frac{3}{8} = 6,$$

$$2\frac{1}{2} \div \frac{1}{32} = 32 \times 2\frac{1}{2} = 80.$$

EXERCISE 116

Find the quotient of:

1. $28 \div \frac{1}{2}$.

6. $\frac{3}{4} \div \frac{1}{3}$.

11. $2\frac{1}{2} \div \frac{1}{4}$.

2. $37 \div \frac{1}{3}$.

7. $\frac{3}{4} \div \frac{1}{16}$.

12. $3\frac{1}{3} \div \frac{1}{6}$.

3. $49 \div \frac{1}{4}$.

8. $\frac{4}{5} \div \frac{1}{35}$.

13. $5\frac{1}{4} \div \frac{1}{8}$.

4. $56 \div \frac{1}{5}$.

9. $\frac{5}{8} \div \frac{1}{72}$.

14. $6\frac{3}{4} \div \frac{1}{12}$.

5. $93 \div \frac{1}{7}$.

10. $\frac{7}{8} \div \frac{1}{25}$.

15. $25\frac{1}{3} \div \frac{1}{32}$.

16. If a sheet of veneer is $\frac{1}{16}$ of an inch thick, how many sheets pressed together will have a thickness of $1\frac{1}{8}$ inches?

17. If a sheet of blotting paper is $\frac{1}{32}$ of an inch thick, how many sheets are there in a pile that is $15\frac{5}{8}$ inches thick?

18. If each book is $\frac{1}{12}$ of a foot thick, how many books are there in a single pile of books $8\frac{1}{3}$ feet high?

19. If a bicycle is traveling at the rate of a mile in $\frac{1}{3}$ of an hour, how many miles will it travel in $3\frac{3}{4}$ hours?

20. How many weights of $\frac{1}{16}$ of a pound (or 1 ounce) each will it take to weigh $17\frac{3}{4}$ pounds?

21. If a wagon wheel makes $\frac{1}{2}$ of a revolution in going 1 foot, how many feet will it go in making $17\frac{3}{4}$ revolutions?

211. Dividing by a Fraction. We have found (§ 210) that when we have to divide by $\frac{1}{3}$ we may multiply by 3. We also know that when we multiply the divisor by 2 we divide the quotient by 2.

That is, $6 \div \frac{1}{3} = 3 \times 6$ (§ 210),

and $6 \div \frac{2}{3} = \text{half as much } (\S 65) = \frac{3 \times 6}{2} = 9.$

That is, $6 \div \frac{2}{3} = \frac{3}{2}$ of 6.

Therefore, *to divide by a common fraction, multiply by the inverted divisor.*

That is, $15 \div \frac{3}{5} = \frac{5}{3}$ of $15 = 25$. Similarly, $\frac{3}{8} \div \frac{5}{6} = \frac{3}{5}$ of $\frac{3}{\frac{8}{4}} = \frac{9}{20}$.

An inverted fraction is also called the *reciprocal* of the fraction. That is, the reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$. The product of a fraction and its reciprocal is always one; thus, $\frac{6}{5}$ of $\frac{5}{6} = 1$.

EXERCISE 117

Find the quotient of:

- | | | | |
|----------------------------|-----------------------------|---------------------------------------|--|
| 1. $3 \div \frac{2}{3}$. | 11. $15 \div \frac{3}{4}$. | 21. $\frac{2}{3} \div \frac{3}{4}$. | 31. $2\frac{1}{3} \div \frac{3}{4}$. |
| 2. $5 \div \frac{3}{4}$. | 12. $15 \div \frac{3}{4}$. | 22. $\frac{3}{4} \div \frac{2}{3}$. | 32. $3\frac{2}{3} \div \frac{3}{4}$. |
| 3. $7 \div \frac{4}{5}$. | 13. $16 \div \frac{4}{5}$. | 23. $\frac{3}{8} \div \frac{5}{8}$. | 33. $5\frac{1}{8} \div \frac{1}{2}$. |
| 4. $9 \div \frac{3}{5}$. | 14. $21 \div \frac{3}{5}$. | 24. $\frac{7}{8} \div \frac{5}{8}$. | 34. $5\frac{3}{8} \div \frac{3}{4}$. |
| 5. $8 \div \frac{3}{8}$. | 15. $24 \div \frac{2}{5}$. | 25. $\frac{3}{8} \div \frac{3}{4}$. | 35. $7\frac{5}{8} \div \frac{3}{8}$. |
| 6. $5 \div \frac{5}{8}$. | 16. $30 \div \frac{3}{8}$. | 26. $\frac{3}{4} \div \frac{7}{8}$. | 36. $8\frac{5}{8} \div \frac{5}{8}$. |
| 7. $8 \div \frac{7}{8}$. | 17. $40 \div \frac{5}{8}$. | 27. $1\frac{5}{2} \div \frac{3}{4}$. | 37. $6\frac{7}{8} \div \frac{5}{8}$. |
| 8. $9 \div \frac{5}{8}$. | 18. $20 \div \frac{7}{8}$. | 28. $1\frac{7}{2} \div \frac{2}{3}$. | 38. $7\frac{3}{8} \div \frac{7}{8}$. |
| 9. $8 \div \frac{2}{5}$. | 19. $32 \div \frac{5}{8}$. | 29. $\frac{3}{8} \div 1\frac{7}{2}$. | 39. $9\frac{5}{8} \div \frac{3}{4}$. |
| 10. $9 \div \frac{2}{7}$. | 20. $75 \div \frac{5}{8}$. | 30. $\frac{5}{8} \div 1\frac{1}{2}$. | 40. $8\frac{5}{2} \div 1\frac{7}{6}$. |

212. Dividing by a Mixed Number. To divide by a mixed number it is generally best to reduce the divisor to an improper fraction and proceed as in § 211.

For example, to divide 10 by $3\frac{1}{3}$ is the same as to divide 10 by $\frac{10}{3}$, or to take $\frac{3}{10}$ of 10. But $\frac{3}{10}$ of 10 = 3; hence $10 \div 3\frac{1}{3} = 3$.

Therefore, *to divide by a mixed number, reduce the divisor to an improper fraction and multiply the dividend by the reciprocal of the divisor.*

$$\text{Thus} \quad 17\frac{1}{2} \div 3\frac{1}{4} = \frac{35}{2} \div \frac{13}{4} = \frac{35}{2} \times \frac{4}{13} = \frac{70}{13} = 5\frac{5}{13}.$$

EXERCISE 118

Find the quotient of:

1. $3 \div 7\frac{1}{2}$.

10. $48 \div 12\frac{1}{2}$.

19. $\frac{5}{8} \div 7\frac{3}{4}$.

2. $5 \div 4\frac{3}{4}$.

11. $72 \div 16\frac{2}{3}$.

20. $\frac{3}{18} \div 6\frac{2}{3}$.

3. $9 \div 6\frac{1}{2}$.

12. $26 \div 33\frac{1}{3}$.

21. $\frac{5}{18} \div 16\frac{2}{3}$.

4. $7 \div 5\frac{1}{8}$.

13. $48 \div 66\frac{2}{3}$.

22. $12\frac{1}{2} \div 6\frac{1}{4}$.

5. $5 \div 4\frac{3}{8}$.

14. $50 \div 37\frac{1}{2}$.

23. $12\frac{1}{2} \div 3\frac{1}{3}$.

6. $5 \div 6\frac{7}{8}$.

15. $\frac{3}{4} \div 2\frac{1}{2}$.

24. $12\frac{1}{2} \div 6\frac{2}{3}$.

7. $8 \div 3\frac{3}{4}$.

16. $\frac{3}{8} \div 3\frac{1}{3}$.

25. $16\frac{2}{3} \div 12\frac{1}{2}$.

8. $25 \div 3\frac{1}{2}$.

17. $\frac{4}{5} \div 2\frac{2}{3}$.

26. $33\frac{1}{3} \div 66\frac{2}{3}$.

9. $62 \div 6\frac{3}{4}$.

18. $\frac{3}{8} \div 5\frac{1}{4}$.

27. $37\frac{1}{2} \div 6\frac{1}{4}$.

28. Find the cost per ton if $3\frac{1}{4}$ tons of hay cost $\$37\frac{3}{8}$.

29. How many strips of cloth, each $2\frac{1}{2}$ yards long, can be cut from a piece that is 45 yards long?

30. A dealer paid $\$247\frac{1}{2}$ for $7\frac{1}{2}$ dozen pairs of shoes. How much did they cost per dozen? How much per pair?

31. A dealer paid $\$224\frac{1}{4}$ for $3\frac{1}{4}$ dozen pairs of shears. How much did they cost per dozen? How much per pair?

213. To find what Fraction One Number is of Another. For example, 3 is what fraction of 4?

Since $1 = \frac{1}{4}$ of 4,
therefore $3 = 3 \times \frac{1}{4}$ of 4, or $\frac{3}{4}$ of 4.

The result therefore equals $3 \div 4$.

As a further example, $\frac{3}{7}$ is what part of $\frac{2}{5}$?

Since $1 = \frac{5}{2}$ of $\frac{2}{5}$,
therefore $\frac{3}{7} = \frac{3}{7}$ of $\frac{5}{2}$ of the second fraction
 $= \frac{3}{7}$ of the second fraction.

The result therefore equals $\frac{3}{7} \div \frac{2}{5}$.

Therefore, to find what fraction one number is of a second, divide the first number by the second.

The result may be a mixed number. It may also be integral, as when we find what fraction 33 is of $16\frac{1}{2}$, the answer being 2.

Special emphasis should be given to this important case, and the class should be asked to suggest original problems based upon it.

EXERCISE 119

Find what fraction the first number is of the second:

- | | | | |
|--------------------------|---------------------------|--------------------------------------|-------------------------------------|
| 1. 5, 15. | 7. $17, 8\frac{1}{2}$. | 13. $\frac{2}{3}, \frac{3}{5}$. | 19. $3\frac{2}{3}, 5\frac{1}{2}$. |
| 2. 19, 57. | 8. $39, 9\frac{3}{4}$. | 14. $\frac{3}{4}, \frac{5}{8}$. | 20. $4\frac{1}{4}, 5\frac{3}{8}$. |
| 3. 15, 35. | 9. $9\frac{2}{3}, 47$. | 15. $\frac{7}{8}, 1\frac{5}{16}$. | 21. $6\frac{2}{3}, 3\frac{1}{3}$. |
| 4. 29, 87. | 10. $7\frac{3}{8}, 59$. | 16. $1\frac{5}{12}, 1\frac{3}{16}$. | 22. $8\frac{1}{3}, 16\frac{2}{3}$. |
| 5. 17, 85. | 11. $51\frac{1}{8}, 91$. | 17. $1\frac{7}{16}, 3\frac{5}{2}$. | 23. $33\frac{1}{3}, 100$. |
| 6. $16\frac{2}{3}, 50$. | 12. $3\frac{3}{2}, 99$. | 18. $3\frac{9}{2}, 4\frac{5}{8}$. | 24. $66\frac{2}{3}, 100$. |

Find what part of a dollar is represented by the following:

- | | | |
|---------------|----------------------------|----------------------------|
| 25. 10 cents. | 29. 75 cents. | 33. $6\frac{1}{4}$ cents. |
| 26. 20 cents. | 30. $12\frac{1}{2}$ cents. | 34. $37\frac{1}{2}$ cents. |
| 27. 50 cents. | 31. $33\frac{1}{3}$ cents. | 35. $62\frac{1}{2}$ cents. |
| 28. 25 cents. | 32. $66\frac{2}{3}$ cents. | 36. $87\frac{1}{2}$ cents. |

37. If a man has to drive $17\frac{1}{2}$ mi., what part of the distance has he gone when he has driven $3\frac{1}{2}$ mi.?

38. If 2 lb. of flour are produced from $2\frac{1}{2}$ lb. of wheat, the weight of the flour is what part of the weight of the wheat?

39. A cubic inch of copper weighs $5\frac{3}{8}$ oz. This is what part of the weight of a piece of copper wire weighing $10\frac{3}{16}$ oz.?

40. Two men are working on the same job for the same wages. One works 15 days and the other works 21 days. If the total wages are \$99, how much does each man receive?

41. Two men are working on the same job for the same wages. One works $4\frac{7}{8}$ days and the other $5\frac{3}{4}$ days. What part of the total wages should each receive?

42. From a barrel containing 30 gal. of oil $1\frac{7}{8}$ gal. leaked out. What fractional part of all the oil remained?

43. If a boy is making a 25-mile trip on his bicycle, what fractional part of his journey has been completed when he has gone $18\frac{3}{4}$ mi.?

44. What part of a barrel ($31\frac{1}{2}$ gal.) is 1.23 gal.?

45. If a man by working 8 hr. can finish $\frac{3}{8}$ of a piece of work, what part of the work can he finish in 3 hr.?

46. In $6\frac{7}{8}$ lb. of beefsteak there are $4\frac{1}{8}$ lb. of water. What part of the beefsteak is water?

47. If a piece of bronze weighing $7\frac{1}{2}$ lb. contains $5\frac{1}{4}$ lb. of copper, what part of the bronze is copper?

48. If a mass of lead and zinc weighing $16\frac{1}{8}$ lb. contains $10\frac{3}{4}$ lb. of lead, what fractional part of the whole mass is lead?

49. If a book has 540 pages, what part is finished when 216 pages have been read?

50. In Ex. 49 what part of the book still remains to be finished? Add the answers to Exs. 49 and 50.

214. To find the Whole when a Fractional Part is Given. For example, if $\frac{3}{8}$ of a yard of cloth costs 30 cents, what does 1 yard cost?

Since $\frac{3}{8}$ of a yard costs 30 cents,
 then $\frac{1}{8}$ of a yard costs $\frac{1}{3}$ of 30 cents, or 10 cents,
 and $\frac{8}{8}$ of a yard, or a whole yard, costs 8×10 cents, or 80 cents.

A short way to obtain this result is evidently to multiply 30 cents by $\frac{8}{3}$, that is, by the reciprocal (§ 211) of the fraction. Special emphasis should be given to this important case.

Therefore, to find the whole when a fractional part is given, multiply the given part by the reciprocal of the fraction.

EXERCISE 120

1. \$16 is $\frac{4}{7}$ of what sum of money?
- ✓ 2. \$45 is $\frac{5}{8}$ of what sum of money?
3. 84 mi. is $\frac{7}{9}$ of how many miles?
4. $17\frac{1}{2}$ ft. is $\frac{2}{3}$ of how many feet?
5. $243\frac{2}{3}$ lb. is $\frac{4}{5}$ of how many pounds?
- ✓ 6. If $\frac{5}{8}$ of an acre costs \$35, what does 1 acre cost?
- ✓ 7. If $\frac{7}{8}$ of a roll of carpeting costs \$84, what is the cost of the whole roll?
8. A dealer sold 12 yd. of cloth, which was $\frac{4}{15}$ of the whole piece. How many yards were there in the piece?
9. A dealer sold $28\frac{1}{2}$ yd. of cloth, which was $\frac{2}{3}$ of the whole piece. How many yards were there in the piece?
10. \$75 is $\frac{1}{4}$ more than (that is, $\frac{5}{4}$ of) what sum of money?
11. \$75 is $\frac{1}{4}$ less than (that is, $\frac{3}{4}$ of) what sum of money?
12. 119 ft. is $\frac{1}{16}$ more than what length?
13. 155 ft. is $\frac{1}{32}$ less than what length?
14. \$1107 is $\frac{1}{8}$ more than what sum of money?
15. \$1001 is $\frac{1}{8}$ less than what sum of money?

Given the following whole numbers, which are the indicated fractional parts of certain other numbers, find the other numbers :

16. $48, \frac{1}{2}$.

23. $102, \frac{1}{2}$.

30. $225, \frac{5}{8}$.

17. $48, \frac{3}{4}$.

24. $102, \frac{3}{4}$.

31. $222, \frac{6}{7}$.

18. $56, \frac{7}{8}$.

25. $120, \frac{7}{8}$.

32. $235, \frac{5}{8}$.

19. $72, \frac{8}{9}$.

26. $125, \frac{5}{8}$.

33. $333, \frac{9}{10}$.

20. $96, \frac{8}{9}$.

27. $130, \frac{5}{8}$.

34. $1200, \frac{6}{7}$.

21. $99, \frac{9}{10}$.

28. $140, \frac{7}{8}$.

35. $1440, \frac{1}{3}$.

22. $27, \frac{9}{16}$.

29. $168, \frac{8}{9}$.

36. $1760, \frac{1}{5}$.

37. If 45 qt. will fill 48 bottles, each bottle requires what part of a quart ?

38. What part of $8\frac{3}{4}$ qt. of sirup will it take to fill a bottle containing $\frac{7}{8}$ qt. ?

39. What part of $9\frac{1}{2}$ qt. is a cupful, the cup holding $\frac{3}{8}$ qt. ?

40. What part of 28 in. is $\frac{1}{2}$ in. ? $\frac{1}{4}$ in. ? $\frac{3}{4}$ in. ? $\frac{7}{8}$ in. ? 7 in. ? $5\frac{3}{4}$ in. ? $4\frac{3}{4}$ in. ?

41. What part of $7\frac{1}{4}$ in. is $1\frac{7}{8}$ in. ?

42. What part of 675 lb. of butter is used to fill a jar containing $18\frac{3}{4}$ lb. ?

43. A man is traveling $9\frac{3}{4}$ mi. What part of the distance has he covered when he has gone $3\frac{1}{4}$ mi. ?

44. If a man had to travel 69 mi. and has already traveled $7\frac{3}{8}$ mi., what part of his journey is finished ?

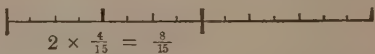
45. A boy had 50 mi. to ride on his bicycle. He has already ridden $6\frac{3}{4}$ mi. What part of his journey is finished ?

46. A figure having a certain number of equal sides has a perimeter of $34\frac{1}{2}$ in. Each side is $5\frac{1}{2}$ in. long. This is what part of the perimeter ? How many sides has the figure ?

215. Multiplying and Dividing the Terms of a Fraction. The following important principles should now be reviewed:

Multiplying the numerator by any number multiplies the fraction by that number.

That is, $\frac{2 \times 4}{15} = 2 \times \frac{4}{15}$.



This is seen by studying the accompanying figure.

Dividing the numerator by any number divides the fraction by that number.

That is, $\frac{4 \div 2}{5} = \frac{4}{5} \div 2$.

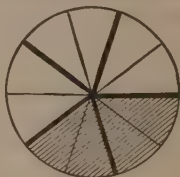


Show by these squares that $\frac{4}{5}$ of the rectangle divided by 2 is equal to $\frac{4 \div 2}{5}$ of the rectangle, or $\frac{2}{5}$ of the rectangle.

Multiplying the denominator by any number divides the fraction by that number.

That is, $\frac{3}{2 \times 5} = \frac{3}{5} \div 2$.

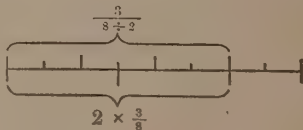
Show by this circle that $\frac{3}{2 \times 5}$, or $\frac{3}{10}$, equals $\frac{3}{5} \div 2$.



Dividing the denominator by any number multiplies the fraction by that number.

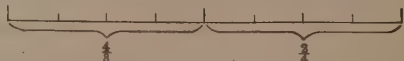
That is, $\frac{3}{8 \div 2} = \frac{3}{4} = 2 \times \frac{3}{8}$.

Show by this line that $\frac{3}{8 \div 2}$, or $\frac{3}{4}$, equals $2 \times \frac{3}{8}$.



Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

That is, $\frac{2}{4} = \frac{2 \times 2}{2 \times 4} = \frac{4}{8}$;



$\frac{4}{8} = \frac{4 \div 2}{8 \div 2} = \frac{2}{4}$, and both $\frac{4}{8}$ and $\frac{2}{4}$ equal $\frac{1}{2}$.

EXERCISE 121

DRILL WORK IN FRACTIONS

Find the sum of:

- | | | | |
|-----------------------------------|------------------------------------|------------------------------------|------------------------------------|
| 1. $\frac{1}{2}, \frac{3}{8}$. | 6. $\frac{1}{8}, \frac{3}{16}$. | 11. $\frac{1}{2}, \frac{7}{32}$. | 16. $\frac{3}{4}, \frac{7}{24}$. |
| 2. $\frac{1}{4}, \frac{3}{16}$. | 7. $\frac{3}{8}, \frac{7}{16}$. | 12. $\frac{1}{2}, \frac{15}{32}$. | 17. $\frac{3}{4}, \frac{1}{24}$. |
| 3. $\frac{1}{4}, \frac{5}{16}$. | 8. $\frac{3}{8}, \frac{15}{16}$. | 13. $\frac{1}{4}, \frac{25}{32}$. | 18. $\frac{5}{8}, \frac{5}{24}$. |
| 4. $\frac{1}{4}, \frac{9}{16}$. | 9. $\frac{5}{8}, \frac{15}{16}$. | 14. $\frac{3}{4}, \frac{31}{32}$. | 19. $\frac{7}{8}, \frac{23}{24}$. |
| 5. $\frac{3}{4}, \frac{13}{16}$. | 10. $\frac{7}{8}, \frac{13}{16}$. | 15. $\frac{3}{8}, \frac{5}{32}$. | 20. $\frac{5}{16}, \frac{1}{4}$. |
21. To the sum of $\frac{1}{2}, \frac{1}{4}$, and $\frac{3}{8}$, add the sum of $\frac{3}{5}, \frac{7}{15}$, and $\frac{2}{3}$.
22. To the sum of $\frac{3}{4}, \frac{5}{8}$, and $\frac{1}{16}$, add the sum of $\frac{1}{2}, \frac{7}{8}$, and $\frac{1}{16}$.

Find the sum of:

- | | | |
|-------------------------------------|--|--|
| 23. $8\frac{3}{4}, 5\frac{1}{3}$. | 31. $62\frac{5}{6}, 29\frac{2}{3}$. | 39. $5\frac{1}{2}, 6\frac{2}{3}, 4\frac{2}{15}$. |
| 24. $6\frac{3}{4}, 8\frac{2}{3}$. | 32. $75\frac{7}{8}, 62\frac{5}{16}$. | 40. $6\frac{7}{8}, 3\frac{1}{2}, 5\frac{1}{6}$. |
| 25. $2\frac{1}{3}, 7\frac{1}{8}$. | 33. $92\frac{5}{6}, 75\frac{4}{15}$. | 41. $4\frac{1}{3}, 3\frac{3}{5}, 5\frac{1}{30}$. |
| 26. $6\frac{1}{3}, 5\frac{3}{8}$. | 34. $73\frac{2}{3}, 68\frac{7}{15}$. | 42. $2\frac{7}{8}, 5\frac{1}{6}, 6\frac{5}{24}$. |
| 27. $8\frac{2}{3}, 9\frac{5}{8}$. | 35. $52\frac{4}{5}, 75\frac{1}{15}$. | 43. $3\frac{5}{8}, 2\frac{5}{12}, 6\frac{7}{24}$. |
| 28. $7\frac{2}{3}, 8\frac{7}{8}$. | 36. $48\frac{9}{10}, 73\frac{1}{12}$. | 44. $4\frac{7}{8}, 3\frac{2}{3}, 5\frac{1}{24}$. |
| 29. $3\frac{3}{4}, 5\frac{5}{12}$. | 37. $52\frac{1}{6}, 61\frac{7}{12}$. | 45. $3\frac{1}{6}, 5\frac{3}{12}, 7\frac{1}{4}$. |
| 30. $4\frac{3}{8}, 6\frac{5}{16}$. | 38. $37\frac{4}{15}, 32\frac{1}{10}$. | 46. $9\frac{2}{15}, 13\frac{13}{30}, 15\frac{1}{20}$. |

Subtract as indicated:

- | | | |
|--------------------------------------|----------------------------|---------------------------------------|
| 47. $\frac{3}{4} - \frac{1}{8}$. | 54. $32 - 4\frac{1}{8}$. | 61. $9\frac{7}{16} - 7\frac{3}{8}$. |
| 48. $\frac{5}{8} - \frac{1}{2}$. | 55. $63 - 5\frac{3}{4}$. | 62. $8\frac{9}{16} - 3\frac{3}{8}$. |
| 49. $\frac{5}{8} - \frac{3}{16}$. | 56. $72 - 8\frac{9}{10}$. | 63. $6\frac{1}{16} - 4\frac{5}{8}$. |
| 50. $\frac{7}{8} - \frac{5}{16}$. | 57. $48 - 3\frac{7}{12}$. | 64. $7\frac{1}{16} - 3\frac{7}{8}$. |
| 51. $\frac{1}{2} - \frac{5}{32}$. | 58. $67 - 5\frac{3}{16}$. | 65. $9\frac{3}{2} - 6\frac{3}{4}$. |
| 52. $\frac{7}{8} - \frac{5}{32}$. | 59. $71 - 6\frac{5}{24}$. | 66. $8\frac{2}{3} - 5\frac{3}{8}$. |
| 53. $\frac{15}{32} - \frac{3}{64}$. | 60. $80 - 9\frac{5}{8}$. | 67. $23\frac{1}{8} - 8\frac{3}{16}$. |

Find the value of :

68. $9\frac{3}{16} - 7\frac{1}{2}$.

69. $42\frac{5}{8} - 8\frac{5}{12}$.

70. $39\frac{3}{8} - 2\frac{1}{12}$.

71. $48\frac{1}{2} - 6\frac{5}{12}$.

72. $73\frac{1}{3} - 17\frac{7}{12}$.

73. $69\frac{2}{3} - 48\frac{3}{16}$.

74. $75\frac{1}{16} + 1\frac{3}{16} - 9\frac{3}{32}$.

75. $\frac{1}{2} + \frac{3}{4} + \frac{1}{4} + \frac{1}{2} - \frac{1}{3}$.

76. $\frac{2}{3} + \frac{2}{3} + \frac{1}{3} + \frac{4}{3} - \frac{1}{2}$.

77. $\frac{3}{4} + \frac{7}{8} - \frac{2}{3}$.

78. $\frac{4}{5} + \frac{5}{8} - \frac{3}{4}$.

79. $\frac{3}{16} + \frac{7}{8} - \frac{3}{4}$.

80. $\frac{1}{2} + \frac{7}{8} - \frac{1}{4} - 1$.

81. $\frac{7}{8} + \frac{3}{4} - \frac{1}{2} - 1$.

82. $\frac{2}{5} + 1\frac{1}{5} + 2 - \frac{1}{3}$.

83. $\frac{2}{3} + \frac{1}{6} + 5\frac{1}{2} - \frac{1}{2}$.

84. $20 + 1\frac{3}{5} - 2\frac{1}{3} - 6$.

85. $30 - 3\frac{3}{4} - 1\frac{7}{8} - 5$.

86. If we add $12\frac{1}{2}$ to $16\frac{2}{3}$, and then subtract $9\frac{1}{6}$ from the sum, what is the result?

Find the value of :

87. $\frac{5}{8}$ of 68.

90. $1\frac{1}{6}$ of 110.

93. $\frac{5}{84}$ of 144.

88. $\frac{7}{8}$ of 96.

91. $1\frac{2}{3}$ of 120.

94. $\frac{7}{64}$ of 128.

89. $\frac{4}{5}$ of 90.

92. $1\frac{5}{6}$ of 240.

95. $1\frac{5}{8}$ of 192.

Multiply as indicated :

96. $75 \times \frac{4}{5}$.

100. $\frac{3}{4}$ of $\frac{3}{4}$.

104. $\frac{3}{32}$ of $\frac{5}{8}$.

97. $64 \times \frac{7}{16}$.

101. $\frac{5}{8}$ of $\frac{3}{8}$.

105. $\frac{5}{32}$ of $1\frac{3}{10}$.

98. $96 \times \frac{5}{12}$.

102. $\frac{7}{16}$ of $\frac{8}{9}$.

106. $\frac{3}{64}$ of $\frac{8}{9}$.

99. $81 \times \frac{5}{9}$.

103. $\frac{7}{15}$ of $\frac{3}{20}$.

107. $\frac{5}{48}$ of $1\frac{2}{3}$.

Find the following products :

108. $5\frac{3}{8} \times 6\frac{7}{8}$.

114. $7\frac{3}{4} \times 63\frac{7}{8}$.

120. $63 \times 429\frac{3}{4}$.

109. $6\frac{2}{7} \times 7\frac{7}{8}$.

115. $4\frac{2}{9} \times 28\frac{1}{8}$.

121. $48\frac{2}{3} \times 48\frac{3}{16}$.

110. $6\frac{1}{8} \times 9\frac{5}{7}$.

116. $8\frac{5}{7} \times 31\frac{1}{16}$.

122. $63\frac{1}{5} \times 127\frac{1}{12}$.

111. $7\frac{2}{3} \times 7\frac{7}{8}$.

117. $5\frac{2}{7} \times 48\frac{1}{16}$.

123. $35\frac{1}{5} \times 228\frac{9}{32}$.

112. $9\frac{2}{3} \times 8\frac{5}{8}$.

118. $4\frac{1}{3} \times 51\frac{9}{16}$.

124. $61\frac{1}{24} \times 240\frac{2}{5}$.

113. $52 \times 607\frac{3}{4}$.

119. $28 \times 635\frac{1}{4}$.

125. $37\frac{7}{24} \times 480\frac{48}{100}$.

Perform the divisions indicated:

- | | | |
|---|--|--|
| 126. $\frac{2}{3} \div \frac{2}{3}$. | 133. $\frac{2}{3} \div \frac{4}{5}$. | 140. $\frac{3}{7} \div \frac{2}{3}$. |
| 127. $\frac{2}{3} \div \frac{3}{7}$. | 134. $\frac{7}{8} \div \frac{4}{5}$. | 141. $\frac{4}{5} \div \frac{7}{8}$. |
| 128. $\frac{6}{7} \div \frac{3}{4}$. | 135. $\frac{3}{4} \div \frac{6}{7}$. | 142. $\frac{3}{4} \div 5\frac{1}{2}$. |
| 129. $\frac{5}{7} \div 2\frac{1}{8}$. | 136. $\frac{2}{3} \div 4\frac{2}{3}$. | 143. $7\frac{6}{7} \div 1\frac{3}{4}$. |
| 130. $9\frac{2}{3} \div 3\frac{1}{2}$. | 137. $10\frac{4}{5} \div 3\frac{1}{2}$. | 144. $5\frac{5}{7} \div 1\frac{1}{7}$. |
| 131. $29\frac{1}{5} \div \frac{1}{3}$. | 138. $3\frac{9}{10} \div 2\frac{1}{3}$. | 145. $2\frac{4}{5} \div 3\frac{1}{2}$. |
| 132. $4\frac{7}{8} \div 2\frac{2}{3}$. | 139. $4\frac{6}{7} \div 1\frac{3}{4}$. | 146. $16\frac{2}{7} \div 2\frac{1}{8}$. |

Find the value of:

- | | |
|--|---|
| 147. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$. | 151. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of 1 cu. ft. |
| 148. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of 1 cu. ft. | 152. $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of 1 cu. in. |
| 149. $\frac{1}{2}$ of 5 times 8 sq. ft. | 153. $\frac{2}{3}$ of 15 times 48 sq. ft. |
| 150. $\frac{7}{8}$ of 4 times 16 sq. in. | 154. $\frac{3}{7}$ of 13 times 14 sq. in. |
| 155. Multiply the sum of $\frac{2}{3}$ and $\frac{5}{8}$ by the sum of $\frac{3}{4}$ and $\frac{7}{8}$. | |
| 156. Divide 7 by the sum of $\frac{3}{4}$ and $\frac{5}{16}$. | |
| 157. Divide the sum of $\frac{4}{5}$ and $\frac{5}{8}$ by 79. | |
| 158. Divide the product of $\frac{2}{3}$ and $\frac{4}{5}$ by the product of $\frac{3}{8}$ and $\frac{8}{9}$. | |

In the following, the second number is what part of the first number?

- | | | |
|--|--|--|
| 159. $2\frac{2}{7}$, $1\frac{6}{11}$. | 166. $3\frac{1}{3}$, $1\frac{2}{3}$. | 173. 17, $8\frac{1}{2}$. |
| 160. 82, $9\frac{1}{3}$. | 167. $41\frac{2}{3}$, $5\frac{1}{3}$. | 174. $66\frac{2}{3}$, $16\frac{2}{3}$. |
| 161. 243, 27. | 168. 100, $37\frac{1}{2}$. | 175. $45\frac{2}{3}$, $7\frac{2}{3}$. |
| 162. $16\frac{5}{8}$, $2\frac{3}{8}$. | 169. $42\frac{1}{2}$, $14\frac{1}{8}$. | 176. $62\frac{1}{8}$, $7\frac{7}{8}$. |
| 163. $66\frac{2}{3}$, $33\frac{1}{3}$. | 170. 225, $37\frac{1}{2}$. | 177. $22\frac{1}{2}$, $16\frac{2}{3}$. |
| 164. $41\frac{7}{8}$, $21\frac{1}{8}$. | 171. $37\frac{1}{2}$, $33\frac{1}{3}$. | 178. 421, $60\frac{1}{7}$. |
| 165. $33\frac{1}{3}$, $16\frac{2}{3}$. | 172. 427, $47\frac{1}{3}$. | 179. 325, $29\frac{6}{11}$. |

216. Practical Uses of Cancellation. We have often seen that cancellation is useful in simplifying the work. It is of particular value in connection with mixed numbers.

What is the product of $1\frac{1}{5}$, $45\frac{1}{2}$, $33\frac{1}{3}$, and $16\frac{7}{8}$?

Reducing the mixed numbers to improper fractions, $45\frac{1}{2} = \frac{91}{2}$, $33\frac{1}{3} = \frac{100}{3}$, $16\frac{7}{8} = \frac{135}{8}$. Proceeding as in § 182, and canceling, the product is $1706\frac{1}{4}$.

$$\frac{1 \times 91 \times \overset{25}{\cancel{100}} \times \overset{3}{\cancel{135}}}{\cancel{15} \times 2 \times \overset{9}{\cancel{3}} \times \overset{2}{\cancel{8}}} = \frac{6825}{4} = 1706\frac{1}{4}$$

EXERCISE 122

Multiply:

1. $20\frac{1}{2} \times 32\frac{1}{5} \times 16\frac{7}{8}$.
2. $15\frac{2}{3} \times 16\frac{7}{8} \times 33\frac{1}{3}$.
3. $27\frac{1}{5} \times 24\frac{1}{2} \times 37\frac{1}{2}$.
4. $42\frac{2}{3} \times 66\frac{2}{3} \times 75\frac{1}{6}$.
5. $35\frac{1}{6} \times 22\frac{1}{2} \times 12\frac{4}{5}$.
6. $7\frac{3}{5} \times 2\frac{1}{7} \times 5\frac{1}{4} \times 21\frac{1}{3}$.
7. $2\frac{1}{2} \times 3\frac{3}{4} \times 4\frac{4}{5} \times 5\frac{5}{6}$.
8. $3\frac{1}{2} \times 4\frac{2}{3} \times 5\frac{3}{4} \times 6\frac{4}{5} \times 7\frac{5}{6}$.
9. $8\frac{2}{3} \times 7\frac{1}{2} \times 5\frac{7}{6} \times 1\frac{3}{4}$.
10. $2\frac{2}{3} \times 3\frac{4}{5} \times 5\frac{6}{7} \times 6\frac{7}{8} \times 7\frac{1}{2}$.

11. Divide the product of 4, 7, 19, 21, and 64 by the product of 14, 38, 28, and 3.

12. Divide the product of 3, 9, 15, 38, 42, and 75 by the product of 19, 7, 3, 5, 27, 60, and 8.

13. Divide the product of 2, 3, 4, 5, 6, 7, 8, and 9 by the product of 10, 12, 14, 15, and 16.

14. How many times as much will it cost to transport 12 boxes of merchandise weighing 300 lb. each 200 mi., as to transport 6 boxes weighing 400 lb. each 75 mi., at the same rate per pound for each mile?

15. How many times as much will it cost to transport 3 boxes of merchandise weighing 400 lb. each a given distance, as to transport 2 boxes weighing 300 lb. each the same distance, at the same rate per pound?

16. A bushel contains $2150\frac{1}{2}$ cu. in. The capacity of an orange box, 12 in. by 12 in. by $26\frac{3}{8}$ in., is how many cubic inches more than $1\frac{3}{4}$ bu.?

17. A bushel contains $2150\frac{1}{2}$ cu. in. In shipping Georgia peaches a crate 8 in. by $12\frac{1}{2}$ in. by 22 in. is used. This contains how many cubic inches more than 1 bu.?

18. A bushel contains $2150\frac{1}{2}$ cu. in. In New Jersey a bushel cranberry box is 12 in. by $8\frac{3}{4}$ in. by 22 in. How does this differ in capacity from the bushel?

19. California apples and pears for local use are shipped in a 50-pound box 22 in. by 10 in. by 12 in. How does this differ in capacity from the bushel of $2150\frac{1}{2}$ cu. in.?

20. A California cantaloupe crate is 38 in. long, 16 in. wide, and 15 in. deep. This is how many cubic inches?

21. The California orange box is sometimes $26\frac{1}{2}$ in. by $11\frac{1}{4}$ in. by $11\frac{1}{4}$ in., and sometimes 22 in. by $7\frac{3}{4}$ in. by $17\frac{1}{2}$ in. Which holds the more, and how many cubic inches more?

22. Dried fruits are shipped in 50-pound or 25-pound boxes. The 50-pound box is $15\frac{1}{4}$ in. by 9 in. by 9 in. How many cubic inches in the box?

23. A 25-pound box for shipping dried fruit is sometimes $13\frac{3}{4}$ in. by $9\frac{3}{4}$ in. by $5\frac{3}{4}$ in., and sometimes 16 in. by 9 in. by 5 in. Which box holds the more, and how much more?

24. For shipping fruit out of the state, the sizes of some of the boxes used in California are as follows:

Plums, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by 4 in.

Cherries, $16\frac{1}{2}$ in. by $10\frac{3}{8}$ in. by $2\frac{1}{2}$ in.

Pears, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by $8\frac{1}{2}$ in.

Peaches, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by $4\frac{7}{8}$ in.

Find the number of cubic inches in each box.

EXERCISE 123

DRILL WORK

Add as indicated:

- | | | |
|---------------------------------------|---------------------------------------|--|
| 1. $47\frac{1}{2} + 93\frac{1}{3}$. | 5. $72\frac{1}{4} + 83\frac{1}{6}$. | 9. $29\frac{3}{4} + 47\frac{1}{5}$. |
| 2. $62\frac{1}{5} + 35\frac{1}{3}$. | 6. $85\frac{4}{5} + 16\frac{2}{3}$. | 10. $87\frac{5}{8} + 19\frac{2}{3}$. |
| 3. $67\frac{4}{9} + 29\frac{1}{6}$. | 7. $89\frac{3}{7} + 23\frac{1}{2}$. | 11. $67\frac{4}{5} + 49\frac{1}{2}$. |
| 4. $39\frac{9}{10} + 28\frac{3}{4}$. | 8. $87\frac{3}{5} + 62\frac{7}{10}$. | 12. $37\frac{4}{7} + 18\frac{1}{14}$. |

Subtract as indicated:

- | | | |
|---------------------------------------|--|---------------------------------------|
| 13. $79\frac{4}{5} - 22\frac{1}{2}$. | 16. $137\frac{1}{2} - 86\frac{2}{3}$. | 19. $48\frac{3}{4} - 23\frac{7}{8}$. |
| 14. $92\frac{1}{4} - 37\frac{2}{5}$. | 17. $30\frac{2}{3} - 19\frac{7}{8}$. | 20. $82\frac{1}{2} - 68\frac{7}{8}$. |
| 15. $72\frac{2}{3} - 56\frac{1}{8}$. | 18. $41\frac{3}{8} - 16\frac{5}{8}$. | 21. $72 - 46\frac{3}{4}$. |

Multiply as indicated:

- | | | |
|--|---|--|
| 22. $27 \times 14\frac{2}{3}$. | 28. $32 \times 92\frac{1}{2}$. | 34. $67 \times 74\frac{2}{3}$. |
| 23. $18 \times 96\frac{3}{4}$. | 29. $30\frac{1}{2} \times 68$. | 35. $40\frac{1}{3} \times 96$. |
| 24. $32\frac{2}{3} \times 85$. | 30. $41\frac{3}{4} \times 96$. | 36. $75 \times 60\frac{4}{5}$. |
| 25. $12\frac{1}{2} \times 12\frac{1}{2}$. | 31. $16\frac{1}{3} \times 2\frac{1}{5}$. | 37. $3\frac{1}{4} \times 6\frac{4}{5}$. |
| 26. $15\frac{3}{4} \times 8\frac{4}{5}$. | 32. $32 \times 35\frac{1}{4}$. | 38. $32 \times 40\frac{3}{4}$. |
| 27. $16 \times 417\frac{3}{4}$. | 33. $21 \times 220\frac{3}{7}$. | 39. $\frac{2}{3}$ of $\frac{7}{8}$ of $1\frac{5}{7}$. |

Find the following:

- | | |
|--|--|
| 40. $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{5}{7}$. | 44. $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of 2 ft. |
| 41. $\frac{3}{5}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of 1 ft. | 45. $\frac{7}{8}$ of $\frac{3}{5}$ of $\frac{2}{3}$ of 5 in. |
| 42. $\frac{1}{4}$ of 15 times 8 ft. | 46. $\frac{4}{5}$ of 25 times 48 ft. |
| 43. $\frac{3}{8}$ of 9 times 16 in. | 47. $\frac{3}{16}$ of 12 times 24 in. |

Find the number of which:

- | | | |
|----------------------------|------------------------------|---------------------------------------|
| 48. 23 is $\frac{1}{3}$. | 51. \$150 is $\frac{3}{5}$. | 54. 63 is $\frac{7}{8}$. |
| 49. 75 is $\frac{3}{8}$. | 52. 38 is $\frac{2}{3}$. | 55. 125 is $\frac{2}{3}\frac{5}{2}$. |
| 50. 222 is $\frac{9}{7}$. | 53. 45 is $\frac{9}{10}$. | 56. \$2.25 is $\frac{5}{8}$. |

Divide, using cancellation :

57. $2 \times 3 \times 6$ by $5 \times 9 \times 4$.

58. $3 \times 14 \times 20$ by $15 \times 21 \times 7$.

59. $4 \times 15 \times 27$ by $9 \times 5 \times 24$.

60. $7 \times 21 \times 77 \times 6$ by $49 \times 33 \times 42$.

61. $20 \times 25 \times 32 \times 48$ by $27 \times 40 \times 15 \times 12$.

62. $32 \times 27 \times 44 \times 39$ by $55 \times 72 \times 26 \times 8 \times 3$.

63. $125 \times 108 \times 51 \times 8$ by $40 \times 45 \times 84 \times 12 \times 3$.

Divide as indicated :

64. $\frac{4}{5} \div \frac{2}{3}$.

77. $\frac{3}{8} \div \frac{3}{7}$.

90. $\frac{5}{7} \div 2\frac{1}{5}$.

65. $1\frac{3}{4} \div \frac{3}{7}$.

78. $\frac{4}{5} \div \frac{5}{4}$.

91. $6\frac{2}{3} \div 3\frac{1}{3}$.

66. $\frac{3}{7} \div \frac{3}{8}$.

79. $\frac{3}{4} \div 2\frac{1}{2}$.

92. $9\frac{1}{5} \div 1\frac{1}{3}$.

67. $\frac{7}{8} \div \frac{2}{3}$.

80. $\frac{9}{10} \div \frac{4}{5}$.

93. $3\frac{7}{8} \div 4\frac{2}{3}$.

68. $1\frac{5}{12} \div \frac{7}{8}$.

81. $\frac{7}{16} \div 3\frac{3}{2}$.

94. $4\frac{5}{9} \div 12\frac{1}{3}$.

69. $\frac{5}{6} \div 3\frac{7}{12}$.

82. $\frac{3}{40} \div 2\frac{6}{5}$.

95. $10\frac{7}{8} \div 9\frac{1}{4}$.

70. $\frac{7}{8} \div 1\frac{7}{12}$.

83. $\frac{3}{4} \div 1\frac{1}{18}$.

96. $2\frac{7}{8} \div 3\frac{1}{8}$.

71. $\frac{2}{3} \div \frac{5}{8}$.

84. $5\frac{1}{4} \div 19\frac{3}{8}$.

97. $3\frac{9}{10} \div 2\frac{3}{10}$.

72. $\frac{5}{8} \div \frac{3}{8}$.

85. $7\frac{3}{10} \div 1\frac{1}{5}$.

98. $4\frac{9}{10} \div 12\frac{5}{8}$.

73. $\frac{3}{4} \div \frac{4}{5}$.

86. $1\frac{2}{3} \div 3\frac{2}{3}$.

99. $5\frac{4}{7} \div 2\frac{3}{4}$.

74. $\frac{5}{6} \div \frac{3}{4}$.

87. $40\frac{3}{8} \div 33\frac{1}{2}$.

100. $15\frac{5}{7} \div 3\frac{2}{7}$.

75. $\frac{3}{8} \div 1\frac{5}{12}$.

88. $13\frac{9}{10} \div 2\frac{2}{3}$.

101. $12\frac{4}{5} \div 2\frac{1}{5}$.

76. $\frac{9}{24} \div 1\frac{5}{16}$.

89. $14\frac{5}{7} \div 2\frac{3}{7}$.

102. $46\frac{3}{7} \div 5\frac{1}{4}$.

103. At $12\frac{1}{2}$ ¢ each, find the cost of 147 balls.

104. At $16\frac{2}{3}$ ¢ each, find the cost of 114 notebooks.

105. At $33\frac{1}{3}$ ¢ each, find the cost of 102 handkerchiefs.

106. At \$ $1.33\frac{1}{3}$ a volume, find the cost of 165 books.

107. At \$ $1.16\frac{2}{3}$ a yard, find the cost of 142 yd. of lace.

108. At \$ $1.12\frac{1}{2}$ each, find the cost of 112 yd. of velvet.

EXERCISE 124

MISCELLANEOUS PROBLEMS

1. At 32¢ a gallon, what will $12\frac{1}{2}$ gal. of vinegar cost?

In each of the following examples estimate the approximate answer and write it down. After the problem is solved compare the approximate answer with the real answer. This is one of the best checks against absurd results. Thus $3\frac{1}{2} \times 22\text{¢}$ is a reasonable approximation for $3\frac{3}{8} \times 22\text{¢}$ in Ex. 2.

2. At 22¢ a pound, what will $3\frac{5}{8}$ lb. of steak cost?

3. At 18¢ a gallon, what will $6\frac{1}{4}$ gal. of oil cost?

4. At 26¢ a pound, what will $6\frac{7}{8}$ lb. of butter cost?

5. At $12\frac{1}{2}\text{¢}$ a yard, what will $8\frac{3}{4}$ yd. of ribbon cost?

6. At \$2.50 a yard, what will $7\frac{5}{8}$ yd. of velvet cost?

7. If 6 yd. of silk cost \$6.75, what is the price per yard?

8. If 8 doz. men's hose cost \$55, what is the price per dozen?

9. A man drives 39.6 mi. in $5\frac{1}{2}$ hr. What is the rate per hour?

10. If 48 doz. boys' hose cost \$57.60, what is the price per dozen?

11. At 14¢ a gross, how many gross of buttons can be bought for \$3.22?

12. A dealer buys 200 lb. of raisins for \$19. How much do they cost a pound?

13. A grocer pays \$10.08 a gross for canned soup. How much does he pay per can?

14. If 20 electric lights cost 10¢ per hour, how long can we burn 4 such lights for 20¢?

15. A dealer pays \$18.83 for 7 rocking chairs. How much does he pay for each?

16. A dealer sells 17 pails of mackerel for \$40.80. What is the price per pail?

17. A man pays \$1110 a year for the rent of a city house. How much does he pay a month?

18. A grocer buys a 50-gallon barrel of Porto Rico molasses for \$11.50. How much does it cost per gallon?

19. A grocer sells a half chest of tea, 140 lb. to the chest, for \$19.60. What is the price per pound?

20. A merchant sold a 40-yard piece of cloth for \$45, gaining $12\frac{1}{2}\%$ a yard. How much did the piece cost him?

21. A grocer buys $33\frac{1}{3}$ doz. 1-pound cans of cocoa at 43¢ each and sells them at 58¢ each. How much does he gain?

22. A dry-goods dealer sold 75 yd. of cloth for \$112.50, thereby gaining \$28.50. How much did it cost him per yard?

23. If a family uses 3 bbl. of flour in a year, and can buy the flour at \$5.50 a barrel or at 3¢ a pound, how much will be saved by purchasing by the barrel? (Allow 196 lb. to the barrel.)

24. If a family uses $1\frac{1}{4}$ gal. of kerosene in 5 weeks, and the kerosene costs 15¢ a gallon, how much will be used in 52 weeks, and how much will it all cost?

25. If gas costs \$1 per 1000 cu. ft., and a family uses 2320 cu. ft. in October and 2475 cu. ft. in November, how much is the gas bill for the two months?

26. At the beginning of the month the gas meter registers 14,260, and at the end of the month 17,140. How much is the gas bill for the month, at \$1.25 per 1000 cu. ft.?

27. A grocer buys six 50-pound boxes of dried apples at $4\frac{1}{2}\%$ a pound and sells them at 6¢ a pound. How much does he gain in all? What fractional part of the cost does he gain?

28. A salesman receives \$15 a week. He works 49 weeks a year. His expenses are \$396 a year. How much does he save a year? What fractional part of his income does he save?

29. I bought 6 lb. of prunes @ $12\frac{1}{2}\text{¢}$, 8 lb. of dried apricots @ $12\frac{1}{2}\text{¢}$, $2\frac{1}{2}$ lb. of raisins @ 16¢ , $1\frac{1}{2}$ lb. of figs @ 22¢ , and $\frac{1}{2}$ lb. of English walnuts @ 18¢ , and gave the grocer a \$5 bill. How much change should I receive?

30. If 1 cu. ft. of water weighs 62 lb. 8 oz., what part of 1 oz. does 1 cu. in. of water weigh?

31. A gallon being 231 cu. in., what does 1 gal. of water weigh? 1 qt. of water?

32. A certain kind of stone is $2\frac{7}{8}$ times as heavy as water. What does $2\frac{1}{2}$ cu. ft. of this stone weigh?

33. Another kind of stone is $3\frac{1}{8}$ times as heavy as water. What does $3\frac{3}{4}$ cu. ft. of this stone weigh?

34. If good milk is 1.03 times as heavy as water, what is the weight of a can of such milk containing $3\frac{1}{2}$ cu. ft., the can itself weighing 6 lb. 4 oz.?

35. Ice is 0.92 as heavy as water. How much does 1 cu. ft. weigh? How much will a block of ice weigh that is $3\frac{1}{2}$ ft. long, $2\frac{1}{2}$ ft. wide, and 14 in. thick?

36. A water tank for fire purposes is placed on top of a factory. It is $16\frac{1}{2}$ ft. long, $8\frac{1}{4}$ ft. wide, and $4\frac{3}{4}$ ft. deep. When filled within 2 in. of the top, what is the weight of the water?

37. I bought 8 cans of peaches @ \$3.75 a dozen, 15 cans of tomatoes @ \$1.37 $\frac{1}{2}$ a dozen, 9 cans of soup @ \$1.50 a dozen, $3\frac{1}{2}$ lb. of raisins @ 18¢ a pound, $2\frac{1}{2}$ lb. of nuts @ 17¢ a pound, $1\frac{1}{2}$ gal. of sirup @ 60¢ a gallon, $1\frac{1}{2}$ doz. cakes of soap @ 42¢ a dozen. How much was my bill? Write the bill.

38. If a boy rides $23\frac{3}{4}$ mi. on a bicycle in $2\frac{3}{4}$ hr., what is his rate per hour?

39. In buying $2\frac{1}{2}$ doz. cans of peaches, how much is gained by buying @ \$3.74 a dozen over buying at the rate of 3 cans for a dollar?

40. What is the cost of $6\frac{1}{4}$ lb. of mackerel @ 16¢, $3\frac{1}{2}$ lb. of codfish @ 12¢, and 4 cans of salmon at the rate of 2 cans for a quarter?

41. I bought $\frac{1}{2}$ gal. of olive oil @ \$3.30 a gallon, but $\frac{1}{4}$ of what I bought leaked out. What rate per gallon do I pay for what is left?

42. In buying 48 cans of tomatoes, how much is gained by buying two cases of 2 doz. each, @ \$2.89 a case, over buying at the rate of 3 cans for 40¢?

43. I bought 3 gal. of best maple sirup @ \$1.12 $\frac{1}{2}$ a gallon, and $2\frac{1}{2}$ gal. of New Orleans molasses @ 62¢ a gallon, and gave the grocer a \$5 bill. How much change did I receive? (Neglect fractions less than $\frac{1}{2}$ ¢, although in a bill a fraction of a cent is usually called a cent.)

44. I bought 8 lb. of prunes @ 12 $\frac{1}{2}$ ¢, 4 lb. of figs @ 19¢, $2\frac{1}{2}$ lb. of candles @ 12 $\frac{1}{2}$ ¢, 12 lb. of sugar @ 6 $\frac{1}{2}$ ¢, and $2\frac{1}{2}$ lb. of English walnuts @ 18¢, and gave the grocer a \$5 bill. How much change did I receive?

45. I bought 10 lb. of white beans @ 6 $\frac{1}{4}$ ¢, $3\frac{1}{2}$ lb. of split peas @ 7¢, 3 cans of soup @ 12 $\frac{1}{2}$ ¢, and $2\frac{1}{2}$ lb. of paraffin candles @ 12 $\frac{1}{2}$ ¢. How much was my bill?

46. If we buy 12 lb. of sugar at 5 $\frac{3}{4}$ ¢, $3\frac{1}{2}$ lb. of tea at 60¢, and 12 lb. of coffee at 37 $\frac{1}{2}$ ¢, how much is the bill?

47. If 12 cups of flour make 4 loaves of bread, how many loaves will a barrel of flour make? Allow 3 cups of flour to the pound, and 196 lb. of flour to the barrel.

48. If a recipe will provide enough for 5 persons, and you wish to provide for 8 persons, what part must you add to the recipe? Instead of 10 teaspoons you would need how many?

49. Providing $1\frac{1}{2}$ potatoes for each person at a meal, how many potatoes would you need for 8 persons?

50. If there are 40 potatoes in a peck, and you use 12 of them, what fractional part of a peck do you use?

51. If a family uses $\frac{2}{3}$ of a peck of potatoes in one day, how many pecks will it use in a month of 30 days?

52. Allowing 4 cups to the quart, a quart of cranberries divided among 16 pupils in a class would allow what part of a cup to each member?

53. Cranberries are cooked with half as many cups of sugar as of berries, and half as many cups of water as of sugar. If you wish to cook 3 pt. of cranberries, how much sugar will you use? How much water?

54. If flour costs \$6.25 a barrel (196 lb.), or $3\frac{1}{2}$ ¢ a pound when bought by the pound, how much is saved on 196 lb. of flour in buying by the barrel?

55. If a family uses 12 cups of flour in each baking of bread, and there are 2 bakings a week, and we allow $\frac{1}{4}$ lb. to the cup, how many pounds of flour would be used in a year?

56. If we pay 25¢ for a box of domino sugar weighing $3\frac{1}{2}$ lb., how much do we pay per pound?

57. A gallon of water contains 231 cu. in., and 1 cu. ft. of water weighs $62\frac{1}{2}$ lb. If the water in a tank weighs 14,437 $\frac{1}{2}$ lb., how many gallons are there?

On pages 226-230 many industrial problems are included, requiring the use of denominate numbers and fractions, as suggested by the New York State Syllabus. If further work of this nature is needed at this time, pages 99-142 may be reviewed in whole or in part.

CHAPTER XI

PERCENTAGE

217. Per Cent. Another name for hundredths is *per cent*.

Thus 0.01, or $\frac{1}{100}$, is the same as 1 per cent; $0.00\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{100}$, is the same as $\frac{1}{2}$ per cent; 0.06 is the same as 6 per cent.

That part of arithmetic which treats of per cent is called *percentage*.

218. Symbol for Per Cent. The symbol for per cent is written thus: %.

We may read 0.06 either "6 hundredths" or "6 per cent." In the same way, we may think of 6% as either "6 per cent" or "6 hundredths," although it is read "6 per cent."

The expression 800% means $\frac{800}{100}$ and equals the whole number 8; 225% equals the mixed number 2.25, or $2\frac{1}{4}$; $\frac{1}{2}$ % means $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$, and is read either " $\frac{1}{2}$ per cent" or, quite commonly, " $\frac{1}{2}$ of 1%."

219. Relation to Fractions. Since 6% means $\frac{6}{100}$, which equals 0.06, or $\frac{3}{50}$, we see that we may express per cent as a decimal fraction or as a common fraction.

It is often convenient to use one form, and often another. Thus, if we are multiplying by 13.5%, it is more convenient to think of the multiplier as 0.135; but if we are multiplying by $33\frac{1}{3}$ %, it is better to think of it as $\frac{1}{3}$ instead of $0.33\frac{1}{3}$.

220. Per Cents as Common Fractions. Since $62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}$, therefore

To express per cent as a common fraction, write the number indicating the per cent for the numerator and 100 for the denominator, and reduce this fraction to lowest terms.

For example, 50% means 0.50, or $\frac{50}{100}$, and this equals $\frac{1}{2}$. Likewise, 25% means 0.25, or $\frac{25}{100}$, and this equals $\frac{1}{4}$.

EXERCISE 125

Reduce to a common fraction, integer, or mixed number:

- | | | | |
|---------|---------|-----------|-------------------------|
| 1. 10%. | 6. 4%. | 11. 100%. | 16. $33\frac{1}{3}\%$. |
| 2. 20%. | 7. 5%. | 12. 200%. | 17. $37\frac{1}{2}\%$. |
| 3. 30%. | 8. 6%. | 13. 300%. | 18. $66\frac{2}{3}\%$. |
| 4. 40%. | 9. 8%. | 14. 150%. | 19. $62\frac{1}{2}\%$. |
| 5. 50%. | 10. 9%. | 15. 275%. | 20. $16\frac{2}{3}\%$. |
21. How much is $\frac{1}{5}$ of 35 ? 20% of 35 ? 20% of 350 ?
 22. How much is $\frac{1}{4}$ of 40 ? 25% of 40 ? 25% of 400 ?
 23. How much is $\frac{1}{2}$ of 66 ? 50% of 66 ? 50% of 660 ?
 24. How much is $\frac{3}{4}$ of 48 ? 75% of 48 ? 75% of 480 ?
 25. How much is $\frac{4}{5}$ of 90 ? 80% of 90 ? 80% of 900 ?

221. Per Cents as Decimals. Since 25.5% and 0.255 have the same value, therefore

To express as a decimal a number written with the per cent sign, omit the sign and move the decimal point two places to the left.

Thus $1\frac{1}{2}\% = 0.01\frac{1}{2}$, or 0.015 ; $125\% = 1.25$; $0.6\% = 0.006$.

EXERCISE 126

Express as decimals :

- | | | | |
|---------|-------------------------|-----------|------------|
| 1. 25%. | 6. $12\frac{1}{2}\%$. | 11. 125%. | 16. 37.5%. |
| 2. 30%. | 7. $33\frac{1}{3}\%$. | 12. 150%. | 17. 16.5%. |
| 3. 50%. | 8. $37\frac{1}{2}\%$. | 13. 175%. | 18. 34.8%. |
| 4. 75%. | 9. $66\frac{2}{3}\%$. | 14. 250%. | 19. 62.5%. |
| 5. 80%. | 10. $87\frac{1}{2}\%$. | 15. 375%. | 20. 78.3%. |
21. Express $\frac{1}{2}$ as a decimal. Express $\frac{1}{2}\%$ as a decimal.
 22. Express $\frac{1}{4}$ as a decimal. Express $\frac{1}{4}\%$ as a decimal.

222. Decimals as Per Cents. Since per cent means hundredths, to express a decimal as per cent we have to consider only how many hundredths the decimal represents.

Thus $0.3 = 0.30 = 30 \text{ hundredths} = 30\%$.

$0.375 = 0.37\frac{1}{2} = 37\frac{1}{2} \text{ hundredths} = 37\frac{1}{2}\%, \text{ or } 37.5\%.$

$1.25 = 1\frac{25}{100} = 125\%.$

Therefore, to express a decimal as per cent, write the per cent sign after the number of hundredths.

EXERCISE 127

Express as per cents :

1. 0.25. 3. 0.5. 5. 1.50. 7. 0.375. 9. $0.3\frac{1}{2}$.

2. 0.75. 4. 0.8. 6. 2.5. 8. 0.625. 10. $0.6\frac{2}{3}$.

11. A pint is what per cent of a quart ? of a gallon ?

12. A dime is what per cent of a dollar ? of a cent ?

223. Common Fractions as Per Cents. Since per cent means hundredths, therefore

To express a common fraction as per cent, reduce it to hundredths, omit the denominator, and write the numerator followed by the per cent sign.

This is easily done by reducing the common fraction to a decimal and proceeding as in § 222.

Thus $\frac{4}{5} = 0.80 = 80\%$; $\frac{1}{4} = 1\frac{25}{100} = 1.25 = 125\%.$

EXERCISE 128

Express as per cents :

1. $\frac{1}{2}$. 3. $\frac{3}{4}$. 5. $\frac{3}{8}$. 7. $\frac{3}{8}$. 9. $3\frac{3}{4}$.

2. $\frac{1}{4}$. 4. $\frac{1}{8}$. 6. $\frac{1}{8}$. 8. $2\frac{5}{8}$. 10. $6\frac{7}{8}$.

11. A foot is what per cent of a yard ? of 2 yards ?

12. An ounce is what per cent of a pound ? of 2 pounds ?

224. Important Per Cents. Certain per cents are used so frequently that their equivalent common fractions should be remembered. These are as follows:

$$50\% = \frac{1}{2} \quad 83\frac{1}{3}\% = \frac{5}{6}$$

$$25\% = \frac{1}{4} \quad 20\% = \frac{1}{5}$$

$$75\% = \frac{3}{4} \quad 40\% = \frac{2}{5}$$

$$12\frac{1}{2}\% = \frac{1}{8} \quad 60\% = \frac{3}{5}$$

$$6\frac{1}{4}\% = \frac{1}{16} \quad 80\% = \frac{4}{5}$$

$$3\frac{1}{8}\% = \frac{1}{32} \quad 5\% = \frac{1}{20}$$

$$37\frac{1}{2}\% = \frac{3}{8} \quad 8\frac{1}{3}\% = \frac{1}{12}$$

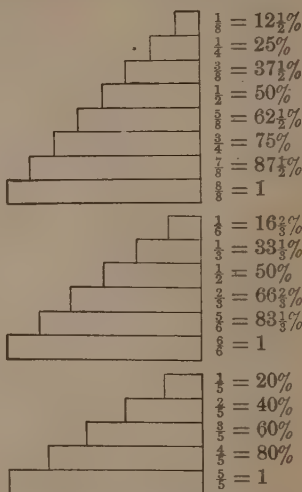
$$62\frac{1}{2}\% = \frac{5}{8} \quad 10\% = \frac{1}{10}$$

$$87\frac{1}{2}\% = \frac{7}{8} \quad 30\% = \frac{3}{10}$$

$$16\frac{2}{3}\% = \frac{1}{6} \quad 70\% = \frac{7}{10}$$

$$33\frac{1}{3}\% = \frac{1}{3} \quad 90\% = \frac{9}{10}$$

$$66\frac{2}{3}\% = \frac{2}{3} \quad 100\% = 1$$



To take $87\frac{1}{2}\%$ of 648 is, therefore, the same as to take $\frac{7}{8}$ of 648.

The similarity of these per cents to the corresponding decimal fractions given on page 87 should be noted.

EXERCISE 129

Find:

1. 50% of \$274.
2. 25% of \$372.
3. $12\frac{1}{2}\%$ of \$512.
4. $6\frac{1}{4}\%$ of \$3376.
5. $37\frac{1}{2}\%$ of \$17.76.
6. $62\frac{1}{2}\%$ of \$11.92.
7. $87\frac{1}{2}\%$ of \$34.48.
8. $3\frac{1}{8}\%$ of \$739.20.
9. $66\frac{2}{3}\%$ of \$80.07.
10. $83\frac{1}{3}\%$ of \$71.10.
11. 20% of \$735.15.
12. 40% of \$611.25.
13. 60% of \$734.15.
14. $112\frac{1}{2}\%$ of 72; of 88.
15. $137\frac{1}{2}\%$ of 96; of 88.
16. $116\frac{2}{3}\%$ of 72; of 96.

17. How much is $\frac{1}{8}$ of \$2640? $12\frac{1}{2}\%$ of \$2640?

18. How much is $\frac{3}{8}$ of \$3360? $37\frac{1}{2}\%$ of \$3360?

19. How much is $\frac{5}{8}$ of \$4320? $62\frac{1}{2}\%$ of \$4320?

20. How much is $\frac{7}{8}$ of \$5768? $87\frac{1}{2}\%$ of \$5768?

21. An inch is what fraction of a foot? what per cent of a foot?

22. A pint is what fraction of a gallon? what per cent of a gallon?

23. Five cents is what fraction of a dollar? what per cent of a dollar?

24. If you wish to find $33\frac{1}{3}\%$ of a number, what fractional part of the number do you wish to find?

25. If a man spends one fifth of his income for rent, what per cent does he spend for this purpose?

26. What per cent of the perimeter is one side of a square? What per cent are two sides? three sides?

27. If there are 30 pupils in a class and 50% are boys, what fraction are boys? How many boys are there?

28. If there are 450 pupils in a school and 52% are boys, what decimal fraction of the pupils are boys? How many boys are there? How many girls are there?

29. If a city has a population of 18,400, and $22\frac{1}{2}\%$ are pupils in school, what decimal fraction of the population is in school? How many are in school?

30. If a shop employs 1680 workers, of whom $37\frac{1}{2}\%$ are men, 50% are women, and the rest are boys, how many are there of each?

31. A merchant has \$9600 invested in his business. The first year he made a profit of $6\frac{1}{4}\%$, the second year $12\frac{1}{2}\%$, and the third year $16\frac{3}{4}\%$. How much did he make in the three years?

225. Terms used in Percentage. The number of which some per cent is to be taken is called the *base*.

The number of hundredths of the base is called the *rate*.

For example, in 25% of \$300, \$300 is the base and 25% is the rate.

Sometimes 25 is called the *rate per cent*, 25% being called the *rate*, but these two terms are commonly used to mean the same thing.

The result found by taking a certain per cent of the base is called the *percentage*.

Therefore, *the percentage is the product of the base and the rate.*

226. To find Some Per Cent of a Number. Required to find $23\frac{3}{4}\%$ of 275.

$$23\frac{3}{4}\% = 0.23\frac{3}{4}.$$

Multiplying by $0.23\frac{3}{4}$, we have $65.31\frac{1}{4}$.

If we wish, we may write $23\frac{3}{4}\%$ as 0.2375 and then multiply.

We may also, if we choose, write the product 65.3125 .

275

.23 $\frac{3}{4}$

206 $\frac{1}{4}$

825

550

65.31 $\frac{1}{4}$

Therefore, *to find a required per cent of a number, multiply the number by the given rate.*

EXERCISE 130

Find the following:

1. 10% of 80.

6. 6% of \$75.

11. $33\frac{1}{3}\%$ of 66.

2. 25% of 48.

7. 4% of \$50.

12. $66\frac{2}{3}\%$ of 99.

3. 75% of 96.

8. 5% of \$80.

13. $37\frac{1}{2}\%$ of 48.

4. 15% of 60.

9. 3% of \$90.

14. $62\frac{1}{2}\%$ of 168.

5. 40% of 40.

10. $4\frac{1}{2}\%$ of \$80.

15. $16\frac{2}{3}\%$ of 360.

16. In a school of 300 pupils 11% are in the sixth grade. How many pupils are in the sixth grade?

17. In a box of 192 oranges $6\frac{1}{4}\%$ have spoiled. How many have spoiled?

18. From a barrel of flour weighing 196 lb., 25% has been sold. How many pounds have been sold?

19. A contractor agreed to build a stone wall 72 ft. long. He has already built 20% of it. How many feet has he still to build?

20. A trolley car has a run of 56 mi. a day. When it has run 15% of this distance, how many miles has it run?

21. How much will 48% of a cubic foot of steel weigh if a cubic foot weighs 490 lb.?

22. If a steel car when full carries 96,000 lb. of coal, how much does it carry when it is $62\frac{1}{2}\%$ full?

23. If out of 240 problems in arithmetic you solved $93\frac{3}{4}\%$ without any errors, how many did you solve correctly?

24. If a locomotive weighing 116 tons can exert a pull equal to 22% of its weight, how many tons of pull can it exert?

25. A ball team has played 32 games this season and has lost $37\frac{1}{2}\%$ of them. How many games has it won?

26. A cubic foot of water weighs $62\frac{1}{2}$ lb. Ice is 92% as heavy as water. What does $25\frac{3}{4}$ cu. ft. of ice weigh?

27. The diameter of a circle is about $31\frac{1}{3}\%$ as long as the circumference. What is the diameter of a circle whose circumference is 15 ft.?

28. A meter is $9\frac{1}{3}\frac{2}{3}\%$ longer than a yard. How many inches in a meter?

29. A knot, used at sea, is $15\frac{1}{3}\%$ longer than a mile. How many feet are there in a knot?

30. A sidewalk 112 ft. long and 4 ft. 2 in. wide is to be made $12\frac{1}{2}\%$ longer and 20% wider. How many feet will it be extended? How much wider will it be? By how many square feet will its area be increased?

227. The Per Cent One Number is of Another. Because the percentage is the product of the base and the rate (§ 225), therefore

The rate equals the percentage divided by the base.

For example, what per cent of 5 is 4?

Since 4 is the product of 5 by some per cent, therefore $4 \div 5$, or 80%, equals the required per cent.

EXERCISE 131

Find what per cent the second number is of the first :

- | | | |
|------------|--------------|---------------------|
| 1. 5, 3. | 6. 100, 6. | 11. \$125, \$6.25. |
| 2. 10, 5. | 7. 150, 25. | 12. \$150, \$9.00. |
| 3. 12, 4. | 8. 175, 25. | 13. \$180, \$9.00. |
| 4. 25, 5. | 9. 225, 75. | 14. \$250, \$7.50. |
| 5. 75, 15. | 10. 250, 50. | 15. \$475, \$23.75. |

16. A man bought a horse for \$150 and sold it for \$175. How much did he gain? What per cent of the cost did he gain?

17. A man bought a farm for \$6500 and sold it for \$7150. How much did he gain? What per cent of the cost did he gain?

18. A boy weighed 75 lb. a year ago and he now weighs 82½ lb. What per cent of 75 lb. has he gained?

19. A village had a population of 1200 five years ago. It now has a population of 1332. What has been the per cent of increase?

20. A poultry raiser has 231 chickens this year. Last year he had 220 chickens. What is the per cent of gain this year? The year before he had 200 chickens. What was the per cent of gain last year?

21. A man saved \$750 more this year than last year. Last year he saved \$2000. What is the per cent of increase?

22. A man weighed 160 lb. in September and 10 lb. more the next April. What per cent had he gained on his weight in September?

23. A farm of 300 acres has 48 acres of woodland. What per cent of the farm is woodland?

24. In a certain school 22 of the pupils are in the sixth grade. There are 275 pupils in the school. What per cent are in the sixth grade?

25. A contractor has agreed to erect a building for \$48,000. He has been paid \$18,000 on account. What per cent of the contract price has he been paid?

26. In an arithmetic class it is planned to solve 550 problems in a certain number of days. All but 44 have been solved. What per cent of the problems remain to be solved? What per cent of them have been solved?

27. A baseball team has won 32 games and lost 28 games. What per cent of the total number of games has it won? What per cent has it lost?

28. A cubic foot of ice weighs $57\frac{1}{2}$ lb. A block of ice containing 4 cu. ft. loses $28\frac{3}{4}$ lb. by melting. What per cent does it lose?

29. A cubic foot of water weighs $62\frac{1}{2}$ lb. From a tank containing 16 cu. ft. there is drawn off an amount of water that would weigh 125 lb. What per cent of the water is drawn off?

30. To a class of 32 pupils 8 more are admitted. What per cent of the original number are admitted? What per cent of the present number are admitted? If 10 pupils are now taken out, what per cent are taken out?

228. One Number is a Given Per Cent of what Number? Because the percentage is the product of the base and the rate (§ 225), therefore

The base equals the percentage divided by the rate.

For example, if 15% of a number is 390, what is the number?

$$390 \div 15\% = 39,000 \div 15 = 2600. \text{ Ans.}$$

The simple equation may be used in cases of this kind, it being explained that $2x$ means 2 times some number that is to be found.

For example, $0.15x = 390.$

Therefore $x = 2600,$

by dividing these equals by 0.15

EXERCISE 132

Of what numbers are the following the given per cents?

- | | | |
|--------------|-----------------------------|--------------|
| 1. 720, 12%. | 4. 175, 25%. | 7. \$84, 6%. |
| 2. 300, 15%. | 5. 360, 75%. | 8. \$75, 5%. |
| 3. 690, 23%. | 6. 480, $12\frac{1}{2}\%$. | 9. \$88, 4%. |

10. What is the number of which \$75 is 125%? That is, what number increased by 25% equals \$75?

11. What number increased by $33\frac{1}{3}\%$ equals \$480?

12. What number increased by 50% equals \$600?

13. \$260 is 4% more than (or 104% of) what number?

14. \$420 is 5% more than what number?

15. \$375 is 75% of what number? That is, what number decreased by 25% equals \$375?

16. What number decreased by 25% equals \$900?

17. \$435 is 6% of what sum of money?

18. What number increased by $33\frac{1}{3}\%$ equals \$500?

19. A city increased 14% in population in ten years. The increase was 3332. What was the population before this increase?

20. A city increased 10% in population in ten years. The increase was 5200. What was the population both before and after this increase?

21. The weekly pay roll in a certain shop was increased 8%. The increase amounted to \$192. What was the pay roll before the increase?

22. The attendance at a certain school has increased 14, which is 5% of the attendance last year. What was the attendance last year?

23. A lady paid 95¢ less for groceries this week than last week. This was a decrease of 10%. How much did she pay last week? this week?

24. A man purchased this year $2\frac{1}{2}$ tons of coal less than he did last year, a reduction of $12\frac{1}{2}\%$. How many tons did he purchase last year? this year?

25. In a certain month there were seven stormy days, or 25% of the total number of days. What month was it?

26. A man sold a farm at a profit of $16\frac{3}{4}\%$ on the cost, thereby making \$540. What did the farm cost him?

27. A man has a meadow to mow. After he has mowed 24 acres, $37\frac{1}{2}\%$ of the work is done. How many acres has he then to mow?

28. A man buying a farm is told that if he will pay cash he can get it for $6\frac{1}{4}\%$ less, and can thereby save \$500. What would the farm then cost him?

29. A state is building a certain number of miles of road this year. It has already finished $33\frac{1}{3}\%$ of it, or 280 miles. How many miles is it to build this year?

EXERCISE 133

PROBLEMS WITHOUT NUMBERS

1. How do you find some required per cent of a number ?
2. How do you express per cent as a decimal fraction ?
3. How do you express a decimal fraction as per cent ?
4. How do you express per cent as a common fraction ?
5. How do you express a common fraction as per cent ?
6. Name eight of the most important per cents, and the corresponding common fractions.
7. If you know the base and rate, how do you find the percentage ?
8. How do you find what per cent one number is of another ?
9. How do you find the number of which a given number is a certain per cent ?
10. If you know your weight a year ago and also to-day, how do you find the gain ? the per cent of gain ?
11. If you know how many pounds you have gained, and the per cent, how do you find your weight a year ago ?
12. Knowing how much a man paid for his farm, and the amount for which he is selling it, how can you find the per cent of gain or loss ?
13. Knowing that a farmer paid a certain amount for a team of horses, and sold them at a certain per cent of profit, how do you find the selling price ?
14. If a boy had a kite string of a certain length, and lost a certain per cent of it, how do you find the amount left ?
15. If a boy had a fishing line of a certain length, and you know how many feet of it he lost, how do you find the per cent of loss ?

CHAPTER XII

PROFIT AND LOSS

229. Profit and Loss. In business transactions profit and loss are often computed as certain per cents of the cost of the property. The following may be taken as typical problems in profit and loss, including the marking of goods:

(1) At what price must a merchant mark goods that cost \$275, so as to gain 20%?

If he gains 20% of \$275 he gains $\frac{1}{5}$ of \$275, or \$55. Therefore he must mark the goods \$275 + \$55, or \$330.

(2) If goods marked \$75 are sold at a bargain sale for 15% off the marked price, at what price are they sold?

If the goods are sold at 15% off the marked price, they are sold for \$75 less 15% of \$75, or \$75 - \$11.25, or \$63.75.

(3) If some goods are damaged so that they are marked down to \$120, or 25% below cost, how much did they cost?

100% of the cost = the cost.

25% of the cost = the loss.

75% of the cost = the selling price = \$120.

1% of the cost = $\frac{1}{75}$ of \$120.

100% of the cost = $1\frac{2}{3}$ (or $\frac{4}{3}$) of \$120, or \$160.

(4) A dealer sold a hat that cost \$2.40 so as to gain 25%. The selling price was 20% less than the marked price. What was the marked price?

He gained 25% (or $\frac{1}{4}$) of \$2.40, or \$0.60. Therefore the selling price was \$2.40 + \$0.60, or \$3. Now show that

80% of the marked price = the selling price = \$3.

1% of the marked price = $\frac{1}{80}$ of \$3.

100% of the marked price = $1\frac{3}{4}$ (or $\frac{7}{4}$) of \$3, or \$3.75.

EXERCISE 134

For how much must a merchant sell goods that he bought at the price here given, so as to gain the per cent specified?

- | | | |
|---------------|-------------------------------|-----------------------------------|
| 1. \$30, 10%. | 9. \$240, $12\frac{1}{2}\%$. | 17. \$480, $16\frac{2}{3}\%$. |
| 2. \$45, 12%. | 10. \$375, 15%. | 18. \$540, $33\frac{1}{3}\%$. |
| 3. \$75, 15%. | 11. \$675, 16%. | 19. \$720, $12\frac{1}{2}\%$. |
| 4. \$25, 12%. | 12. \$175, $8\frac{1}{2}\%$. | 20. \$172.80, $12\frac{1}{2}\%$. |
| 5. \$36, 14%. | 13. \$190, $9\frac{1}{4}\%$. | 21. \$373.20, $16\frac{2}{3}\%$. |
| 6. \$65, 15%. | 14. \$250, 12%. | 22. \$45,550, $17\frac{1}{2}\%$. |
| 7. \$72, 18%. | 15. \$575, 15%. | 23. \$27,475, 22%. |
| 8. \$85, 25%. | 16. \$735, 18%. | 24. \$75,250, 25%. |

If goods marked as here shown are sold at a bargain sale for the given per cent off, find the selling price:

- | | | |
|----------------|--------------------------------|-----------------------------------|
| 25. \$60, 15%. | 31. \$176, $6\frac{1}{4}\%$. | 37. \$975, 20%. |
| 26. \$85, 18%. | 32. \$712, $12\frac{1}{2}\%$. | 38. \$960, 25%. |
| 27. \$72, 22%. | 33. \$972, $16\frac{2}{3}\%$. | 39. \$981, $33\frac{1}{3}\%$. |
| 28. \$75, 10%. | 34. \$720, $6\frac{1}{4}\%$. | 40. \$225.40, 25%. |
| 29. \$65, 12%. | 35. \$840, $12\frac{1}{2}\%$. | 41. \$27,250, 20%. |
| 30. \$96, 15%. | 36. \$636, $16\frac{2}{3}\%$. | 42. \$22,400, $17\frac{1}{2}\%$. |

If goods are sold for the sum specified, which is the given per cent below cost, find the cost:

- | | | |
|-------------------|-----------------------------------|-----------------------------------|
| 43. \$71.25, 5%. | 50. \$735, $12\frac{1}{2}\%$. | 57. \$358.80, 22%. |
| 44. \$72.16, 12%. | 51. \$506, 12%. | 58. \$568.80, 21%. |
| 45. \$58.48, 14%. | 52. \$382.50, 15%. | 59. \$226.20, 13%. |
| 46. \$66.75, 11%. | 53. \$848.40, $12\frac{1}{2}\%$. | 60. \$11,780, 20%. |
| 47. \$72.98, 11%. | 54. \$736.89, $12\frac{1}{2}\%$. | 61. \$12,300, 18%. |
| 48. \$140, 20%. | 55. \$207.20, 30%. | 62. \$14,025, 15%. |
| 49. \$726, 12%. | 56. \$265.60, 17%. | 63. \$14,700, $12\frac{1}{2}\%$. |

Given the cost and rate of gain as follows, find the amount of gain :

64. \$275, 15%. 69. \$275.40, 15%. 74. \$4275.50, 8%.
 65. \$365, $12\frac{1}{2}\%$. 70. \$346.50, 16%. 75. \$2936.40, 12%.
 66. \$4723, 14%. 71. \$492.75, 18%. 76. \$3275.75, 16%.
 67. \$3965, 16%. 72. \$575.50, 24%. 77. \$4206.03, $33\frac{1}{3}\%$.
 68. \$7287, 22%. 73. \$812.50, 16%. 78. \$3742.20, $16\frac{2}{3}\%$.

Given the cost and rate of loss as follows, find the amount of loss :

79. \$275, 6%. 82. \$362.50, 4%. 85. \$4725.25, 8%.
 80. \$345, 4%. 83. \$427.50, 8%. 86. \$2632.50, 12%.
 81. \$463, 8%. 84. \$732.25, 12%. 87. \$4170.20, 15%.

88. Some goods were sold for \$1015.30 at a loss of $31\frac{1}{4}\%$. Find the loss.

89. Beans bought @ \$1.60 a bushel are sold @ 7¢ a quart. Find the per cent of gain or of loss.

90. A dealer sold some goods that cost \$875 so as to gain 15%. What was the selling price?

91. A man bought a farm for \$4800 and sold it at a gain of 25%. What was the selling price?

92. A merchant's stock of goods cost him \$8450. It was damaged 45% by fire. What was it then worth?

93. Papers bought at the rate of 60 for 50¢ are sold for 1¢ each. Find the per cent of gain and the cost of 90 papers.

94. A box of 180 oranges is bought for \$5.40. The oranges are sold at 50¢ per dozen. Find the gain per cent.

95. A dealer buys 320 yd. of cloth at $87\frac{1}{2}\%$ a yard. At what price per yard must he sell the cloth to make a profit of \$40?

— 96. A merchant marked his goods by private signs so as to conceal the exact cost. He used this scheme:

B L A C K	H O R S E
1 2 3 4 5	6 7 8 9 0

An article worth between \$1 and \$2 was marked $\frac{BLE}{BKE}$, indicating the cost and the selling price. What was the cost? the selling price? the gain? the per cent of gain?

97. Using the key words of Ex. 96, what is the per cent of gain on a hat that cost LEE and sells for LKE?

98. Using EDUCATIONX for 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, what is the per cent of profit on an arithmetic that costs AA and sells for OX?

99. Using the key of Ex. 98, how must a dealer mark a pair of shoes that cost UUX so as to sell them at a profit of $33\frac{1}{3}\%$?

100. Using ARITHMETiC as the key, and X and Y as duplicate symbols for 0, how may a dealer mark goods that cost tTX and IiY so as to make a profit of 25% on the former and $33\frac{1}{3}\%$ on the latter?

The class may be encouraged to devise similar key words and make examples to correspond.

— 101. A merchant sold some goods that cost \$175 so as to gain 20%. The selling price was 20% less than the marked price. What was the marked price?

9 102. A dealer sold some furniture for which he had paid \$240 so as to gain $12\frac{1}{2}\%$. The selling price was 25% less than the marked price. What was the marked price? What per cent was the marked price above cost?

— 103. A grocer buys 50 bu. of potatoes at 60¢ a bushel and retails them at 20¢ a peck. Find his whole gain and his gain per cent.

104. A merchant sold some goods at 10% below cost, this being 20% below his marked price. The goods cost \$300. What was the marked price?

105. A wholesale dealer paid \$2.50 per barrel for apples. He sold them to a retailer at a gain of 8%, and the retailer sold them at a gain of $16\frac{2}{3}\%$. What was the retail price?

106. A clothier marked a suit of clothes that cost him \$13.20 so as to gain 40%. He then deducted from the marked price $16\frac{2}{3}\%$. What was the actual selling price?

107. A man bought a horse for \$150 and a carriage for \$120. He sold them, gaining 8% on the horse and losing 12% on the carriage. Find his net gain or loss.

108. A farmer received \$160 for some produce. It passed through the hands of a shipper, a commission merchant, and a retailer, who gained respectively $12\frac{1}{2}\%$, $3\frac{1}{3}\%$, and 20% on what it cost them. What was the retail price?

109. Some goods cost a merchant \$100. At what price must he mark them so that he can take off 20% and still make a profit of 20%?

110. Some goods cost a merchant \$240. At what price must he mark them so that he can take off 25% and still make a profit of 25%?

111. At what per cent above cost must a merchant mark his goods so that he can take off 20% and still make a profit of 20%?

112. A man sold two houses for \$9600 each. On one he gained 20% and on the other he lost 20%. Did he gain or lose on the whole transaction, and how much?

113. A grocer bought 625 doz. pound packages of a certain food for \$900, less $16\frac{2}{3}\%$ discount. He sold each package for 16¢. How much did he gain in all?

114. A merchant marks goods $12\frac{1}{2}\%$ above cost and sells them 20% below the marked price. Find his per cent of loss. Write the analysis.

115. A dealer bought some goods for \$2400 and sold them for \$3000. What was the per cent of gain on the purchase price of the goods?

116. A merchant's stock of goods cost him \$12,500. It was damaged by fire so that he sold out the entire stock for \$7500. What was the per cent of loss?

117. A merchant paid \$2250 for a carload of furniture. He sold \$1250 worth at a profit of 40%, but the rest having been damaged by fire was sold at a loss of 50%. Did he gain or lose on the entire carload; and how much?

118. A cattle dealer bought a lot of cows for \$1250. He sold them at a profit of 30% and invested the proceeds in cattle which he was obliged to sell at a loss of 30%. Did he gain or lose on the whole transaction, and how much?

119. A dealer in horses bought a carload of ponies for \$1250. He sold them at a loss of 20%, and with the proceeds he bought some horses that he sold at a profit of 20%. Did he gain or lose on the whole transaction, and how much?

120. A merchant invested \$4500 in goods and sold them at a profit of $33\frac{1}{3}\%$. He invested the proceeds again, selling this stock at a profit of 28%. How much did he make on the two transactions?

121. A man bought some land and sold it off in building lots. On two thirds of it he gained $33\frac{1}{3}\%$, and on one third of it he gained $66\frac{2}{3}\%$. On which did he gain the more?

122. A man bought some property for \$2400 and sold it at a profit of $12\frac{1}{2}\%$. He invested the proceeds in some more property and sold this at a profit of $12\frac{1}{2}\%$. What per cent did he gain on his original investment of \$2400?

CHAPTER XIII

THE EQUATION

230. The Equation Method. There is a better method for solving many of the problems of arithmetic than any that have thus far been given. It is known as the *equation method*.

231. Equation. An indicated equality between two expressions is called an *equation*.

For example, $2 + x = 5$ is an equation.

232. Unknown Quantity. In the equation $2 + x = 5$, x is called the *unknown quantity*.

We may indicate an unknown quantity by x , or by some initial, as d for dollars, or n for number. We may also use $x\%$, or any other convenient symbol.

233. Indicated Multiplication. We indicate the multiplication of $5 \times x$ by writing simply $5x$.

234. Solving an Equation. To find the value of the unknown quantity in an equation is to *solve* the equation.

235. How to Solve. In the equation $2 + x = 5$, we may subtract 2 from these two equal quantities, and we have $x = 5 - 2$, or $x = 3$.

To solve an equation, do to one side what is necessary to leave x by itself, and then perform the same operation on the other side.

Thus, if	$3x = 15$, divide both sides by 3.
Then	$x = 5$.
If	$\frac{1}{3}x = 7$, multiply both sides by 3.
Then	$x = 21$.

236. Applications of the Equation. The following are illustrations of the special application of the equation to percentage:

1. 25 is what per cent of 225?

Let $x\%$ = the required rate per cent.

Then $x\% \times 225 = 25$.

Dividing by 225, $x\% = \frac{25}{225}$
 $= \frac{1}{9}$, or $11\frac{1}{9}\%$.

2. \$8.40 is 12% of what number?

Let x = the required number.

Then $0.12 x = \$8.40$.

Dividing by 0.12, $x = \$8.40 \div 0.12$
 $= \$70$.

EXERCISE 135

Find what per cent the first number is of the second:

1. 25, 750.

6. $1\frac{1}{2}$, $7\frac{1}{2}$.

2. 37, 148.

7. \$1.80, \$3.

3. 5 ft., $37\frac{1}{2}$ ft.

8. 7 ft., 7 yd.

4. 528 ft., 2 mi.

9. \$4.90, \$29.40.

5. \$8.60, \$77.40.

10. 73 ft., 194 yd. 2 ft.

11. A man's income is \$1650 a year, and he spends \$693. What per cent of his income does he save?

12. In a certain village 576 out of the 1200 pupils in school are boys. What per cent are boys? girls?

13. The purity of gold is measured in carats, or 24ths, 18 carats meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in an 18-carat ring?

14. What is the per cent of pure gold in a watch case that is 16 carats fine? in a chain that is 12 carats fine? in a ring that is 14 carats fine?

15. A horse and carriage together cost \$375, and the carriage cost twice what the horse did. What did the horse cost?

We may explain the work as follows:

Let x = the *number of dollars* the horse cost.

Then $2x$ = the *number of dollars* the carriage cost,

and $x + 2x$ = the *number of dollars* both cost, which is 375.

Therefore $3x = 375$, because x (that is, $1x$) and $2x$ are $3x$.

Therefore $x = 125$, by dividing both sides by 3.

Therefore the horse cost \$125.

The actual work of this problem is as follows:

$$3x = 375.$$

$$x = 125.$$

16. After increasing 6%, a certain sum amounts to \$318. What is the sum?

We may explain the work as follows:

Let x = the *number of dollars* in the sum.

Then $0.06x$ = the *number of dollars* increase.

Adding, $1.06x$ = the *number of dollars* after the increase.

Therefore $1.06x = 318$.

Therefore $x = 318 \div 1.06 = 300$.

Therefore the sum is \$300, as is easily proved.

The actual work of this problem is as follows:

$$1.06x = 318.$$

$$x = 300.$$

17. A man sold a carriage for \$120, which was 20% less than it cost. How much did it cost?

We may explain the work as follows:

Let x = the *number of dollars* of cost.

Then $0.20x$ = the *number of dollars* lost,

and $x - 0.20x$ = the *number of dollars* received.

Therefore $0.80x = 120$, because $1x - 0.20x = 0.80x$.

Therefore $x = 120 \div 0.80 = 150$.

Therefore the cost was \$150, as is easily proved.

The actual work of this problem is as follows:

$$0.80x = 120.$$

$$x = 150.$$

18. Of what length is 112 ft. 6 in. exactly 45%?

19. After gaining 12%, a certain sum amounts to \$392.
What is the sum?

20. If to $\frac{1}{5}$ of a certain number 730 is added, the result is 855. What is the number?

21. A boy lost $\frac{2}{3}$ of his marbles and had 18 left. How many marbles had he at first?

22. A man sold $\frac{3}{4}$ of his cattle and had 68 left. How many cattle had he at first?

23. A certain sum was increased 50% and then amounted to \$505.50. What was the sum?

24. A man sold a house for \$2250, which was 20% less than it cost. How much did it cost?

25. A man saves \$675.20 a year, which is 32% of his income. How much is his income?

26. A school has 20% of its pupils in the sixth grade, which numbers 29. How many are there in the school?

27. If a school is in session 50% of the days of a certain year, and is in session 183 days, is that a leap year or not?

28. With \$2250 a man bought 20 acres of land from Mr. A, who thereby gained 25% on what it cost him. How much did Mr. A pay per acre for the land?

29. A man sold a farm of 100 acres at the rate of \$56.25 an acre. He lost 10% on the cost of the farm. How much did the farm cost him?

30. A man buys some lumber and sells it for \$60, thereby gaining 20% on the cost. The \$60 is what per cent of the cost? What is the cost?

31. A certain school has 119 boys, which is 85% of the number of girls. How many girls are there in the school? What is the total number of pupils?

32. A man sells 41 sheep, which is $33\frac{1}{3}\%$ of all he owned. He received \$5.50 a head. At this rate, what is the value of the sheep remaining in his flock?

33. A man's salary has been increased 22% this year. It is now \$1525. What was it last year?

$$1.22x = \text{what number?}$$

34. If I have added 60% to the size of my farm, and now have 240 acres, how many did I have before the increase?

35. If you weigh 78.1 lb., and have increased 42% in weight in the last 5 years, how much did you weigh five years ago?

36. If our regular army has been increased 21% , and now numbers 73,810, how many did it number before the increase?

37. If a library has increased in size $16\frac{2}{3}\%$ in the past five years, and now has 8323 volumes, how many volumes had it before the increase?

38. A dealer bought two horses at the same price. He sold one, at a profit of 20% , for \$102. The other he sold at a loss of 10% . How much did he receive for the latter?

39. A certain village, after gaining 10% on its population in 1910, had 10% less population than another village of 4180 inhabitants. What was the population of the first village in 1910?

40. A furniture dealer bought 5 sets of furniture at \$65 a set, 3 sets at \$90, and 2 sets at \$40. The \$40 sets he sold at a profit of 30% , and the \$65 sets at a profit of 20% , but on the \$90 sets he lost 10% . What per cent did he gain on the lot?

41. If you have read all but 40% of a book, and have read 234 pages, how many pages has the book?

$$x - 0.40x = 0.60x. \quad 0.60x = \text{what number?}$$

42. If the width of a schoolhouse is 10% less than the length, and the width is 46.8 ft., what is the length?

43. If you lose 10% of a kite string, and have 585 ft. left, how long was the string? How many feet did you lose?

44. How much is a man's income if he pays 12% of it for rent at the rate of \$15 a month?

45. If an automobile going 14 mi. an hour increases its speed 37%, at what rate will it then be traveling?

46. If the boys in a certain school are 45% of the total, and there are 66 girls, how many boys are there?

47. What sum of money increased by $33\frac{1}{3}\%$ of itself becomes \$600? What sum of money decreased by $33\frac{1}{3}\%$ of itself becomes \$600?

48. A stenographer can write 105 words a minute, which is 25% more than she could write three months ago. What was her rate then?

49. A stenographer can write 95 words a minute. Three months ago she wrote 20% less than now. What was her rate then?

50. A cubic foot of platinum weighs 1220 lb., and a cubic foot of pressed gold weighs 10 lb. less. Platinum is what per cent heavier than pressed gold?

51. A bank averaged \$1200 a day of new deposits in November, and 26% more a day in December. What were the average daily deposits in December?

52. A clerk's salary is \$12 a week. If he spends 20% of it for rent, 10% for clothes, and 25% for other expenses, and saves the rest, how much does he save in 52 wk.?

CHAPTER XIV

TRADE DISCOUNT

237. Trade Discount. A reduction from the marked price of an article, from the amount of a bill of goods, or from the amount of a debt is called *trade discount*.

Trade discount is also called *commercial discount*.

Discounts are usually reckoned at some common fraction of, or at some rate per cent of, the amount from which the discount is made.

238. Cash Discount. A discount that is taken off because a purchaser is paying cash at once instead of waiting to make the payment later, is called a *cash discount*.

The amount of the bill after the discount has been made is called the *net amount*, or the *net price*.

The price given in a printed list is called the *list price*.

What is the net amount of a bill of \$125 after a discount of 20% is made?

Amount of bill	\$125
Less 20%, or $\frac{1}{5}$, of \$125	25
Net amount	\$100

Instead of speaking of "20% off," merchants often speak of " $\frac{1}{5}$ off." It is not so convenient to use common fractions for 12%, 7%, etc.

EXERCISE 136

Find the discount on the following at the rates stated:

- | | | |
|----------------|---------------|----------------|
| 1. \$250, 10%. | 4. \$575, 4%. | 7. \$1250, 2%. |
| 2. \$375, 20%. | 5. \$675, 6%. | 8. \$1400, 8%. |
| 3. \$450, 15%. | 6. \$825, 3%. | 9. \$2250, 12% |

Find the net prices of goods billed as follows, with discounts as stated :

10. \$250, 20%. 13. \$475, 8%. 16. \$480, $16\frac{2}{3}\%$.

11. \$175, 10%. 14. \$560, 12%. 17. \$720, $12\frac{1}{2}\%$.

12. \$360, 10%. 15. \$775, 15%. 18. \$810, $33\frac{1}{3}\%$.

19. A sewing machine was marked \$50 but was sold at $12\frac{1}{2}\%$ discount. What was the net price?

20. A piano was marked \$800 but was sold at 8% discount. What was the net price?

21. A dealer bought some goods for \$975.25, less 20%. What was the net price?

22. A lady bought a \$50 cloak at a bargain sale at 25% discount. What did it cost her?

23. A dealer offers some goods for \$75, or 4% less for cash. How much does a purchaser save by paying cash?

24. A dealer offers some goods for \$80, or for \$72 if the purchaser pays cash. What per cent discount does he offer?

25. A dealer offers some goods for \$10.80 less than the regular price, a discount of 12%. What is the regular price?

26. The regular price of some goods is \$496, but they cost a dealer 85% of this sum. How much is the discount?

27. A dealer buys a bill of goods amounting to \$1875. He is allowed a discount of 8% on this bill. How much is the discount?

28. A dealer buys a bill of goods amounting to \$2250. He is allowed a discount of 6% on this bill. What is the net amount of the bill?

29. A dealer buys a bill of goods amounting to \$2800, but by paying cash he settles it for \$2576. What is the rate of discount allowed?

30. A dealer buys a bill of goods, and by paying cash he saves \$94.50. The rate of discount being 7%, what is the original bill? How much does he pay?

31. A dealer buys a bill of goods amounting to \$2575, but by paying cash he settles it for \$231.75 less. What is the rate of discount allowed?

32. The discount on some goods listed at \$460 is \$115. What is the rate of discount?

33. The discount on some goods is \$41.60 at 8%. What is the list price?

34. The discount on some goods is \$2.05 at $12\frac{1}{2}\%$. What is the list price?

35. A bicycle was marked \$45, but was sold at 12% discount. What was the net price?

36. A piano was listed at \$550, but was sold at 30% discount. What was the selling price?

37. A jobber bought some goods listed at \$2450. He was allowed a discount of 22%. What was the net price?

38. A merchant bought 850 yd. of muslin at 6¢ a yard, less 18%. How much did it all cost?

39. A merchant bought 975 yd. of lawn for \$146.25, less 20%. How much did he pay per yard?

40. A grocer bought 36 doz. cans of soup listed at \$1.75 a dozen. He was allowed a discount of 25%. What was the net price?

41. A dealer sold some boxes of soap, 60 cakes to the box, listed at \$2.25 a box. He allowed a discount of 20%. How much did he receive per cake?

42. What per cent of profit will a man make by paying \$17.60 for an article, with a discount of 20% from the list price, and selling it at the list price?

43. Find the net amount of a bill for \$470, the discount being 18%.

44. On a bill for \$1760 a discount of \$105.60 is allowed. What is the rate of discount?

45. A grocer bought 24 doz. pound packages of macaroni for \$31.20, less 15%. He sold it for 15¢ per package. How much did he gain in all?

46. A merchant sold at $12\frac{1}{2}\%$ discount some goods that were marked \$272.80. What was the selling price?

47. A grocer bought 160 small bags of flour, $12\frac{1}{4}$ lb. to the bag, listed at 4¢ a pound. He was allowed a discount of 15%. What did the flour cost him?

48. Find the net amount of a bill for \$880 subject to a discount of $12\frac{1}{2}\%$; of $6\frac{1}{4}\%$; of 25%; of $2\frac{1}{2}\%$.

49. On a bill of \$1650 a discount of $8\frac{1}{2}\%$ was allowed. What was the net price?

50. Find the net cost of a piano marked \$675, a discount of $8\frac{1}{2}\%$ being allowed.

51. What is the marked price of an automobile the discount on which is \$140 at 8%?

52. If the marked price of an automobile is \$1850 and the selling price is \$1702, what is the rate of discount?

53. Which is the better for the purchaser, and how much better: a discount of $16\frac{2}{3}\%$ on a bill for \$842.50, or a flat reduction of \$140?

54. Which is the better for the seller, and how much better: a discount of $12\frac{1}{2}\%$ on a bill for \$1275, or a flat reduction of \$159?

55. Which is the better for the purchaser, and how much better: a discount of $6\frac{1}{4}\%$ on a bill for \$625.75, or a flat reduction of \$39.25?

239. Bills Discounted. A bill for goods purchased by a dealer and subject to discount is usually made out as follows:

New York, Aug. 1, 1914

Mr. Thos. K. Ryder, Syracuse, N.Y.

Bought of Hibbard and Company

Terms: 4%

Dealers in Stationery

July 5	60 gro. Pens @ 79¢	\$47	40		
	12 doz. Penholders @ 8¢		96		
	40 gro. Lead pencils @ \$3 ⁴⁰	136			
		\$184	36		
	Less 4%	7	37	\$176	99

*Received payment
Hibbard & Co.*

For bills not subject to trade discount, see page 171. /

EXERCISE 137

Write bills similar to the above, inserting places, dates, and names, making the extensions and footing, and then deducting the discount:

1. 25 M (or 25,000) envelopes @ \$2.30 per M, 480 lb. paper @ 14¢, 16 doz. bottles ink @ 36¢ per dozen, 12 doz. bottles mucilage @ 35¢ per dozen. Discount 5%.

2. 60 doz. cans soup @ \$2.96 per dozen, 20 doz. jars meat extract @ \$4.16 per dozen, 20 doz. cartons raisins @ \$2.39 per dozen, 40 doz. baskets washed figs @ \$2.42 per dozen. Discount 8%.

3. Supply the proper numbers in the spaces marked with an asterisk (*) in the following bill:

Ithaca, N. Y., March 7, 1915

Mr. Robert J. Richards, Elmira, N. Y.

Bought of Thompson Bros.

Terms: 6% 30 da., net 60 da.

March 7	50 gro. Pens	@ 78¢	\$39			
	15 doz. Notebooks	@ \$1. ⁷⁰ _—	*			
	4 doz. Rulers	@ 1. ³² _—	*			
	30 gro. Lead pencils	@ 3. ³⁵ _—	*			
			*			
	Less 6%		*		*	

Make out bills for the following:

4. Bought 40 arithmetics @ 45¢, 65 geographies @ 90¢, 25 readers @ 35¢. Discount 12%.

5. Bought 3 doz. locks @ \$4.25, 4 doz. bolts @ \$1.10, 2½ doz. hinges @ \$2.35. Discount 30%.

6. Bought 4 doz. spoons @ \$15, 8 doz. plated tablespoons @ \$5, 6½ doz. teaspoons @ \$10.50. Discount 8%.

7. Bought 3 doz. pencils @ 25¢, ½ doz. rulers @ \$1.00, 1 pt. ink @ 25¢, 75 notebooks @ 15¢, 3 doz. bottles of ink @ 38¢, 4 lb. paper @ 30¢. Discount 10%.

8. Bought 575 yd. cotton flannel @ 38½¢, 684 yd. cotton @ 7½¢, 364 yd. buckram @ 9½¢, 648 yd. sheeting @ 26¾¢, 327 yd. Java canvas @ 62½¢, 340 yd. taffeta @ 96¢, 480 yd. lawn @ 23¢. Discount 12%.

9. 8 doz. fountain pens @ \$21.75 per dozen, 20 doz. boxes fine note paper @ \$4.80 per dozen, 8 gross erasers @ \$3.60 per gross. Terms: 6%.

10. 25 doz. cartons prunes @ \$1.38 per dozen, 8 doz. packages citron @ \$2.72 per dozen, 2 gro. cartons dates @ \$8.80 per gross, 2 gro. gelatin @ \$9.60 per gross, 60 bbl. flour @ \$5.48. Terms: 6%.

11. 8 gro. macaroni @ \$13.78 per gross, 10 doz. bottles olive oil @ \$2.38 per dozen, 3 gro. packages beans @ \$17.80 per gross, 8 gro. packages oatmeal @ \$27.70 per gross, 70 gal. sirup @ 42¢ per gallon. Terms: 8%.

12. 5 gro. baking powder @ \$23.80 per gross, 8 doz. jars mustard @ \$1.91 per dozen, 8 doz. boxes table salt @ \$1.22 per dozen, 16 doz. boxes cinnamon @ \$3.61 a dozen, 18 doz. cans molasses @ \$1.96 per dozen. Terms: 6%.

13. 40 M envelopes @ \$2.70 per M, 3 doz. fountain pens @ \$18.50 per dozen, 8 doz. bottles ink @ 32¢ per dozen, 5 gross erasers @ \$3.30 per gross. Terms: 8%.

14. 80 bbl. flour @ \$5.23, 60 gal. molasses @ 38¢, 75 gal. fancy sirup @ 50¢, 3 doz. cartons rye flour @ \$2.64, 260 lb. coffee @ 9.65¢. Terms: 6%.

15. 10 doz. hatchets @ \$10.75 per dozen, 6 doz. pr. hinges @ \$4.35 per dozen, 5 doz. carpenter's squares @ \$34.50 per dozen, $\frac{1}{2}$ gross locks @ \$42.50 per gross, 8 doz. files @ \$6.25 per dozen. Terms: 40%.

16. 500 yd. cotton flannel @ 38 $\frac{1}{4}$ ¢, 600 yd. cotton @ 7 $\frac{1}{3}$ ¢, 360 yd. buckram @ 9 $\frac{1}{2}$ ¢, 640 yd. sheeting @ 26 $\frac{3}{4}$ ¢, 320 yd. Java canvas @ 62 $\frac{1}{2}$ ¢. Terms: 12%.

17. D. P. Howard purchased from Brown Bros. 84 gross bone buttons @ 18 $\frac{1}{2}$ ¢, 72 doz. pearl buttons @ 9 $\frac{3}{4}$ ¢, 690 yd. cambric @ 17 $\frac{1}{2}$ ¢, 360 yd. cashmere @ 82 $\frac{1}{4}$ ¢, 400 yd. chambray @ 21 $\frac{3}{4}$ ¢. Terms: 12%.

240. Discount Series. The changes in cost of production from time to time, and the variation due to the credit of the buyer, and the quantity of goods purchased, give rise frequently to several discounts known as a discount series.

Goods that are selling at a discount of 20% may be produced at a lower cost owing, for example, to an unusually large cotton crop. An additional discount of 10% may then be allowed. Likewise a person whose credit is exceptional, who buys an especially large quantity, or who pays cash, may be entitled to a still further discount of 5%.

In a discount series the first denotes the discount from the list price, the second denotes the discount from the remainder, and so on.

Thus \$100 less 20%, 10%, and 5%, means that 20% is taken from \$100, leaving \$80; then 10% from \$80, leaving \$72; then 5% from \$72, leaving \$68.40. The order is, however, immaterial.

When there is a fraction of a cent in a remainder, the seller usually counts it a whole cent in his favor.

The regular discount is quoted first, and then the special discounts.

Advantages of a discount series are that it saves reprinting elaborate catalogues, allows of easy change, enables the dealer easily to offer different discounts to different customers, and allows for several options to purchasers.

If a bill is due at once it usually bears the words "Terms cash." If due in 60 days without discount, it may bear the marks "N/60" or "Net 60 da." If due in 60 days, but with a special discount of 3% if paid in 10 days, it may read "3/10, N/60," or "3% 10 da., net 60 da."

EXERCISE 138

Given these list prices and discounts, find the net prices :

- | | |
|--------------------------------------|---|
| 1. \$225, 20% and 6% | 7. \$480, 25%, 10%, 5%. |
| 2. \$368, 25% and 5%. | 8. \$560, 25%, 20%, 3%. |
| 3. \$480, $12\frac{1}{2}\%$ and 3%. | 9. \$729, $33\frac{1}{3}\%$, 10%, 10%. |
| 4. \$350, 15% and 4%. | 10. \$280, 25%, 10%, 10%. |
| 5. \$560, 25% and 10%. | 11. \$2760, $33\frac{1}{3}\%$, 20%, 10%. |
| 6. \$270, $33\frac{1}{3}\%$ and 10%. | 12. \$5550, $33\frac{1}{3}\%$, 25%, 15%. |

13. How much will 240 ft. of iron pipe cost if listed at \$1.50 a foot, a discount of 40% and 10% being allowed?

14. How much will a gross of jars of mixed pickles cost at \$6 a dozen, a discount of 20% and 10% being allowed?

15. The list price of 2-pound cartons of prunes is \$24 per 100. A discount of 20% and 10% is allowed. What is the net price of 300 cartons?

16. A book dealer can buy an encyclopedia for \$60 less 20% and 10%, or for the same price less 15% and 15%. Which is the better for him, and how much better?

17. A grocer buys 100 dozen jars of table salt listed at \$1.50 a dozen, a discount of 25% and 8% being allowed. If he sells the salt at 12¢ a jar, how much does he gain?

18. A dry goods dealer buys 960 yd. of silk listed at \$1.50 a yard, a discount of $33\frac{1}{3}\%$ and 5% being allowed. If he sells it at \$1.50 a yard, how much does he gain?

19. A dealer buys 36 doz. pocket knives listed at \$8.40 a dozen, a discount of 25% and 10% being allowed. If he sells them at 75¢ apiece, how much does he gain?

20. A furniture dealer buys 16 bedroom sets listed at \$36 each, a discount of $16\frac{2}{3}\%$ and 10% being allowed. If he sells the furniture at \$36 a set, how much does he gain?

21. A hardware dealer bought a shipment of stoves. After deducting a discount of $33\frac{1}{3}\%$ and 25%, and paying \$9.75 freight and \$2.50 cartage, the stoves cost him \$252.25. What was the list price of the stoves?

22. A dry goods dealer bought a shipment of cloth. After deducting a discount of 25% and 10%, and paying \$8.50 for freight and cartage, the cloth cost him \$410. What was the list price?

241. Bills with Several Discounts. The following is a common form of a bill of goods purchased from a wholesale dealer and subject to a discount series.

Binghamton, N. Y., March 15, 1917

Mr. R. D. Seanlan, Cortland, N. Y.

Bought of Lane & Company, Jewelers

Terms: 20%, 10%

Feb.	10	8 doz. Spoons @ \$14.75	\$118	00		
		7 doz. Forks @ \$4.20	29	40		
			\$147	40		
		Less 20%, 10%	41	27	\$106	13

EXERCISE 139

Make out bills for the following and discount them :

- 36 doz. files at \$6.25. Discounts 30%, 20%.
- 960 yd. silk at \$1.40. Discounts 25%, 10%.
- 4 doz. pr. hinges at \$4.60; 25 doz. table knives at \$8.40. Discounts 20%, 10%.
- 12 doz. locks at \$4.20; 6 doz. mortise locks at \$4.60. Discounts 25%, 8%.
- 360 yd. taffeta at 98¢; 4 gro. pompons at \$144; 3 doz. pieces braid at \$20.40. Discounts 10%, 5%.
- 60 lb. brads at 6¢; 200 bolts at \$4.80 per C; 6 doz. knives at \$7.20. Discounts 40%, 20%, 10%.
- 480 yd. silk at \$1.60; 640 yd. lawn at 22¢; 960 yd. taffeta at 96¢. Discounts 10%, 5%, 5%.

8. 2 doz. tennis rackets @ \$21, 8 doz. balls @ \$3, $\frac{3}{4}$ doz. tennis nets @ \$19.20. ^{per doz.} Discounted @ 12%, 5%, 2%.

9. $\frac{1}{2}$ doz. rugs at \$84, ^{per doz.} 750 yd. carpet @ 82¢, 3 doz. has-socks @ \$6. ^{per doz.} Discounted @ $\frac{1}{5}$, $\frac{1}{4}$.

10. 5 doz. bottles ink @ \$3, 300 lb. paper @ 30¢, 2 doz. bottles mucilage at \$4, 6 gro. pencils @ \$2.75. Discounted @ 3%, 2%, 1%.

11. 18 brass bedsteads @ \$17.25; 15 woven wire mattresses @ \$3.45; 3 oak sideboards @ \$17.25. Discounted @ $\frac{1}{8}$, $\frac{1}{10}$.

12. 50 M envelopes @ \$2.10, 3 doz. fountain pens @ \$19.50, 8 doz. bottles ink @ 34¢, 5 gro. erasers @ \$3.38. Terms: 8% 30 da., net 60 da.

13. 75 bbl. flour at \$5.23, 63 gal. molasses @ 39¢, 75 gal. fancy sirup at 58¢, 3 doz. cartons flour @ \$2.64, 260 lb. coffee @ 9.65¢. Discounted @ 6%.

14. 8 doz. hatchets @ \$10.75, 6 doz. pr. hinges @ \$4.35, 5 doz. carpenter's squares @ \$34.50, $\frac{1}{2}$ gro. locks @ \$42.50, 3 doz. pr. shears @ \$7.50, 8 doz. files @ \$6.25. Discounted @ 40%, 10%, 5%.

15. 21 Ohio steers, 1226 lb. (average), @ \$5.10 (per 100 lb.); 20 do. (ditto = the same), 1247 lb., @ \$5.10; 20 do., 1112 lb., @ \$4.95. Discounted @ $\frac{1}{10}$.

16. 20 Illinois steers, 1157 lb. (see Ex. 15), @ \$4.60; 28 do., 996 lb., @ \$4.12 $\frac{1}{2}$; 20 Kentucky steers, 1230 lb., @ \$4.80; 7 Indiana steers, 1078 lb., @ \$4.70. Discounted @ $\frac{1}{10}$, $\frac{1}{10}$.

17. 75 180-pound bags Western potatoes @ \$1.48, 50 150-pound do. @ \$1.28, 125 168-pound do. @ \$1.42, 1620 lb. Jersey potatoes in bulk @ \$1.47 per 180 lb. Discounted @ $\frac{1}{8}$, $\frac{1}{10}$.

242. Single Discount equal to a Series. It is often convenient to find a single discount equal to a series.

For example, find the single discount equal to the discount series 20%, 10%, and 10%.

Let the list price be represented by 100	% of the list price.
Then deducting 20% of this, or	20 % of the list price,
we have	80 % of the list price.
Deducting 10% of this, or	8 % of the list price,
we have	72 % of the list price.
Deducting 10% of this, or	7.2 % of the list price,
we have	64.8 % of the list price.

Then $100\% - 64.8\% = 35.2\%$, the single discount.

EXERCISE 140

Find the single discount equal to the discount series:

- | | |
|-----------------------------|-----------------------------------|
| 1. 10%, 8%. | 9. 25%, 15%, 10%. |
| 2. 15%, 6%. | 10. 25%, 20%, 10%. |
| 3. 18%, 3%. | 11. 30%, 20%, 10%. |
| 4. 14%, 9%. | 12. $33\frac{1}{3}\%$, 10%, 5%. |
| 5. 20%, 10%. | 13. $33\frac{1}{3}\%$, 10%, 10%. |
| 6. 30%, 10%. | 14. $16\frac{2}{3}\%$, 10%, 10%. |
| 7. 25%, $12\frac{1}{2}\%$. | 15. 20%, $12\frac{1}{2}\%$, 10%. |
| 8. 10%, 10%, 10%. | 16. 25%, $12\frac{1}{2}\%$, 10%. |

17. Compare the single discount equal to the series 10% and 5%, with the sum of 10% and 5% minus the product of 10% and 5%.

18. By the method suggested in Ex. 17, find the single discount equal to the series 20%, 5%.

19. How does the single discount equal to the discount series 20%, 10%, and 5%, compare with that equal to the discount series 10%, 5%, and 20%? Prove it.

EXERCISE 141

REVIEW

1. Find the net amount of a bill for \$880 subject to discounts of $2\frac{1}{2}\%$, 5% , and 10% .

2. On a bill of \$620 trade discounts of 5% and 5% are allowed. What is the net price?

3. Find the net cost of a piano marked \$450, with trade discounts of 3% , 8% , and 10% .

4. What is the list price of an article whose net cost is \$6.15 after discounts of 20% and 25% are deducted?

5. What per cent of profit will a man make by paying \$17.10 for an article, with discounts of 20% , 10% , and 5% from the list price, if he sells it at the list price?

6. Furniture is bought at trade discounts of 5% and 16% from the list price, and the net cost is \$518.70. Find the list price of the furniture.

7. Find the net amount of a bill of \$470, the trade discounts being 20% and 5% . What single rate of discount is equivalent to these discounts?

8. What single discount is equivalent to a discount series of 40% , 20% , 10% , and 5% ?

9. Which is the better for the purchaser, and how much better: two successive trade discounts of 25% and 10% on a bill of \$500, or a single discount of $33\frac{1}{3}\%$?

10. Which is the better for the seller, and how much better: trade discounts of 20% , 12% , and 3% , on a bill of \$800, or a single discount of $31\frac{3}{4}\%$?

11. Which is the better for the purchaser, and how much better: trade discounts of 10% , 10% , and 10% , on a bill of \$625.75, or a single discount of $27\frac{1}{10}\%$?

EXERCISE 142

PROBLEMS WITHOUT NUMBERS

1. If you know the list price of some goods and the single rate of discount, how do you find the net price?

2. If you know the list price and net price, how do you find the rate of discount?

3. If you know the net price and the rate of discount, how do you find the list price?

4. If you know the list price and the several rates in a discount series, how do you find the net price?

5. If you have a discount series, is it right to add the several rates to find the single rate of discount? State the reason.

6. If you know the several rates in a discount series, how do you find the single rate equivalent to them?

7. If you know the several rates in a discount series, and the net price, how do you find the list price?

8. Does it make a difference in the net price if you vary the order in a discount series? State the reason.

9. If the rate of discount is given in per cent, how can you express it as a common fraction?

10. If the rate of discount is given as a common fraction, how can you express it as per cent?

11. If a discount series is given in per cents, how can you express it as a single common fraction?

12. If a discount series is given in common fractions, how can you express it as a single rate per cent?

13. If a dealer marks some goods a certain rate above cost, and then takes off that same rate as a discount from the marked price, does he gain or lose? Why?

CHAPTER XV

COMMISSION

243. Commission or Brokerage. A sum paid by one person to another for transacting business is called *commission* or *brokerage*.

For example, if a man sells a house for another man, at 2% commission, and the price received is \$5000, the amount of the commission is 2% of \$5000, or \$100.

Similarly, if a man collects a bill of \$250 for another man, at 5% commission, he receives 5% of \$250, or \$12.50.

244. Principal and Agent. The person for whom another transacts business is called the *principal*. The person who transacts the business is called the *agent*.

245. Commission Merchant and Broker. An agent who has actual possession of the goods of his principal is called a *commission merchant*.

An agent who makes sales or purchases for his principal without having actual possession of the goods is called a *broker*.

Commission and brokerage are usually reckoned at a certain per cent of the money involved in the transaction. If goods are bought, the commission is a certain per cent of the cost; if goods are sold, the commission is a certain per cent of the amount received; if a collection is made, the commission is a certain per cent of the amount collected.

Brokerage is often a fixed sum per bushel (as of wheat purchased), a fixed sum per 100 bales (as of cotton), a fixed sum per tierce (as of lard), and so on.

In all problems, unless otherwise directed, discard any fraction of a cent less than $\frac{1}{2}$ in a final result; when a fraction of a cent is $\frac{1}{2}$ or more, count it as a full cent.

EXERCISE 143

Find the commissions on sales or purchases of the following amounts, at the rates specified :

- | | | |
|----------------|--------------------------------|--------------------------------|
| 1. \$275, 2%. | 9. \$2750, $1\frac{1}{2}\%$. | 17. \$8000, $\frac{1}{8}\%$. |
| 2. \$345, 3%. | 10. \$3275, $2\frac{1}{2}\%$. | 18. \$7500, $\frac{1}{4}\%$. |
| 3. \$675, 4%. | 11. \$2760, $3\frac{1}{2}\%$. | 19. \$6200, $\frac{1}{2}\%$. |
| 4. \$725, 5%. | 12. \$4175, $4\frac{1}{2}\%$. | 20. \$3520, $\frac{3}{8}\%$. |
| 5. \$875, 6%. | 13. \$6375, $3\frac{1}{3}\%$. | 21. \$4800, $1\frac{1}{8}\%$. |
| 6. \$4125, 2%. | 14. \$6540, 5%. | 22. \$8000, $1\frac{1}{4}\%$. |
| 7. \$7250, 3%. | 15. \$8650, 6%. | 23. \$2400, $1\frac{1}{4}\%$. |
| 8. \$9325, 4%. | 16. \$7775, 8%. | 24. \$4800, $2\frac{1}{2}\%$. |

Find the net amounts remaining after deducting commissions on sales as follows, at the rates specified :

- | | | |
|--------------------------------|--------------------------------|-------------------------------|
| 25. \$4800, $\frac{1}{8}\%$. | 32. \$2800, $2\frac{1}{4}\%$. | 39. \$275.50, 4%. |
| 26. \$6400, $\frac{1}{4}\%$. | 33. \$5600, $2\frac{1}{8}\%$. | 40. \$612.50, 8%. |
| 27. \$3500, $\frac{1}{8}\%$. | 34. \$6400, $3\frac{1}{2}\%$. | 41. \$721.50, 6%. |
| 28. \$400, 2%. | 35. \$1500, $1\frac{1}{2}\%$. | 42. \$5600, $\frac{1}{8}\%$. |
| 29. \$600, 4%. | 36. \$2200, $1\frac{1}{4}\%$. | 43. \$4200, $\frac{1}{4}\%$. |
| 30. \$850, 5%. | 37. \$3600, $1\frac{3}{4}\%$. | 44. \$3200, $\frac{3}{8}\%$. |
| 31. \$1250, $4\frac{1}{2}\%$. | 38. \$4800, $3\frac{3}{4}\%$. | 45. \$6400, $\frac{5}{8}\%$. |

46. After paying 5% for collecting a debt of \$1200, how much remains ?

47. After paying 3% for selling some property for \$3200, how much remains ?

48. An agent received \$84, or 2%, for selling some property. What was the selling price ?

49. An agent received \$75, or $2\frac{1}{2}\%$, for selling some property. What was the selling price ?

50. An agent sold 325 bu. of wheat @ 80¢ a bushel, charging a commission of 5%. Find the net proceeds.

51. An agent charged \$6.63 for selling 52 bu. of tomatoes @ 85¢ a bushel. Find the rate of his commission. $\times \div$

52. What is the commission at $3\frac{1}{2}\%$ on a sale of potatoes to the amount of \$1270? $\times$

53. A real-estate agent charged \$246.25 for selling a piece of property for \$9850. What was the rate of his commission? $\div$

54. A real-estate agent charged \$312.50 for selling a piece of property for \$12,500. What was the rate of commission? $\div$

55. An agent sold 44 wagons @ \$135 each and paid his principal \$5197.50 as the proceeds of the sale. Find the rate of the agent's commission. $\times - \div$

56. An agent bought for a dealer 240 baskets of peaches @ 75¢ a basket, charging $3\frac{1}{2}\%$ commission. Find the total cost to the dealer. $\times \times +$

57. A commission merchant sold 45 bbl. of apples @ \$2.80 a barrel, deducting 6¢ a barrel for storage and a commission of 5%. Find the net proceeds of the sale. $-$

58. An agent sold some produce for \$935, charging 3% commission. How much did he remit to his principal? $\times -$

59. An agent sold some produce, charging 3% for his services. His commission amounted to \$28.05. How much did he remit to his principal, after deducting his commission? $\div -$

60. An agent sold some produce, retaining his commission of \$3.69 and remitting the proceeds, \$119.31. What was the rate of commission? $+ \div$

61. A commission merchant sold some produce, and, after deducting his commission, remitted to his principal \$906.95. His commission amounted to \$28.05. What was the selling price? What was the rate of his commission? $+ \div$

— 62. What is an architect's commission on a house costing \$9472, at 5%? $\times$

— 63. An architect's commission on a house costing \$8750 is \$437.50. What is the rate? $\div$

— 64. What is an auctioneer's commission for selling \$358 worth of furniture, at 6%? $\times$

65. A real-estate dealer sells 168 acres of land at \$40 an acre. How much is his commission at $2\frac{1}{2}\%$? $\times$

— 66. A lawyer charged \$40 for collecting a debt of \$750. What is the rate? $\div$

— 67. An agent sells our school \$65 worth of maps, and his firm pays him $7\frac{1}{2}\%$ commission. How much is this? $\times$

68. A farmer sends some butter and eggs to the city, and they are sold for \$72.50. He is charged 8% by his agent. What is the net amount received by the farmer? $\times$

69. What is the commission on 87 tubs of butter, each containing 50 lb., if sold at 27¢ a pound, the rate of commission being $3\frac{1}{2}\%$? $\times$

70. An agent sold a house for a man, charging 5% commission. He sent to his employer \$4512.50 as the proceeds. At what price did he sell the house? $\div$

71. A commission merchant sold for a farmer some produce for \$260, and sent to the farmer \$239.20 as the proceeds. What was the rate of commission? $\div$

72. An agent sold some land for a man and received \$9500. He sent to his employer \$9215 as the proceeds. What was the rate of commission? $\div$

73. A salesman's sales in a department store in a city amounted, during the six days of one week, to \$257.70, \$103.10, \$175.02, \$132.30, \$302.25, \$50.63. He receives $2\frac{1}{2}\%$ on his sales. How much did he receive that week? $\times$

74. What is the commission at $7\frac{1}{2}\%$ on a sale of an automobile for \$1250? X

75. What is the commission at $2\frac{1}{2}\%$ for collecting a debt of \$750? X

76. What is the net amount remaining after deducting a commission of 3% from a sale amounting to \$850? X -

77. A broker bought 850 bales of cotton for a customer, charging \$5 per hundred bales as brokerage. What was the brokerage?

78. A broker bought 750 tierces of lard for a customer, charging $2\frac{1}{2}\%$ a tierce. A tierce being 340 lb., how many pounds did he buy, and what was the brokerage?

79. Lard selling at \$7.03 per tierce, how much will 1250 tierces cost? How many pounds are there? What is the brokerage on the purchase at $2\frac{1}{2}\%$ per tierce?

80. A commission merchant sells for a factory 6900 lb. of cheese at 12¢ a pound, charging 5% commission. What sum should he remit to his principal? \ X -

81. A commission merchant sold 1700 doz. eggs for a dealer, charging him 5% on the sale, and remitting \$339.15. At what price per dozen did he sell the eggs? - + +

82. A commission merchant sold 160 tubs of butter, 50 lb. to the tub, for \$1920, charging 5% commission. What did the commission amount to per pound? X X X

83. A commission merchant sold 240 tubs of butter, 50 lb. to the tub, for \$3360. His commission being $1\frac{3}{4}\%$ a pound, what was his rate of commission? X X X

84. A broker buys 1750 bags of coffee for a customer at 10.43¢ per pound. There being 130 lb. to the bag, and the broker charging \$10 per 250 bags, how many pounds are bought and what is the total cost including the brokerage?

85. Given the net proceeds \$438.44, and the commission \$13.56, find the rate of commission. $\div$ $\div$

86. A real-estate agent charged \$446.25 for selling a piece of property for \$12,750. What was the rate of his commission? $\div$

87. The net proceeds of the sale of 244 bbl. of flour is \$1124.23, the commission being 3%. Find the selling price of the flour per barrel. $\div$ $\div$

88. An agent bought for a dealer 360 baskets of peaches @ $82\frac{1}{2}\phi$ a basket, charging $3\frac{3}{4}\%$ commission. Find the total cost to the dealer. $\times$ $\times$ $+$

89. An agent sold 82 horses @ \$112.50 each and paid his principal \$8763.75 as the proceeds of the sale. Find the rate of the agent's commission. $\times$ $-$ $\div$

90. A real-estate agent charged \$562.50 for selling a piece of property. The rate of commission being $4\frac{1}{2}\%$, at what price did he sell the property? $\div$

91. A commission merchant sold 225 bbl. of apples @ \$2.87 a barrel, charging 7¢ a barrel for storage and a commission of 5%. Find the net proceeds of the sale. $\div$ $-$ $-$

92. A commission merchant sold some produce, charging $3\frac{1}{2}\%$ for his services. He remitted to his principal \$1621.20. At what price did he sell the produce? $-$ $\div$

93. A manufacturer sent his agent \$3177.81 to pay for some leather, and the commission of $\frac{1}{2}\%$. How much did the agent invest and how much was his commission?

94. A broker bought 700 bales of cotton for a customer @ 12.1¢ per pound. Taking the bales as weighing 500 lb. each, and the brokerage at \$5 per hundred bales, find the number of pounds purchased and the total cost including the brokerage.

95. On which will the net amount after deducting the commission be greater, a sale of \$980 less 3% commission, or one of \$970 less 2% commission? How much greater? $\times \times -$

96. A clerk receives \$4 a week and 2% commission on goods sold. If he sells \$372 worth of goods in a week, what is his income for the week? $\times +$

97. On which will the net amount after deducting the commission be greater, a sale of \$1500 less 4% commission, or one of \$1600 less 10% commission? If there is any difference, what is it? $\times \times -$

98. A clerk receives \$10 a week and $1\frac{1}{2}\%$ commission on goods sold. If he sells \$492 worth of goods in a week, how much is his income for the week? $\times +$

99. A city grocer sends a man out to his customers' residences every morning to take orders. If the man receives 8% commission on all orders, and if he takes orders to the amount of \$9800 a year, how much does he earn? $\times$

100. An agent sells some property for one man for \$7500 on a commission of 2%, and some for another man for \$6000 on a commission of $2\frac{1}{2}\%$. On which does he receive the greater commission, and how much commission does he receive in all? $\times \times - +$

101. A saleswoman is offered her choice of \$8.50 a week, \$5 a week with 1% on all sales, or 4% on all sales. She takes the last plan, and her sales for 52 weeks average \$265 a week. How much does she gain over each of the other two plans? $\times \times \times + \times - -$

102. In a large dry-goods store in New York a woman is employed to "shop" for out-of-town customers. She receives from the dealers a commission of 5% on all purchases. Her purchases amount to \$422 a month, on the average. How much is her commission during the year? $\times \times$

EXERCISE 144

PROBLEMS WITHOUT NUMBERS

1. If you know the amount paid by an agent for some property, and his rate of commission, how do you find the amount of his commission? *74c*

2. If you know the amount received by a broker for goods sold, and his rate of brokerage, how do you find the net amount remitted to his principal? *61c*

3. If you know the net amount of a sale remitted by an agent to his principal, and the rate of his commission, how do you find the price at which the property was sold? *76c*

4. If you know the net amount left after the commission has been deducted, and the price at which the property was sold, how do you find the rate of commission? *77c*

5. If you know the amount collected by an agent, and the rate of his commission, how do you find the net amount to be remitted by him to his principal? *78c*

6. If you know an agent's rate of commission for collecting some money, and the net amount remitted by him to his principal, how do you find the amount collected? *79c*

7. If you know the price per acre at which an agent sells some land, the number of acres, and the rate, how do you find his commission? *80c*

8. If you know the price per acre at which an agent sells some land, the number of acres, and the commission, how do you find the rate of commission? *81c*

9. If a farmer sends a certain number of barrels of apples to a commission merchant, and these are sold at a certain price per barrel, and the freight per barrel and the rate of commission are known, how do you find the amount due the farmer? *82c*

CHAPTER XVI

INTEREST

246. Interest. Money paid for the use of money is called *interest*.

Interest is computed as a certain per cent of the money used. Thus, if a man borrows \$500 for a year, at 6%, the interest is 6% of \$500, or \$30.

247. Principal. Money loaned on which interest is paid is called the *principal*.

In the example just given, \$500 is the principal.

The sum of the principal and interest is called the *amount*.

248. Rate of Interest. The rate per cent of the principal paid in interest for one year is called the *rate of interest*.

In the example under § 246, the rate of interest is 6%, and the amount is \$500 + \$30, or \$530.

For example, consider the following :

(1) Find the interest on \$840 at 5% for 2 yr.

Since the rate is 5% a year,

$$\text{Interest for 1 yr.} = 5\% \text{ of } \$840 = \$42.$$

$$\text{Interest for 2 yr.} = 2 \times \$42 = \$84.$$

(2) Find the interest on \$750 at $5\frac{1}{2}\%$ for $3\frac{1}{2}$ yr.

Since the rate is $5\frac{1}{2}\%$ a year,

$$\text{Interest for 1 yr.} = 5\frac{1}{2}\% \text{ of } \$750 = \$41.25.$$

$$\text{Interest for } 3\frac{1}{2} \text{ yr.} = 3\frac{1}{2} \times \$41.25 = \$144.38.$$

Therefore, *to find the interest, given the principal, rate, and time, multiply the principal by the rate, and the product by the number that expresses the time in years.*

EXERCISE 145

Given the principal, rate, and time, find the interest :

- | | |
|----------------------|--------------------------------------|
| 1. \$100, 6%, 2 yr. | 19. \$100, 6%, $2\frac{1}{2}$ yr. |
| 2. \$100, 4%, 3 yr. | 20. \$100, 5%, $2\frac{1}{2}$ yr. |
| 3. \$100, 3%, 4 yr. | 21. \$200, 6%, $3\frac{1}{4}$ yr. |
| 4. \$200, 6%, 3 yr. | 22. \$200, 5%, $2\frac{1}{4}$ yr. |
| 5. \$300, 6%, 2 yr. | 23. \$375, 6%, $2\frac{1}{2}$ yr. |
| 6. \$600, 3%, 2 yr. | 24. \$675, 4%, $3\frac{1}{4}$ yr. |
| 7. \$250, 6%, 2 yr. | 25. \$480, 6%, $2\frac{3}{4}$ yr. |
| 8. \$375, 4%, 3 yr. | 26. \$540, 6%, $3\frac{3}{4}$ yr. |
| 9. \$650, 6%, 5 yr. | 27. \$640, 5%, $2\frac{3}{4}$ yr. |
| 10. \$275, 5%, 2 yr. | 28. \$975, 4%, 2 yr. |
| 11. \$350, 4%, 3 yr. | 29. \$975, $5\frac{1}{2}$ %, 2 yr. |
| 12. \$460, 6%, 2 yr. | 30. \$725, $4\frac{1}{2}$ %, 3 yr. |
| 13. \$525, 5%, 3 yr. | 31. \$835, $3\frac{1}{2}$ %, 2 yr. |
| 14. \$265, 4%, 2 yr. | 32. \$425.50, 6%, $2\frac{1}{2}$ yr. |
| 15. \$425, 5%, 3 yr. | 33. \$375.75, 6%, $3\frac{1}{2}$ yr. |
| 16. \$635, 6%, 4 yr. | 34. \$420.50, 5%, $2\frac{1}{4}$ yr. |
| 17. \$875, 6%, 2 yr. | 35. \$2450, 4%, $3\frac{3}{4}$ yr. |
| 18. \$925, 5%, 5 yr. | 36. \$3575, 5%, $2\frac{3}{4}$ yr. |

37. A man borrowed \$250 and agreed to pay 6% interest. He kept the money a year. How much interest did he pay ?

38. A man borrowed \$250 and agreed to pay 6% interest. He kept the money two years. How much interest did he pay ?

39. How much is the interest on \$400 at 6% for 1 yr. ? for 2 yr. ? for $\frac{1}{2}$ yr. ? for $1\frac{1}{2}$ yr. ? for 2 yr. 6 mo. ?

40. How much is the interest on \$600 at 4% for 1 yr. ? for 2 yr. ? for 6 mo. ? for 1 yr. 6 mo. ? for 2 yr. 6 mo. ?

41. What is the interest on \$200 at 6% for 1 yr.? for 1 mo.? for 2 mo.? for 7 mo.?

42. What is the interest on \$600 at 6% for 1 yr.? for 1 mo.? for 3 mo.? for 10 mo.? for 8 mo.?

43. What is the interest on \$500 at 6% for 2 yr.? for $2\frac{5}{12}$ yr.? for 2 yr. 5 mo.? for 2 yr. 7 mo.?

Given the principal, rate, and time, find the interest:

44. \$300, 2 yr. 5 mo., 4%. 54. \$1200, 1 yr. 7 mo., 6%.

45. \$400, 2 yr. 5 mo., 3%. 55. \$1200, 1 yr. 7 mo., 5%.

46. \$600, 1 yr. 6 mo., 6%. 56. \$1200, 2 yr. 2 mo., 6%.

47. \$600, 1 yr. 8 mo., 6%. 57. \$1500, 2 yr. 2 mo., 6%.

48. \$600, 1 yr. 8 mo., 5%. 58. \$1500, 2 yr. 9 mo., 6%.

49. \$500, 1 yr. 9 mo., 6%. 59. \$1500, 2 yr. 9 mo., 5%.

50. \$600, 1 yr. 9 mo., 5%. 60. \$1800, 3 yr. 8 mo., 6%.

51. \$500, 2 yr. 10 mo., 6%. 61. \$1800, 3 yr. 2 mo., 6%.

52. \$800, 3 yr. 5 mo., 6%. 62. \$1800, 3 yr. 2 mo., 5%.

53. \$900, 2 yr. 3 mo., 5%. 63. \$2500, 4 yr. 3 mo., 6%.

64. What is the interest on \$436, at 6%, for 1 yr.? for 6 mo.? for 4 mo.? for 8 mo.? for 3 yr. 10 mo.?

65. What is the interest on \$360, at 5%, for 1 yr.? for 6 mo.? for 10 mo.? for 1 mo.? for 2 yr. 9 mo.?

66. If the interest on a certain sum is \$93.84 for 1 yr., how much is it for 3 mo.? for 6 mo.? for 1 yr. 9 mo.?

67. If the interest on a certain sum is \$36.48 for 1 yr., how much is it for 1 mo.? for 5 mo.? for 3 yr. 10 mo.?

68. What is the interest on \$540, at 4%, for 1 yr.? for 1 mo.? for $\frac{1}{2}$ mo.? for $2\frac{1}{2}$ mo.? for $5\frac{1}{3}$ mo.?

69. What is the interest on \$175.50, at 4%, for 1 yr.? for 6 mo.? for 4 mo.? for 3 mo.? for 1 mo.? for $2\frac{2}{3}$ mo.?

249. Interest when the Time involves Days. If the time involves days, we may reduce it to a fraction of a year and proceed as in § 248.

For example, find the interest on \$480 for 3 yr. 3 mo. 10 da. at 5%.

$$10 \text{ da.} = \frac{10}{30} \text{ mo.} = \frac{1}{3} \text{ mo.}$$

$$3\frac{1}{3} \text{ mo.} = \frac{10}{3} \text{ of } \frac{1}{12} \text{ yr.} = \frac{5}{18} \text{ yr.}$$

$$3\frac{5}{18} \times 5\% \text{ of } \$480 = \frac{59 \times 5 \times \$480}{18 \times 100} = \$78.67.$$

It is the custom in computing interest in this way to take 30 da. to the month, or 360 da. to the year, and to find the interest to the nearest cent.

EXERCISE 146

Find the interest on :

1. \$100, at 6%, for 2 yr. 6 mo.
2. \$100, at 6%, for 2 yr. 6 mo. 10 da.
3. \$100, at 6%, for 3 yr. 4 mo. 15 da.
4. \$200, at 5%, for 3 yr. 4 mo. 20 da.
5. \$300, at 5%, for 3 yr. 8 mo. 20 da.
6. \$275, at 4%, for 1 yr. 9 mo. 15 da.
7. \$375, at 5%, for 2 yr. 9 mo. 15 da.
8. \$450, at 6%, for 3 yr. 10 mo. 6 da.
9. \$525, at 6%, for 4 yr. 10 mo. 6 da.
10. \$675, at 5%, for 2 yr. 3 mo. 10 da.
11. \$575, at 5%, for 3 yr. 3 mo. 3 da.
12. \$825, at 4%, for 2 yr. 3 mo. 3 da.
13. \$1250, at $5\frac{1}{2}\%$, for 2 yr. 6 mo. 15 da.
14. \$1500, at $4\frac{1}{2}\%$, for 3 yr. 6 mo. 20 da.
15. \$2500, at $3\frac{1}{2}\%$, for 2 yr. 4 mo. 15 da.
16. \$3500, at $3\frac{1}{2}\%$, for 3 yr. 7 mo. 10 da.

250. Difference between Dates. The time between two dates may be found as here shown. For example, required to find the time from April 16, 1914, to February 5, 1919.

			Since February is the second month of the year, and April the fourth, the work is arranged as here shown. We
yr.	mo.	da.	then subtract as with other compound numbers;
1919	2	5	5 — 16 is impossible, but $35 - 16 = 19$. Proceeding
1914	4	16	in the usual manner, the result is 4 yr. 9 mo.
4	9	19	19 da. In such cases allow 30 da. to the month.

EXERCISE 147

Find the difference in time between these dates:

1. Sept. 2, 1914, Oct. 15, 1919.
2. Oct. 5, 1914, Jan. 25, 1920.
3. Aug. 7, 1915, June 4, 1921.
4. Nov. 10, 1916, Jan. 5, 1920.
5. Aug. 23, 1915, Jan. 9, 1920.
6. What is the interest on \$375 from July 4, 1914, to July 4, 1917, at 6%?
7. What is the interest on \$650 from July 20, 1914, to May 20, 1917, at 5%?
8. What is the interest on \$825 from May 12, 1915, to Feb. 12, 1919, at $4\frac{1}{2}\%$?
9. What is the interest on \$360 from June 8, 1916, to April 6, 1918, at 6%?
10. What is the interest on \$480 from March 7, 1916, to April 9, 1918, at 5%?
11. What is the interest on \$750 from Nov. 8, 1916, to Feb. 22, 1918, at 5%?
12. What is the amount of \$650 at interest at 6% from Sept. 27, 1915, to March 12, 1918?

251. Promissory Notes. A written promise to pay a specified sum of money on demand or at a specified time is called a *promissory note*, or simply a *note*.

252. Parties to a Note. The person who signs a note is called the *maker*. The person to whom a note is payable is called the *payee*. The lawful owner of a note is called the *holder*.

A note may be made payable to *bearer*, in which case it is lawful to pay it to any one who presents it to the maker.

It may be made payable to some person named in the note, in which case he or his legal representatives must present it. Such a note is said to be *non-negotiable*.

It may be made payable to the payee, *or order*, in which case the payee may transfer it to another person by writing his own name on the back, thus *indorsing* the note. Such a note is said to be, like one payable to bearer, *negotiable*.

An indorser of a note is responsible for its payment if the maker does not pay it, unless he writes the words "without recourse" above his signature.

253. Form of a Note. A common form of note is as follows:

\$375.25.

Albany, N. Y., June 1, 1913

Six months after date, I promise to pay Frank Mills, or order, Three Hundred Seventy-five and $\frac{25}{100}$ Dollars, with interest at 5%, for value received.

James Lang

Here the maker is James Lang, and the payee is Frank Mills.

If the payee writes across the back,

Payable to the order of John B. Brown,

Frank Mills

he becomes an indorser, and John B. Brown becomes the holder.

If Frank Mills writes simply his name across the back, the note becomes payable to bearer, that is, to any one who may have it.

This note is due on Dec. 1, 1913, which is called the *day of maturity*.

The words "with interest," without specifying the rate, carry the legal rate of the state in which the note is made.

EXERCISE 148

Find the day of maturity and amount due, having given :

<i>Face of Note</i>	<i>Date of Note</i>	<i>Time</i>	<i>Rate of Interest</i>
1. \$300	July 7, 1913	60 da.	6%
2. \$500	Jan. 4, 1913	6 mo.	5%
3. \$400	Feb. 8, 1913	1 yr.	4½%
4. \$751.50	Mar. 7, 1913	3 mo.	6%
5. \$525.25	Apr. 15, 1914	90 da.	6%
6. \$255.50	May 16, 1914	30 da.	5%

Write notes for the following, and compute the interest :

7. \$400	June 6, 1914	6 mo.	6%
8. \$600	Nov. 9, 1914	4 mo.	5%
9. \$750	Jan. 7, 1913	3 mo.	6%

Find the amount due at maturity on the following notes :

10. \$200. ROCHESTER, N.Y., Oct. 20, 1914

Sixty days after date, I promise to pay George A. Wood, or order, Two Hundred Dollars, with interest at 5%. Value received.

JAMES SINCLAIR

11. \$1200. UTICA, N.Y., Mar. 7, 1914

Ninety days after date, I promise to pay James Goodwin, or order, Twelve Hundred Dollars, with interest at 6%. Value received.

JAMES HILTON

12. \$375.50. ELMIRA, N.Y., Apr. 6, 1914

One year after date, I promise to pay Francis Warren, or order, Three Hundred Seventy-five and $\frac{50}{100}$ Dollars, with interest at 5%. Value received.

ALFRED GIBSON

13. F. H. Holmes borrows \$350, at 6%, for 1 yr., from A. B. Seel. He gives a note payable to Mr. Seel or order. Mr. Seel sells the note to B. C. Dean. Make out the note, indorse it, and find the amount due at the end of the year.

Make out and indorse, payable to the order of the buyer, the following notes, and find the amount due at the end of the time stated:

<i>Maker</i>	<i>Payee</i>	<i>Buyer</i>	<i>Face</i>	<i>Rate</i>	<i>Time</i>
14. J. R. Roberts	A. N. Cole	P. R. Jones	\$375	6%	6 mo.
15. M. L. King	A. R. Owen	E. F. Pease	\$750	5%	2 yr.
16. A. K. Jewett	S. M. Wood	E. L. Brown	\$75	6%	10 mo.
17. B. S. Stevens	O. N. House	B. R. Lodge	\$250	5%	3 yr. 6 mo.
18. O. M. Richards	D. E. Dodge	C. King	\$125	6%	1 yr. 2 mo.
19. A. R. Castle	A. J. Jackson	G. F. Ray	\$250	5½%	2 yr. 4 mo.
20. F. E. Taylor	A. B. Baker	G. F. Coe	\$350	4½%	1 yr. 3 mo.
21. F. E. Glover	M. T. Thorn	F. Taylor	\$250	6%	4 yr. 1 mo.

22. A man borrowed \$250 on Jan. 15, for 9 mo., at 6%. Write the note and compute the amount of the principal and interest due on Oct. 15.

23. A man borrowed \$350 on Jan. 27, for 1 yr. 1 mo., at 6%. Write the note and compute the amount of the principal and interest due on Feb. 27 of the following year.

24. A man bought 12 head of cattle on Feb. 1, at \$45 a head, giving his promissory note at 6%. He paid it on the 16th of the following April. What was the amount of principal and interest? *2½ mo.*

25. A man bought 75 acres of land on March 3, at \$85 an acre, giving his promissory note at 5½%. He paid it on the 18th of the following September. What was the amount of principal and interest? *6½ mo.*

254. Problems in Interest. Of the four elements used in computing interest,—principal, rate, time, and interest (or amount, which is only the sum of the principal and interest),—we can find any one if the other three are known.

For, since the interest is the product of three factors (principal, rate, and time), if it is divided by the product of any two of these factors the quotient is the third.

The following examples will illustrate:

(1) What principal will, in 3 yr. 6 mo., yield \$52.50 interest at 6%?

Since the interest for $3\frac{1}{2}$ yr. is \$52.50,
therefore the interest for 1 yr. is $\$52.50 \div 3\frac{1}{2}$, or \$15.

Since 6% of the principal is \$15,
therefore 1% of the principal is \$2.50,
and 100% of the principal is \$250.

(2) What principal will amount to \$570 in 2 yr. 4 mo. at 6%?

The amount of \$1 for 2 yr. 4 mo. at 6% is \$1.14.

Since \$1.14 is the amount of \$1,
\$570 is the amount of $\frac{\$570}{1.14}$, or \$500.

(3) In what time will \$540 yield \$109.80 interest at 6%?

The interest on \$540 at 6% for 1 yr. is \$32.40.

Since \$32.40 is the interest for 1 yr.,
\$109.80 is the interest for $\frac{109.80}{32.40}$ yr., or $3\frac{7}{8}$ yr.

$3\frac{7}{8}$ yr. = 3 yr. $4\frac{1}{2}$ mo. = 3 yr. 4 mo. 20 da.

(4) At what rate will \$600 yield \$81 interest in 3 yr.?

The interest on \$600 at 1% for 3 yr. is \$18.

Since \$18 is the interest at 1%,
\$81 is the interest at $\frac{\$81}{\$18}$ %, or $4\frac{1}{2}$ %.

EXERCISE 149

Find the principal that will yield the following interest :

1. \$20, in 2 yr. 8 mo., at 6%.
2. \$93 $\frac{1}{3}$, in 3 yr. 4 mo., at 5%.
3. \$9.59, in 3 yr. 6 mo., at 4%.
4. \$40.47, in 1 yr. 7 mo., at 6%.
5. \$3.64, in 5 mo. 18 da., at 6%.
6. \$45.90, in 4 yr. 3 mo., at 4 $\frac{1}{2}$ %.

Find the principal that will amount to the following :

7. \$482.40, in 3 yr. 4 mo., at 6%.
8. \$1151.41, in 2 yr. 4 mo., at 4 $\frac{1}{2}$ %.
9. \$1210.50, in 2 yr. 5 mo., at 5%.

Find the time, given the principal, rate, and interest :

- | | |
|-------------------------|---------------------------|
| 10. \$840, 5%, \$147. | 12. \$350, 6%, \$49. |
| 11. \$625, 4%, \$56.25. | 13. \$1250, 6%, \$187.50. |

Find the rate, given the principal, time, and interest :

14. \$330, 1 yr. 2 mo., \$19.25.
15. \$68.50, 3 yr. 6 mo., \$9.59.
16. \$87.50, 2 yr. 8 mo. 24 da., \$14.35.
17. How long will it take \$1 to yield \$1 interest at 6% ?
18. How long will it take \$1 to double itself at 5% ?
19. What sum must be put at interest at 6% so that it shall amount to \$1000 in 16 $\frac{2}{3}$ yr. ?
20. What sum must be put at interest at 5% so that it shall amount to \$1000 in 20 yr. ?

255. Formulas in Interest. We may express the principles involved in the problems on page 285 by using initial letters to represent the elements. In this way we have convenient *formulas*.

Using the initial letters, and letting the time be expressed in years, we have

Interest = principal $\times$ rate $\times$ time (§ 248), or $I = P \times R \times T$;
therefore

$$\text{principal} = \frac{\text{interest}}{\text{rate} \times \text{time}}, \quad \text{or } P = \frac{I}{R \times T};$$

$$\text{rate} = \frac{\text{interest}}{\text{principal} \times \text{time}}, \quad \text{or } R = \frac{I}{P \times T};$$

$$\text{time} = \frac{\text{interest}}{\text{principal} \times \text{rate}}, \quad \text{or } T = \frac{I}{P \times R}.$$

Thus, if the interest on \$200 for 3 yr. at 6% is \$36, we have

$$I = 3 \times 6\% \text{ of } \$200 = \$36,$$

$$P = \frac{\$36}{3 \times 6\%} = \$200,$$

$$R = \frac{\$36}{3 \times \$200} = 6\%,$$

$$T = \frac{\$36}{6\% \text{ of } \$200} = 3.$$

Instead of using capital letters for the initials, we may use small letters if we wish. We shall then have :

$$i = p \times r \times t, \quad p = \frac{i}{r \times t}, \quad r = \frac{i}{p \times t}, \quad t = \frac{i}{p \times r}.$$

The simple equation and the use of the unknown quantity x may profitably be used in the solution of some problems, thus: \$36 is the interest on \$200 for how many years at 6%?

Since $\$36 = x \times 6\% \text{ of } \$200$, we may divide these equals by 6% of \$200, and we have

$$x = \frac{\$36}{6\% \text{ of } \$200} = 3.$$

The same principle is involved as in the preceding solution in which T is found.

EXERCISE 150

1. Given $r = 6\%$, $i = \$90$, and $t = 3$, find p .
2. Given $i = \$64$, $p = \$800$, and $t = 2$, find r .
3. Given $i = \$16$, $p = \$200$, and $r = 4\%$, find t .

Find the missing element, marked x :

p	r	t	i	p	r	t	i
4. \$300	x	4 yr.	\$60	16. \$375	4%	4 yr.	x
5. x	4%	4 yr.	\$64	17. x	5%	2 yr.	\$70
6. \$700	3%	x	\$42	18. \$600	x	3 yr.	\$90
7. \$650	5%	2 yr.	x	19. \$700	6%	2 yr.	x
8. x	5%	3 yr.	\$30	20. \$600	3%	x	\$54
9. \$300	x	3 yr.	\$36	21. x	5%	3 yr.	\$30
10. \$300	3%	x	\$36	22. \$700	x	2 yr.	\$70
11. \$700	5%	3 yr.	x	23. \$600	5%	4 yr.	x
12. x	6%	5 yr.	\$60	24. \$500	4%	x	\$80
13. \$400	x	2 yr.	\$48	25. x	3%	3 yr.	\$36
14. \$500	5%	x	\$75	26. \$250	x	2 yr.	\$20
15. \$800	4%	2 yr.	x	27. \$900	6%	x	\$162

28. A note for \$275 at 6% runs for 3 yr. without any payments being made. What is the amount then due?

29. A note for \$300 at 6% amounts to \$336. How long has the note run?

30. A note for \$400 amounts in 3 yr. to \$460. What is the rate of interest?

31. A note amounts to \$224 in 2 yr. The face of the note is \$200. What is the rate of interest?

32. Write a note for \$275 due 2 yr. 6 mo. after this date, at 6%, and find the amount due at maturity.

EXERCISE 151

REVIEW OF INTEREST

Given the principal, rate, and time, find the interest:

1. \$250, 5%, 4 mo. 10 da.; 8 mo. 20 da.
2. \$520, 6%, 2 yr. 9 mo.; 2 yr. 9 mo. 25 da.
3. \$760, 5%, 4 yr. 6 mo.; 4 yr. 6 mo. 15 da.
4. \$875, 6%, 3 yr. 7 mo.; 3 yr. 7 mo. 16 da.
5. \$1250, 5%, 1 yr. 10 mo.; 1 yr. 10 mo. 5 da.
6. \$2500, $5\frac{1}{2}\%$, 2 yr. 10 mo.; 2 yr. 10 mo. 7 da.
7. \$2275, $5\frac{1}{2}\%$, 3 yr. 11 mo.; 3 yr. 11 mo. 2 da.
8. \$3525, $4\frac{1}{2}\%$, 1 yr. 11 mo.; 1 yr. 11 mo. 3 da.

Given the principal, rate, and time, find the amount:

9. \$175, 6%, 6 mo.; 8 mo.; 1 yr. 2 mo.
10. \$225, 6%, 8 mo. 3 da.; 1 yr. 8 mo. 3 da.
11. \$625, 5%, 4 mo. 3 da.; 2 yr. 4 mo. 3 da.
12. \$1250, 5%, 3 mo. 8 da.; 2 yr. 3 mo. 8 da.
13. \$2275, $5\frac{1}{2}\%$, 10 mo. 14 da.; 3 yr. 10 mo. 14 da.
14. Find the interest on \$275 for 3 yr. 6 mo. at 5%.
15. Find the interest on \$1250 for 7 yr. 11 mo. at 6%.
16. Find the interest on \$630 for 2 yr. 4 mo. 5 da. at 6%.
17. Find the interest on \$345 for 2 yr. 3 mo. 3 da. at 5%.
18. Find the interest on \$290 for 1 yr. 6 mo. 5 da. at 5%.
19. Find the interest on \$350 for 1 yr. 9 mo. 15 da. at 4%.
20. Find the interest on \$650 for 2 yr. 8 mo. 10 da. at 6%.
21. Find the interest on \$1575 for 3 yr. 1 mo. 6 da. at $5\frac{1}{2}\%$; at $4\frac{1}{2}\%$; at 6%.
22. What is the interest on \$375 from July 4, 1915, to May 1, 1918, at 6%?

Given the principal, interest, and time, find the rate :

- | | |
|--------------------------------|---------------------------------|
| 23. \$275, \$27.50, 2 yr. | 30. \$250.50, \$30.06, 2 yr. |
| 24. \$420, \$50.40, 3 yr. | 31. \$480, \$36, 1 yr. 6 mo. |
| 25. \$121.25, \$9.70, 2 yr. | 32. \$240, \$24, 2 yr. 6 mo. |
| 26. \$320.50, \$57.69, 3 yr. | 33. \$325, \$45.50, 3 yr. 6 mo. |
| 27. \$275.25, \$55.05, 5 yr. | 34. \$240, \$10.50, 1 yr. 3 mo. |
| 28. \$3500, \$87.50, 10 mo. | 35. \$160, \$19.80, 2 yr. 3 mo. |
| 29. \$220, \$9.35, 1 yr. 5 mo. | 36. \$480, \$26.60, 1 yr. 7 mo. |

Given the principal, interest, and rate, find the time :

- | | |
|--|---|
| 37. \$240, \$36, 6%. | 42. \$280, \$28, 4%. |
| 38. \$3000, \$155, 2%. | 43. \$350, \$17.50, 3%. |
| 39. \$175, \$12.25, 4%. | 44. \$525, \$47.25, 6%. |
| 40. \$600, \$42.75, 5%. | 45. \$230, \$20.70, $4\frac{1}{2}\%$. |
| 41. \$360, \$19.91, $5\frac{1}{2}\%$. | 46. \$2500, \$37.50, $4\frac{1}{2}\%$. |

47. How long will it take a sum to double itself at 4% ?

48. How long will it take \$220, together with the interest, to amount to \$253, at 6% ?

49. How long will it take \$1750, together with the interest, to amount to \$1968.75, at 5% ?

50. For how long must \$1250 be invested at 5% so that it shall amount, together with the interest, to \$1281.25 ? to \$1385 ? to \$1416.25 ?

51. A girl had \$250 invested for her, at 6%, on her birthday. When she became 21 it amounted, with interest, to \$400. How old was she when it was invested ?

52. A father gave his son his promissory note for \$225, due when the son became 21 years old. The rate of interest was 5%, and when the note became due the principal and interest together amounted to \$303.75. How old was the son when the note was given ?

53. Which yields the better income, \$1675 at 5% or \$1375 at 6%? What is the difference for 2 yr. 6 mo.?

54. A man receives \$2187.50 interest in 2 yr. 6 mo. on an investment of \$17,500. What is the rate of interest?

55. A man has \$250 in a savings bank at 3%. He leaves the principal, but draws out the interest as it is due. How much does he draw out in four years?

56. A man having \$13,500 invested at 5% reinvests \$4000 of it at $5\frac{1}{2}\%$, \$5000 of it at $3\frac{1}{2}\%$, and leaves the rest as before. What is the annual difference in income?

57. A boy has \$300 given to him the day he is 14 years old. His father invests it for him in a 5% promissory note, due on the day he is 21 years old. What is the amount on the day the note is due?

58. A real estate dealer buys 360 acres of farm land at \$30 an acre, and after keeping it 7 months sells it at an advance of \$3.75 an acre. The money being worth to him 5% a year, how much does he gain by the transaction?

59. A man having \$26,750 invested in business has found that his annual profits average 18% a year. He is offered \$35,000 for the business, which he can invest at $4\frac{1}{4}\%$. If he sells out and retires, what will be his loss in income?

60. On April 10 a coal dealer borrowed \$33,250 at 5%, with which he purchased his summer's supply of coal at \$4.75 a ton. He sold the coal at \$5.65 a ton, the buyers paying for unloading and delivery, and paid his debt on November 16. How much did he gain?

61. On October 15 a dealer purchased \$1750 worth of goods for the holiday trade on 30 days' credit. At the end of that time he gave his note for 2 months at 6% a year. He sold the goods for \$2105, and paid the note when due. What per cent did he make on the original purchase price?

EXERCISE 152

PROBLEMS WITHOUT NUMBERS

1. Given the principal, rate, and time, how do you find the interest?
2. Given the principal, rate, and interest, how do you find the time?
3. Given the principal, time, and interest, how do you find the rate?
4. Given the rate, time, and interest, how do you find the principal?
5. How do you find the amount of principal and interest?
6. Given the principal, rate, and time, how do you find the amount of principal and interest?
7. Given the amount of principal and interest, the time, and the principal, how do you find the rate?
8. Given the amount of principal and interest, the rate, and the principal, how do you find the time?
9. How do you find the day of maturity of a promissory note?
10. Given the date of a note, the day of maturity, the principal, and rate, how do you find the interest?
11. How do you find how long it will take any sum of money to double itself at a given rate of interest?
12. How do you find the rate of interest at which a given sum of money will double itself in a given time?
13. Given the face of a note, and its amount in a given number of years, how do you find the rate of interest?
14. Given the amount of a note for a given number of years, and the rate of interest, how do you find the face of the note?

256. Six Per Cent Method. There is a very convenient method of computing the interest at 6%, depending upon the fact that if the rate for 1 yr. is 6%, the rate for 2 mo. is $\frac{1}{6}$ of 6%, or 1%.

Required the interest on \$720 for 5 mo. 17 da. at 6%.

The interest for 2 mo. is 1% of \$720 =	\$7.20
The interest for 2 mo. more =	7.20
The interest for 1 mo. is $\frac{1}{2}$ of \$7.20 =	3.60
The interest for 15 da. is $\frac{1}{2}$ of \$3.60 =	1.80
The interest for 1 da. is $\frac{1}{15}$ of \$1.80 =	.12
The interest for 1 da. more =	.12
The interest for 5 mo. 17 da. =	\$20.04

Therefore, to find the interest for 60 da., move the decimal point two places to the left; for 6 da., three places to the left. From these results find the interest for the required time.

In most cases in which months and days are involved, this is the simplest of all methods. Thus, to find the interest on \$720 for 10 mo. 10 da. at 6%, we have \$7.20 for 2 mo., $5 \times \$7.20$, or \$36, for 10 mo., $\frac{1}{6}$ of \$7.20, or \$1.20, for 10 da. Hence the total interest is \$37.20.

Of all the forms of the six per cent method this is the most easily applied. It is particularly convenient because most bank notes run for 30, 60, or 90 days.

EXERCISE 153

Find the interest at 6% on the following:

- | | |
|------------------------|---------------------------|
| 1. \$240, 4 mo. 6 da. | 9. \$1275, 2 mo. 6 da. |
| 2. \$360, 3 mo. 3 da. | 10. \$3200, 3 mo. 7 da. |
| 3. \$750, 3 mo. 3 da. | 11. \$4500, 4 mo. 9 da. |
| 4. \$475, 3 mo. 6 da. | 12. \$3500, 2 mo. 15 da. |
| 5. \$280, 8 mo. 6 da. | 13. \$2700, 2 mo. 19 da. |
| 6. \$520, 5 mo. 12 da. | 14. \$3350, 8 mo. 17 da. |
| 7. \$450, 2 mo. 15 da. | 15. \$4950, 10 mo. 10 da. |
| 8. \$585, 2 mo. 20 da. | 16. \$6250, 10 mo. 18 da. |

257. Oral Drill. Many easily solved problems should be given for oral work. For example, what is the interest on \$400, at 6%, for 60 da. ? for 30 da. ? for 90 da. ?

Problems of this kind are suggested in Exercise 154, but many more should be given orally by the teacher. They can also easily and advantageously be made up by the pupils, and given rapidly by one pupil to another. Such work should continue throughout the course.

258. Discussion of Solutions. There is an advantage in occasionally discussing a solution without the actual operations being performed. For example, how will you proceed to find the interest on \$1200 for 93 da. at 6% ?

Problems of this kind are suggested in Exercise 154, and similar ones are easily invented as needed. They may profitably be suggested in all subsequent work. The equation and the unknown quantity (page 249) should be used wherever convenient.

259. Approximate Results. As already suggested, there is great value in estimating the result in advance. If the computation leads to a result that is materially different from the one estimated, the work should be carefully examined.

For example, if we have to find the interest on \$490 for 2 yr. 6 mo. at $5\frac{1}{2}\%$, we see that the interest on \$500 for 1 yr. at 5% is \$25, and for $2\frac{1}{2}$ yr. it is \$67.50. This is, therefore, a fair approximation, the correct result being \$67.37 $\frac{1}{2}$. Approximations of this kind should be made in all topics of arithmetic.

EXERCISE 154

State at sight the interest at 6% on the following :

- | | | |
|-------------------|---------------------|--------------------|
| 1. \$500, 60 da. | 7. \$500, 90 da. | 13. \$3000, 60 da. |
| 2. \$600, 30 da. | 8. \$700, 90 da. | 14. \$3000, 30 da. |
| 3. \$800, 90 da. | 9. \$1200, 60 da. | 15. \$3000, 90 da. |
| 4. \$900, 120 da. | 10. \$2000, 30 da. | 16. \$7500, 60 da. |
| 5. \$440, 30 da. | 11. \$4000, 90 da. | 17. \$7500, 30 da. |
| 6. \$860, 30 da. | 12. \$2500, 120 da. | 18. \$7500, 90 da. |

State orally, without solving, how you would proceed to find the interest on the following, at 6% :

- | | |
|--------------------|------------------------|
| 19. \$175, 30 da. | 27. \$275.50, 30 da. |
| 20. \$275, 33 da. | 28. \$326.75, 40 da. |
| 21. \$325, 40 da. | 29. \$421.50, 45 da. |
| 22. \$350, 63 da. | 30. \$627.75, 60 da. |
| 23. \$485, 75 da. | 31. \$421.50, 63 da. |
| 24. \$560, 90 da. | 32. \$2575.25, 90 da. |
| 25. \$675, 93 da. | 33. \$3250.50, 93 da. |
| 26. \$1275, 93 da. | 34. \$4275.40, 120 da. |

State approximately, without completely solving, the interest at 6% on the following :

- | | |
|-------------------|-----------------------|
| 35. \$210, 75 da. | 41. \$1275, 65 da. |
| 36. \$375, 27 da. | 42. \$3250, 70 da. |
| 37. \$450, 35 da. | 43. \$4250, 75 da. |
| 38. \$525, 45 da. | 44. \$517.25, 90 da. |
| 39. \$680, 50 da. | 45. \$635.50, 93 da. |
| 40. \$725, 66 da. | 46. \$40,000, 125 da. |

47. Not using the 6% method, state approximately, without solving, the interest on \$810 for 3 yr. at 5%.

In the same way, state approximately the interest on the following :

- | | |
|-------------------------------------|-------------------------------------|
| 48. \$790, 2 yr., 5%. | 54. \$1025, $2\frac{1}{2}$ yr., 4%. |
| 49. \$795, $2\frac{1}{2}$ yr., 5%. | 55. \$2100, $3\frac{1}{2}$ yr., 5%. |
| 50. \$810, 3 yr., $5\frac{1}{2}$ %. | 56. \$4030, $4\frac{1}{2}$ yr., 5%. |
| 51. \$605, 2 yr., $5\frac{1}{2}$ %. | 57. \$507.25, 2 yr., 5%. |
| 52. \$710, $2\frac{1}{2}$ yr., 5%. | 58. \$796.75, 3 yr., 5%. |
| 53. \$850, 3 yr., 4%. | 59. \$901.96, 2 yr., 6%. |

260. Six Per Cent Method for Other Rates. We may use this method to find interest at the other rates in common use.

Thus, since 5 is $\frac{1}{6}$ less than 6, the interest at 5% is $\frac{1}{6}$ less than the interest at 6%.

Similarly, the interest at $4\frac{1}{2}\%$ is $\frac{1}{4}$ less than the interest at 6%, the interest at 4% is $\frac{1}{3}$ less than the interest at 6%, and so on.

Find the interest on \$1240 for 3 mo. 3 da. at 5%.

Interest for 2 mo. at 6%	\$12.40
Interest for 1 mo. at 6%	6.20
Interest for 3 da. ($\frac{1}{10}$ mo.) at 6%	.62
Total interest at 6%	\$19.22
Deduct $\frac{1}{6}$	3.20
Interest at 5%	\$16.02

EXERCISE 155

Find the interest at 5% on the following:

- | | |
|------------------------|-------------------------|
| 1. \$780, 3 mo. 6 da. | 4. \$1200, 2 mo. 3 da. |
| 2. \$860, 2 mo. 12 da. | 5. \$1500, 3 mo. 3 da. |
| 3. \$940, 1 mo. 18 da. | 6. \$2200, 4 mo. 15 da. |

Find the interest at 4% on the following:

- | | |
|------------------------|--------------------------|
| 7. \$250, 2 mo. | 10. \$1250, 4 mo. |
| 8. \$320, 2 mo. 6 da. | 11. \$1575, 3 mo. 5 da. |
| 9. \$420, 3 mo. 12 da. | 12. \$2500, 2 mo. 12 da. |

Find the interest at $5\frac{1}{2}\%$ on the following:

- | | |
|------------------------|-------------------------|
| 13. \$275, 2 mo. | 15. \$1575, 3 mo. |
| 14. \$575, 4 mo. 3 da. | 16. \$2550, 3 mo. 8 da. |

Find the interest at $4\frac{1}{2}\%$ on the following:

- | | |
|-------------------------|-------------------------|
| 17. \$150, 2 mo. | 19. \$1650, 3 mo. |
| 18. \$975, 3 mo. 10 da. | 20. \$1850, 4 mo. 5 da. |

261. Table of Difference of Time. Bankers always find the difference in time between two dates, and also the interest on any sum, by means of tables. The following is one form of the table of difference of time:

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March . . .	306	337	365	31	61	92	122	153	184	214	245	275
April . . .	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August . . .	153	184	212	243	273	304	334	365	31	61	92	122
September .	122	153	181	212	242	273	303	334	365	30	61	91
October . .	92	123	151	182	212	243	273	304	335	365	31	61
November .	61	92	120	151	181	212	242	273	304	334	365	30
December . .	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from any day of any month to the corresponding day of any month within a year is found opposite the first month and under the second. For example, from August 7 to December 7 is 122 da.; from November 16 to April 23 is 158 da., since to April 16 it is 151 da., and to April 23 it is 7 da. more, or 158 da. in all.

EXERCISE 156

From the table state orally the number of days from :

1. May 7 to Aug. 7.
2. Apr. 6 to Sept. 6.
3. Jan. 5 to Aug. 20.
4. Feb. 7 to Sept. 15.
5. July 4 to Dec. 25.
6. Apr. 15 to Oct. 20.
7. May 20 to Sept. 1.
8. Aug. 15 to Oct. 2.
9. Nov. 16 to July 17.
10. Sept. 20 to Mar. 25.
11. Oct. 21 to Feb. 20.
12. Nov. 25 to Apr. 15.
13. Aug. 27 to Feb. 5.
14. Aug. 19 to Jan. 7.
15. Oct. 7 to Apr. 16.
16. May 13 to Jan. 3.

262. Use of the Table of Time. In computing interest, the table of time (§ 261) is commonly used. We then allow 30 da. to the month and 360 da. to the year.

For example, required the interest on \$420 from July 7 to Nov. 4 at 6%.

The difference of time (§ 261) is 123 da. — 3 da., or 120 da.

Interest at 6% for 60 da. (§ 256) = \$4.20.

Interest at 6% for 120 da. = $2 \times \$4.20 = \$8.40$.

EXERCISE 157

Using the table on p. 297, find the interest at 6% on the following sums from the first date to the second :

- | | |
|------------------------------|------------------------------|
| 1. \$250, Apr. 6, Nov. 6. | 8. \$475, Feb. 4, May 5. |
| 2. \$320, May 4, Nov. 10. | 9. \$240, Mar. 6, Nov. 1. |
| 3. \$220, Jan. 5, Sept. 2. | 10. \$560, Apr. 15, Jan. 10. |
| 4. \$625, June 20, Dec. 17. | 11. \$355, May 1, July 5. |
| 5. \$250, Aug. 16, Feb. 18. | 12. \$750, Aug. 5, Oct. 1. |
| 6. \$375, Sept. 25, Mar. 24. | 13. \$800, Sept. 5, Jan. 2. |
| 7. \$525, Oct. 17, Apr. 4. | 14. \$725, May 7, Oct. 3. |

Also on the following at 5% :

- | | |
|------------------------------|------------------------------|
| 15. \$125, Sept. 5, Feb. 10. | 20. \$430, Oct. 7, Jan. 16. |
| 16. \$326, Oct. 10, Apr. 3. | 21. \$585, Nov. 24, Mar. 7. |
| 17. \$450, July 7, Jan. 20. | 22. \$695, Jan. 16, Aug. 10. |
| 18. \$540, Aug. 15, Nov. 27. | 23. \$843, Apr. 22, Oct. 7. |
| 19. \$315, July 24, Apr. 3. | 24. \$925, Mar. 9, June 15. |

Also on the following at 4% :

- | | |
|------------------------------|-----------------------------|
| 25. \$250, Jan. 1, Sept. 15. | 27. \$840, May 7, Aug. 13. |
| 26. \$375, Mar. 4, Nov. 18. | 28. \$765, Apr. 12, Oct. 4. |

263. Interest Tables. The most common interest tables are based on 360 days to the year, although the government and some banks use tables based on 365 days as being more fair. The following shows part of an interest table :

3 MONTHS, 6%										
Total Days	1000	2000	3000	4000	5000	6000	7000	8000	9000	Days over 3 Months
90	15.00	30.00	45.00	60.00	75.00	90.00	105.00	120.00	135.00	0
91	15.17	30.33	45.50	60.67	75.83	91.00	106.17	121.33	136.50	1
92	15.33	30.67	46.00	61.33	76.67	92.00	107.33	122.67	138.00	2
93	15.50	31.00	46.50	62.00	77.50	93.00	108.50	124.00	139.50	3
94	15.67	31.33	47.00	62.67	78.33	94.00	109.67	125.33	141.00	4
95	15.83	31.67	47.50	63.33	79.17	95.00	110.83	126.67	142.50	5
...	..	..	..	..	..	..	..	..	..	...

For example, find from the table the interest on \$2750 for 94 da. at 6%.

Interest on \$2000 = \$31.33

Interest on 700 = 10.97

Interest on 50 = .78

Interest on \$2750 = \$43.08

Give many oral examples, using \$1000, \$2000, etc.

EXERCISE 158

Using the above table, find the interest at 6% :

- | | | |
|-------------------|-------------------|---------------------|
| 1. \$2670, 91 da. | 4. \$2575, 92 da. | 7. \$5075, 91 da. |
| 2. \$3170, 93 da. | 5. \$3250, 95 da. | 8. \$3775, 93 da. |
| 3. \$4250, 90 da. | 6. \$4050, 92 da. | 9. \$15,750, 90 da. |

Using the tables of § 261, § 263, find the interest at 6% :

10. \$2300, from Mar. 7 to June 7.
11. \$3550, from Nov. 6 to Feb. 9.
12. \$4250, from Dec. 15 to Mar. 18.

264. Exact Interest. When interest is computed for exact days, and at 365 days to the year, it is called *exact interest*.

For example, required the exact interest on \$1241 from Dec. 21 to Feb. 9 at 5%.

The time, found by the table (§ 261), is 50 da. Then $\frac{50}{365} \times 5\%$ of \$1241 is required. Canceling,

$$\frac{50 \times 5 \times \$1241}{365 \times 100} = \$8.50.$$

The common interest is a little greater than the exact interest.

In all such cases always approximate the answer in advance. Occasionally discuss the solutions without computing.

EXERCISE 159

Find the exact interest on the following :

1. \$250 for 75 da. at 6%.
2. \$275 for 85 da. at 5%.
3. \$325 from Apr. 2 to July 1 at 5%.
4. \$350 from May 7 to Sept. 9 at 5%.
5. \$425 from Jan. 6 to Aug. 4 at 6%.
6. \$460 from Feb. 9 to July 9 at 6%.
7. \$500 from May 15 to Dec. 16 at 6%.
8. \$219 from Aug. 14 to Jan. 11 at 5%.
9. \$292 from Sept. 27 to Jan. 25 at 5%.
10. \$438 from July 15 to June 15 at 5%.
11. \$730 from Aug. 17 to Apr. 14 at 4%.
12. \$1460 from May 23 to Mar. 19 at 4%.
13. \$1533 from Feb. 1 to Dec. 28 at $5\frac{1}{2}\%$.
14. \$7300 from Mar. 3 to Oct. 4 at $5\frac{1}{2}\%$.
15. \$3650 from May 23 to Jan. 23 at $4\frac{1}{2}\%$.

265. Partial Payments. Payments of a part of a note are called *partial payments*.

There are several methods of computing interest upon such notes, all recognizing the principle that interest should stop upon the amount paid.

266. Merchants' Rule. This is a convenient rule, although one not generally recognized in law, and is commonly used when settlement is made within a year. It is as follows:

(1) *Find the amount of the note at the date of settlement without regarding payments.*

(2) *Find the amount of each payment with interest from the date of payment to the date of settlement.*

(3) *Subtract the sum of the payment amounts from the total amount.*

For example, a man holds a note for \$380, dated Aug. 16, 1913, on which the following payments are indorsed: Nov. 4, 1913, \$120; Mar. 18, 1914, \$70; May 1, 1914, \$145. Settlement is made June 25, 1914. Find the balance due, interest at 6%.

Time from Aug. 16 to June 25 is 10 mo. 9 da. Int. on \$1 = \$0.0515.

Time from Nov. 4 to June 25 is 7 mo. 21 da. Int. on \$1 = \$0.0385.

Time from Mar. 18 to June 25 is 3 mo. 7 da. Int. on \$1 = \$0.016 $\frac{1}{2}$.

Time from May 1 to June 25 is 1 mo. 24 da. Int. on \$1 = \$0.009.

Int. on \$380 = $380 \times \$0.0515 = \$19.57$. Amount is \$399.57

Int. on \$120 = $120 \times \$0.0385 = \$4.62$. Amount is \$124.62

Int. on \$70 = $70 \times \$0.016\frac{1}{2} = \$1.13$. Amount is 71.13

Int. on \$145 = $145 \times \$0.009 = \$1.31$. Amount is 146.31

Total payment amounts 342.06

Balance due June 25, 1914 \$57.51

The Merchants' Rule is used by bankers and business men on notes running only a few months, or what are called *short-time notes*.

In the following examples find the difference in time in each case by subtraction of compound numbers, as above, and solve by the Merchants' Rule.

EXERCISE 160

Find the balance due :

1. Note for \$230 given May 9, 1914, bearing interest at 6%. Payments: July 1, 1914, \$40; Sept. 16, 1914, \$75; Jan. 2, 1915, \$50. Settlement Feb. 24, 1915.

2. Note for \$375 given Mar. 3, 1914, bearing interest at 5%. Payments: May 8, 1914, \$30; Aug. 16, 1914, \$65; Nov. 20, 1914, \$110. Settlement Jan. 1, 1915.

3. Note for \$600 given May 16, 1915, bearing interest at 5%. Payments: June 1, \$50; July 3, \$25; July 10, \$50. Settlement Nov. 12, 1915.

4. Note for \$1000 given July 18, 1914, bearing interest at 6%. Payments: Sept. 10, \$200; Dec. 20, \$140; April 21, 1915, \$350. Settlement June 2, 1915.

5. Note for \$2760 given Oct. 1, 1914, bearing interest at 5%. Payments: Nov. 1, \$250; Jan. 1, 1915, \$300; Mar. 1, 1915, \$750. Settlement July 16, 1915.

6. Note for \$1000 given Aug. 4, 1916, bearing interest at 6%. Payments: Sept. 10, \$200; Dec. 8, \$200; Feb. 2, 1917, \$250. Settlement May 3, 1917.

7. Note for \$1500 given April 5, 1917, bearing interest at 5%. Payments: June 1, \$500; Aug. 2, \$300; Dec. 1, \$200. Settlement Jan. 4, 1918.

8. Note for \$1250 given July 5, 1918, bearing interest at $5\frac{1}{2}\%$. Payments: Sept. 1, \$275; Dec. 14, \$325; Feb. 8, 1919, \$280. Settlement May 17, 1919.

9. Note for \$80 given Nov. 1, 1919, bearing interest at 6%. Payments of \$10 were made on the first day of each month following. How much interest must be paid with the last payment?

267. United States Rule. The legal rule for partial payments in most of our states is that laid down by the Supreme Court of the United States. It is as follows:

(1) *Find the amount of the principal to the time when the payment, or sum of the payments, is equal to or greater than the interest.*

(2) *From this amount deduct the payment, or sum of the payments.*

(3) *Consider the remainder as a new principal, and proceed as before.*

For example, consider the following promissory note on which partial payments have been made:

\$760. OGDENSBURG, N.Y., Oct. 16, 1914

Three months after date, I promise to pay Henry W. Street, or order, Seven Hundred Sixty Dollars, with interest at 6%. Value received. JAMES JOHNSON

Payments indorsed on the back of the note: Jan. 10, 1915, \$200; Aug. 4, 1917, \$75; May 1, 1918, \$568. What is due Oct. 7, 1919?

Face of note	\$760.00
Int., Oct. 16, 1914, to Jan. 10, 1915, 2 mo. 24 da.	10.64
Amount due Jan. 10, 1915	<u>\$770.64</u>
Deduct payment	200.00
New principal	<u>\$570.64</u>
Int., Jan. 10, 1915, to Aug. 4, 1917, 2 yr. 6 mo. 24 da., is \$87.88, exceeding the payment.	
Int. Jan. 10, 1915, to May 1, 1918, 3 yr. 3 mo. 21 da.	113.27
Amount due May 1, 1918	<u>\$683.91</u>
Deduct sum of payments, \$75 + \$568	643.00
New principal	<u>\$40.91</u>
Int. May 1, 1918, to Oct. 7, 1919, 1 yr. 5 mo. 6 da.	3.52
Amount due Oct. 7, 1919	<u>\$44.43</u>

EXERCISE 161

1. Note for \$425. Date, Oct. 10, 1914. Interest at 6%. Payments: Mar. 14, 1915, \$100; July 8, 1915, \$85; Feb. 24, 1916, \$125. What was due Oct. 10, 1916?

2. Note for \$360. Date, Jan. 10, 1915. Interest at 6%. Payments: Sept. 16, 1915, \$75; Oct. 8, 1916, \$20; Dec. 2, 1916, \$160. What was due Jan. 10, 1917?

3. Note for \$350. Date, May 1, 1915. Interest at 6%. Payments: Dec. 25, 1915, \$50; June 29, 1916, \$5; Aug. 22, 1916, \$15; June 4, 1917, \$100. What was due Apr. 5, 1918?

4. \$375.

WATKINS, N.Y., May 20, 1914

Six months after date, I promise to pay Oliver Higgins Three Hundred Seventy-five Dollars, with interest at 5%.
Value received.

WILLIAM JULIAN

Indorsements: Nov. 20, 1914, \$25; Apr. 6, 1915, \$75; Aug. 23, 1915, \$140. What was due Mar. 12, 1916?

5. \$250.

PLATTSBURG, N.Y., Aug. 24, 1915

One year after date, I promise to pay, for value received, Ira Hoyt, or order, Two Hundred Fifty Dollars, with interest at 6%.

JAMES PATTERSON

Payments: Aug. 24, 1916, \$100; Nov. 15, 1916, \$40; Feb. 3, 1917, \$80; May 10, 1917, \$25. Paid in full, Apr. 16, 1918. What was the last payment?

6. \$620.

PALMYRA, N.Y., Nov. 1, 1915

Six months after date, I promise to pay J. R. Roberts, or order, Six Hundred Twenty Dollars, with interest at 6%.
Value received.

W. J. CAMPBELL

Indorsements: Oct. 6, 1916, \$61.07; Mar. 4, 1917, \$89.03; Dec. 11, 1917, \$107.77; July 23, 1918, \$200.50. What was due Oct. 15, 1918?

EXERCISE 162

REVIEW OF INTEREST

1. Find the interest on \$725 for 187 da. at 5%.

2. A man has \$14,600 at interest at 6%. What is his average daily income from this investment?

3. Find the interest on \$475 from Aug. 14, 1913, to Feb. 2, 1915, at 4%.

4. Find the interest on \$7550 at 6% for 60 da.; for 30 da.; for 90 da.; for 93 da.

In Exs. 5, 6, 7, find the difference in time by § 250.

5. Find the interest on \$660 from Oct. 14 to the following July 1, at 5%.

6. Find the interest on \$720 from Aug. 17 to the following April 15, at $5\frac{1}{2}\%$.

7. Find the amount of principal and interest on \$1440 from Sept. 20 to the following July 1, at 4%.

In Exs. 8, 9, 10, find the difference in time by the table in § 261.

8. Find the interest on \$960 from Nov. 16 to the following Aug. 27, at 6%.

9. Find the interest on \$7200 from July 20 to the following April 7, at 5%.

10. Find the amount of principal and interest on \$420 from June 7 to the following Dec. 1, at 6%.

11. Find the exact interest at 6% on \$650 from Sept. 21, 1911, to Jan. 1, 1912.

12. Find the exact interest at 5% on \$730 from Oct. 15 to the following Aug. 10.

13. Find the exact interest at 4% on \$9125 from Nov. 1 to the following Aug. 5.

14. Find the amount of principal and exact interest on \$1095 from Aug. 9 to the following June 17, at 6%.

15. The interest at 6% for two years on a certain sum is \$93. What is the sum at interest?

16. The interest at 5% for 1 yr. 6 mo. on a certain sum is \$18. What is the sum at interest?

17. The interest at 4% for 2 yr. 3 mo. on a certain sum is \$31.50. What is the sum at interest?

18. What is the sum that will amount, with interest at 6%, to \$517.50 in 2 yr. 6 mo.?

19. What is the sum that will amount, with interest at 5%, to \$793.75 in 1 yr. 2 mo.?

20. What is the sum that will amount, with interest at $5\frac{1}{2}\%$, to \$1006.32 in 3 yr. 7 mo. 6 da.?

21. At $3\frac{1}{2}\%$ interest for a certain time, \$240 amounts to \$281.44. Find the time.

22. At $4\frac{1}{2}\%$ interest for a certain time, \$850 amounts to \$977.50. Find the time.

23. At $5\frac{1}{2}\%$ interest for a certain time, \$940 amounts to \$1022.72. Find the time.

24. At $3\frac{1}{2}\%$ interest for a certain time, \$480 amounts to \$610.90. Find the time.

25. How long will it take \$1200 to earn \$370.15 interest at $5\frac{1}{2}\%$?

26. At what rate will \$1500 yield \$84 interest in 1 yr. 10 mo. 12 da.?

27. At what rate will \$5400 yield \$344.25 interest in 1 yr. 5 mo.?

28. At what rate will \$4200 yield \$784 interest in 3 yr. 1 mo. 10 da.?

268. Bank. A corporation authorized by law to receive and loan money and to perform certain other financial operations is called a *bank*.

269. Commercial Bank. Most banks receive money on deposit and pay it out on written orders, allowing no interest. Such a bank is called a *commercial bank*.

They are also known as *banks of deposit*. They afford a safe place for keeping money. They make their profit by loaning money.

270. Depositing Money. When a reliable person wishes to open an account with a bank he begins by depositing some money. In depositing money a *deposit slip* is filled out each time, as here shown.

SECOND NATIONAL BANK			
BUFFALO, N.Y.			
Deposited for the account of			
<i>J. P. Roberts</i>			
Date <i>Feb. 3</i>		19 <i>15</i>	
Bills		\$75	
Gold		10	
Silver and small coin		20	
Check on <i>1st Nat.</i> Bank		126	75
Check on <i>Chicago Nat.</i> Bank		37	60
Total		\$269	35

The money and deposit slip are then presented to the *receiving teller* or to the *cashier*, together with a bank book in which the teller or cashier enters the amount deposited.

Many banks do not insist on having inserted in a deposit slip the name of the bank on which a check is drawn.

EXERCISE 163

Make out deposit slips for the following deposits, naming some bank in your vicinity :

1. Bills, \$500; gold, \$20; silver, \$12; check on First National Bank, \$90.

Oral problems in summing such items should be given.

2. Bills, \$295; silver, \$27; checks on Farmers Bank, \$275.50; Second National, \$127.62.

3. Bills, \$427; gold, \$25; silver, \$65; checks on Union Trust, \$92.50; Manufacturers Bank, \$326.75.

4. Bills, \$396; gold, \$75; silver, etc., \$73.75; checks, \$72, \$40, \$127.50, \$345.75.

5. Bills, \$426; silver, etc., \$63.30; checks, \$32.75, \$16.50, \$48.90, \$123.45, \$63.50.

6. Bills, \$275; silver, etc., \$50.75; checks, \$27.62, \$48.35, \$62.90, \$91.75, \$126.30.

7. Bills, \$300; silver, etc., \$40; checks, \$37.75, \$52.50, \$46.75, \$75, \$31.90, \$43.

8. Bills, \$250; silver, etc., \$20; checks, \$48, \$70, \$13.62, \$19.83, \$76.75, \$49.32.

9. Bills, \$420; gold, \$75; silver, etc., \$40; checks, \$39.50, \$42.75, \$61.80, \$120.35.

10. Bills, \$37; checks, \$14.75, \$35.20, \$48, \$29.30, \$62.75, \$81.50, \$8.15, \$7.80.

11. Checks, \$48.50, \$16.90, \$17.30, \$21.25, \$48.02, \$37.70, \$42.72, \$30.96, \$48.70, \$9.20.

12. Checks, \$148.62, \$7.90, \$4.08, \$39.70, \$62.80, \$47, \$106.05, \$32.75, \$81.22, \$62.75; \$38.40.

13. Bills, \$428; gold, \$80; silver, etc., \$45.75; checks, \$272.75, \$121.62, \$396.80, \$92.75, \$38.75, \$326.42.

271. Drawing Money. When a person opens an account at a bank he is given a *check book*. Each page has one or more *checks* and *stubs*. On the presentation of a check properly filled out, signed, and indorsed the bank will pay the amount specified, if the drawer's account is sufficient, and the holder is properly identified.

*Stub**Check*

No 575	No. 575 ALBANY, N.Y., Feb. 1, 1915
Date Feb. 1, 1915	FIRST NATIONAL BANK OF ALBANY
To B. H. Roberts	Pay to the order of
For Bal. acct.	B. H. Roberts \$75. ⁷⁵
Amt. \$75. ⁷⁵	Seventy-five and $\frac{75}{100}$ ~~~~~ Dollars
	Rufus J. Perkins

In this case Rufus J. Perkins is the *drawer* and B. H. Roberts is the *payee*. The stub is kept by the drawer as a memorandum.

A check is usually payable to "self", in which case only the drawer can collect it; or to the order of the payee, in which case he must write his name across the back (*indorse* it); or to the payee or bearer, in which case any one can collect it; or to "cash", in which case any one can collect it. Of these forms, the one to the order of the payee, as above shown, is the most common.

In the above case, if B. H. Roberts draws the money he writes simply his name across the back. This is called a *blank indorsement*, or *indorsement in blank*. If he wishes to use it in paying some other person, for instance, Jacob C. Sturgess, he would write:

Pay to the order of

Jacob C. Sturgess

B. H. Roberts

This is called a *full indorsement*, and only Jacob C. Sturgess could collect it, unless he should indorse it in favor of some one else.

EXERCISE 164

1. Make out a check for \$75.60, payable to the order of J. P. Flint. Indorse the check in blank.

2. Make out a check for \$62.75, payable to the order of M. G. Thompson. Write the stub. Indorse the check to the order of R. P. Hull.

3. If a man had a balance of \$273.75 at the beginning of last month, and has since given checks for \$15.63, \$28.96, \$14.60, \$5.75, \$12.30, and \$38.09, and made no deposits, what is his balance now?

4. If a merchant had a balance of \$496.75 on the first of last month, and has since deposited \$39.80, \$126.70, and \$98.90, and given checks for \$72.83, \$48.65, and \$271.96, what is his balance now?

5. If a merchant had a balance of \$592.80 on the fifteenth of last month, and has since deposited \$98.60, \$48.90, \$5.70, \$19.68, \$72.75, \$37.63, and \$100, and has given checks for \$38.40, \$125.75, \$30.42, and \$75, what is his balance now?

6. A merchant's balance on the first of last week was \$575.72. He has since deposited \$162.75, \$380.96, \$401.75, \$25.60, \$75, and \$129.80. He has given checks for \$272.70, \$92.35, \$178.62, \$325.25, and \$68.70. What is his balance now?

7. A merchant's balance on the first of last week was \$298.72. He has since deposited \$42.80, \$121.62, \$148.90, \$275.26, \$72.50, \$69.75, \$14.02, and \$9.70. He has given checks for \$38.72, \$129.40, \$42.60, \$25.80, \$206.07, and \$42.96. He has also paid by check a bill of \$82.50 less 10% and 5%, and one of \$48.75 less 3%. What is his balance now?

272. Borrowing from Banks. Banks make a business of lending money on short terms to reliable borrowers, taking promissory notes as evidence of the debt. Such a note is made more secure in one of three ways:

1. It is made payable to some responsible person who *indorses* it; that is, he signs his name across the *back*. This makes him responsible to the bank if the maker does not pay it, and if he is promptly notified.

2. It is signed by two or more makers, and is made payable to the bank. Such notes usually read, "We jointly and severally promise to pay," so that the bank may hold all the makers together responsible, or any one of them separately.

3. It is made payable to the order of the bank and is secured by the deposit of stocks, bonds, or other securities. These are called *collateral securities*.

273. Form of a Bank Note. The first of the above forms is as follows:

ONEONTA, N.Y., <i>May 15, 1915</i>	
<i>Sixty days</i> after date, I promise to pay to the order of	
<i>P. M. Stockwell</i> ~~~~~	\$2000. ⁰⁰
<i>Two Thousand</i> ~~~~~	Dollars
at The First National Bank, Oneonta, N.Y.	
Value received.	<i>John Wilson</i>
Due <i>July 14, 1915</i>	

This note is due, or it *matures*, on July 14. Formerly three *days of grace* were allowed on a note, so that the above note would mature on July 17, but this custom has been abolished in most states and is not considered in this book.

Bank notes are usually made for 30, 60, or 90 days.

When the time of a note is expressed in months, calendar months are used in computing the date of maturity; when the time is expressed in days, the exact number of days is used.

274. Bank Discount. The note given in § 273 does not mention any interest. This is because banks generally require the interest to be paid in advance.

Interest paid on a note in advance is called *bank discount*, and such a note is said to be *discounted*.

The rate of discount is understood to be by the year, as in the case of interest, unless there is a statement to the contrary.

275. Proceeds. The balance after deducting the discount from the face of a note is called the *proceeds*.

For example, what are the discount and the proceeds on the note given in § 273, the rate being 6%?

Face of note	\$2000
Discount on \$2000, 60 da., @ 6%	<u>20</u>
Proceeds	\$1980

Many simple oral problems of this kind should be given.

EXERCISE 165

Find the discount and the proceeds on the following :

1. \$250, 30 da., 6%. 6. \$175, 30 da., 5%.
2. \$575, 30 da., 6%. 7. \$1275, 60 da., 5½%.
3. \$625, 60 da., 6%. 8. \$2350, 60 da., 5½%.
4. \$1250, 60 da., 5%. 9. \$275.50, 90 da., 6%.
5. \$2250, 60 da., 5%. 10. \$527.50, 90 da., 6%.

11. Make out a 60-day note for \$1250, dated to-day, payable to R. H. Dodge, or order, at some bank in your vicinity. Discount it at 6% and find the proceeds.

12. Make out a 90-day note for \$725, dated to-day, payable to P. M. Milton, or order, at some bank in your vicinity. Discount it at 6% and find the proceeds.

13. A cattle dealer borrows \$1700 for 30 days. How much discount must he pay, the rate being 5%?

276. Discounting Interest-bearing Notes. It frequently happens that a manufacturer or dealer holds notes of his customers and wishes to get the money on them before they are due. In this case he may indorse the notes and sell them to a bank for the sum due at maturity, less the discount.

For example, a manufacturer has a note of one of his customers for \$350, dated October 15 and due 3 months after date, with interest at 5%. He discounts this note at a bank at 6% on Nov. 16. What are the proceeds?

Face of the note	\$350.00
Interest for 3 mo., 5%	4.38
Amount due at maturity	<u>\$354.38</u>
Discount, 60 da., 6%	3.54
Proceeds	<u>\$350.84</u>

Here the time from November 16 to January 15, 60 days, is the *term of discount*.

Bankers compute the discount for the exact number of days in the term of discount. It should be noticed that *interest is computed on the face of the note at the beginning of the term, discount on the amount of the note at maturity*.

EXERCISE 166

Find the discount and the proceeds:

	Face	Date	Due	Interest	Date of Discount	Rate of Discount
1.	\$250	Jan. 1	July 1	5%	Jan. 15	6%
2.	\$375	Oct. 7	Jan. 7	5%	Oct. 7	6%
3.	\$425	Oct. 18	Feb. 18	6%	Oct. 18	6%
4.	\$550	Sept. 20	Jan. 20	5%	Nov. 1	6%
5.	\$675	Nov. 16	May 16	6%	Dec. 1	5%
6.	\$1250	Aug. 7	Feb. 7	5%	Nov. 15	5%
7.	\$375.50	May 16	Sept. 16	6%	July 1	6%
8.	\$827.25	July 1	Feb. 1	6%	Oct. 1	6%

9. A 60-day note for \$1225, without interest, dated June 20, was discounted the day of its date at 6%. Find the discount.

10. A 60-day note for \$1225, with interest at 5%, dated June 20, was discounted the day of its date at 6%. Find the discount.

11. A 60-day note for \$1225, with interest at 6%, dated June 20, was discounted the day of its date at 5%. Find the discount.

12. A 60-day note for \$1225, with interest at 6%, dated June 20, was discounted on July 20 at 6%. Find the discount.

13. A 30-day note for \$2500, without interest, dated April 1, was discounted 15 days later at 6%. Find the discount and the proceeds.

14. A 90-day note for \$1650, with interest at 5%, is discounted 30 days after date at 6%. Find the proceeds.

15. A 90-day note for \$1650, with interest at 6%, is discounted 30 days after date at 5%. Find the proceeds.

16. A 90-day note for \$1650, with interest at 6%, is discounted the day of its date at 6%. Find the proceeds.

17. A 90-day note for \$1650, without interest, is discounted the day of its date at 6%. Find the proceeds.

18. How much greater are the proceeds on a note for \$675 due 30 days hence, than on one for the same amount due 90 days hence, there being no interest in either case, and the rate of discount being 6%?

19. On which can a man realize the more at a bank where the discount rate is 6%, a 6-months note for \$2550 with interest at 6%, or a 4-months note for \$2600 without interest? How much more?

277. Drafts. A written order by which one person directs another to pay a certain sum of money is called a *draft*.

A check is therefore a kind of draft. Usually the word *check* is used for an order on a bank, signed by a private individual; *draft* is used for an order other than a check, on one private individual or corporation signed by another. A draft may therefore be made by an individual on a company, by one bank on another, by a company on a private individual or firm, and so on.

278. Commercial Drafts. A draft made by one business house on another to secure the payment of a debt is called a *commercial draft*.

The following is a common form of commercial draft:

Accepted Aug. 10, 1915 Robert J. Clements Louisville, Ky.	WATERTOWN, N.Y., Aug. 9, 1915 At ~~this~~ ³⁰ days sight pay to the order of \$975.⁰⁰ Nine Hundred Seventy-five and $\frac{00}{100}$ Dollars To Robert J. Clements J. M. Simons
---	---

In this case J. M. Simons has sold Robert J. Clements \$975 worth of goods on 30 days' credit. When he sends the bill he also sends this draft. Mr. Clements writes "Accepted, Aug. 10, 1915, Robert J. Clements" across the face, and returns it to Mr. Simons. This becomes thereby a promise to pay \$975 "at 30 days sight," that is, on Sept. 9, or 30 days after the acceptance on Aug. 10. Mr. Simons may now discount this at a bank, just as he would a promissory note.

This is coming to be a common way of borrowing money at a bank, replacing the older method by a promissory note. A general name for such evidences of debt as can be sold to banks is *negotiable paper*.

In this draft Mr. Simons is the *drawer*, Mr. Clements the *drawee*, and as the draft is payable to him, Mr. Simons is also the *payee*.

EXERCISE 167

1. A draft dated Aug. 8, at 30 days sight, for \$975, is accepted on Aug. 10, and is discounted on Aug. 16 at 6%. What are the proceeds?
2. A draft dated Sept. 1, at 30 days sight, for \$350, is accepted on Sept. 4, and is discounted on Sept. 7 at 6%. What are the proceeds?
3. A draft dated April 2, at 60 days sight, for \$725, is accepted on April 7, and is discounted on April 12 at 6%. What are the proceeds?
4. A draft dated Oct. 15, at 60 days sight, for \$1225.75, is accepted on Oct. 19, and is discounted on Oct. 25 at 6%. What are the proceeds?
5. A draft dated Feb. 7, at 90 days sight, for \$1675.50, is accepted on Feb. 14, and is discounted on Feb. 22 at 5%. What are the proceeds?
6. A draft dated Aug. 6, at 90 days sight, for \$2170.60, is accepted and discounted on the day of its date. The rate of discount being $5\frac{1}{2}\%$, find the proceeds.
7. A manufacturer sells goods listed at \$725.50, discounted at 10% and 5%, and draws on the buyer at 60 days sight. The buyer accepts the draft and it is discounted 10 days later at 6%. Write the draft and find the proceeds.
8. A manufacturer sells goods listed at \$1672.25, discounted at 8% and 3%, and draws on the buyer at 60 days sight. The buyer accepts the draft and it is discounted 5 days later at 5%. Write the draft and find the proceeds.
9. A wholesale house sells goods listed at \$2737.50, discounted at 15%, 8%, and 3%, and draws on the buyer at 90 days sight. The buyer accepts the draft and it is discounted 30 days later at 5%. Write the draft and find the proceeds.

10. Robertson Bros. draw on J. P. Shipley at sight for \$37.50. The charges for collection are 10%. This is what per cent of the face?

Here the draft is not discounted, but the bank charges the owner for its trouble in collecting. Similarly for other examples on this page. A draft payable at once ("at sight") is called a *sight draft*.

11. S. L. James of Rochester draws a sight draft on L. D. Richards of Gloversville for \$75. The bank charges $\frac{1}{5}\%$ for collecting. What are the proceeds?

12. M. T. Snell of Plattsburg is the drawee of a \$200 sight draft; The Farmers Trust Co. of Troy is the payee; The World Soap Co. is the drawer. Write the draft.

13. A. B. Stanley owes M. S. Stanton for 2 doz. suits @ \$113.50; $1\frac{1}{2}$ doz. suits @ \$130; 2 doz. overcoats @ \$139; $7\frac{1}{2}$ doz. pairs trousers @ \$40. Stanton draws on Stanley at sight for the money, and the bank charges him 50% for collecting. What rate is this?

14. Suppose William Bentley of Boston, Mass., owes you \$250, and you wish to draw upon him for this amount. Write out a sight draft, payable to the order of some bank near your home. If the bank charges $\frac{1}{10}\%$ for collecting, what are the proceeds?

15. R. H. Dudley owes Cayley & Co. for 4 dressers @ \$14, and 6 washstands @ \$3.50, all less 15%, and a bedroom set @ \$34.55 net. They draw on him at sight for the amount, the bank charging 10% for collecting. What is the net amount received by Cayley & Co.?

16. R. J. Doane of Montreal owes A. D. Kane of Albany for 10 steel girders @ \$50, and tells the latter to draw upon him for the amount. Mr. Kane keeps his account at the Iron Exchange Bank. Make out a sight draft for Mr. Kane. What is the bank's charge for collecting, at $0.1\frac{1}{2}\%$?

279. Finding the Face. If it is desired to borrow exactly \$1000 on a 60-day note that is to be discounted at a bank at 6%, the face of the note must evidently be such a sum as discounted will leave \$1000 as the proceeds.

First consider the face of the note as \$1.00

Then the discount, @ 6% for 60 da., is 0.01

Therefore the proceeds are \$0.99

Hence \$1000 must be the proceeds of as many times \$1 as \$1000 ÷ \$0.99, or \$1010.10.

That is, the face of the note must be \$1010.10.

EXERCISE 168

1. What must be the face of a 60-day note in order that when discounted the day of its date at 6% the proceeds shall be \$497.50?

2. What must be the face of a 90-day note in order that when discounted the day of its date at 5% the proceeds shall be \$1777.50?

3. What must be the face of a 30-day note in order that when discounted the day of its date at 6% the proceeds shall be \$500?

4. A manufacturer sold some goods for \$1950 less 10% discount. He took the purchaser's note for 60 days for an amount such that when it was discounted at 6% the proceeds just paid the bill. What was the face of the note?

5. A manufacturer sold some goods for \$2400 less 10% and 5%. He took the purchaser's note for 30 days for an amount such that when it was discounted at 6% the proceeds just paid the bill. What was the face of the note?

6. A wholesale dealer sold some goods for \$3200 less 10% and 3%, due in 90 days. He took the purchaser's note for an amount such that when it was discounted at 6% the proceeds just paid the bill. What was the face of the note?

280. Compound Interest. When interest is added to the principal and becomes a part of it at specified intervals, the total interest is called *compound interest*.

In compound interest, the interest draws interest every time it is added to the principal ; in periodic interest it draws interest only once.

Interest is compounded annually, semiannually, or quarterly, according to agreement, but it cannot be enforced legally.

Most savings banks allow compound interest at a low rate, the interest usually being compounded quarterly or semiannually. Since the interest is deposited when it becomes due, the problem is merely one of computing simple interest each time.

For example, find the amount of \$60 at compound interest for 3 yr. at 5%, the interest being compounded annually. Find also the compound interest.

Interest for 1st year = 5% of \$60 = \$3. Amount = \$63.

Interest for 2d year = 5% of \$63 = \$3.15. Amount = \$66.15.

Interest for 3d year = 5% of \$66.15 = \$3.31. Amount = \$69.46.

\$69.46 (compound amt.) - \$60 (principal) = \$9.46 (compound int.).

If the time were $3\frac{1}{2}$ years, we should find the amount at compound interest for 3 years and add to it the interest upon itself for six months.

If the interest is 5% compounded semiannually for 3 years, it amounts to the same as $2\frac{1}{2}\%$ compounded annually for 6 years.

EXERCISE 169

Find the amount at 4%, interest compounded annually :

- | | | |
|-----------------|-------------------------------|-------------------------------|
| 1. \$250, 3 yr. | 4. \$650, 5 yr. | 7. \$1500, $1\frac{1}{2}$ yr. |
| 2. \$375, 4 yr. | 5. \$875, 6 yr. | 8. \$2650, $3\frac{1}{2}$ yr. |
| 3. \$425, 3 yr. | 6. \$1250, $2\frac{1}{2}$ yr. | 9. \$2575, $4\frac{1}{2}$ yr. |

Find the amount at 3%, compounded semiannually :

- | | | |
|------------------|--------------------------------|--------------------------------|
| 10. \$100, 4 yr. | 13. \$650, 3 yr. | 16. \$2500, $3\frac{1}{2}$ yr. |
| 11. \$250, 3 yr. | 14. \$980, 5 yr. | 17. \$1650, $4\frac{1}{2}$ yr. |
| 12. \$575, 2 yr. | 15. \$1350, $2\frac{1}{2}$ yr. | 18. \$1700, $4\frac{1}{2}$ yr. |

281. Compound Interest Table. The table on page 321 shows the amount of \$1 at compound interest at various rates. The compound interest on \$1 is found by subtracting 1 from the proper number shown in the table.

For example, looking at the column headed "2 Per Cent," \$1 will amount to \$1.02 in 1 yr., \$1.0404 in 2 yr., \$1.06121 in 3 yr., and so on, at 2% compound interest. Similarly, \$1000 will amount to \$1020 in 1 yr., \$1040.40 in 2 yr., \$1061.21 in 3 yr., and so on, at 2% compound interest.

Likewise, under "2½ Per Cent," we see that \$100 will amount to \$115.97 in 6 yr. at 2½% compound interest. Many easy cases of this kind should be given orally by the teacher, the pupils having the table open before them.

Compound interest is not allowed on ordinary promissory notes; but if an investor keeps reinvesting his simple interest as it becomes due, on notes running only a short time, he practically receives compound interest.

(1) What principal will in 10 yr., at 6% compound interest, yield \$1898.04 interest?

The interest on \$1 for 10 yr. at 6% is \$0.79085 (see the table).

Since \$0.79085 is the interest on \$1,

$$\$1898.04 \text{ is the interest on } \frac{\$1898.04}{0.79085}, \text{ or } \$2400.$$

(2) In what time will \$1600, at 4½% compound interest, yield \$1000 interest?

Since \$1600 yields \$1000, \$1 will yield $\frac{1}{1600}$ of \$1000, or \$0.625, in the same time, and \$1 will amount to \$1.625. By the table, \$1 will, in 11 yr. at 4%, amount to \$1.62285, and in 12 yr. to \$1.69588. Hence the required time is a little more than 11 yr.

(3) At what rate, compound interest, will \$1500 yield \$1201.41 interest in 15 yr.?

Since \$1500 yields \$1201.41 interest in 15 yr., \$1 in 15 yr. will yield $\frac{1}{1500}$ of \$1201.41, or \$0.80094, and \$1 will amount in 15 yr. to \$1.80094. In the table, opposite 15 yr., we find in the 4% column that the amount of \$1 is \$1.80094. Therefore the rate is 4%.

Table showing the amount of \$1 at compound interest for :

YEAR	2 PER CENT	2½ PER CENT	3 PER CENT	3½ PER CENT	4 PER CENT
1	1.02000	1.02500	1.03000	1.03500	1.04000
2	1.04040	1.05063	1.06090	1.07123	1.08160
3	1.06121	1.07689	1.09273	1.10872	1.12486
4	1.08243	1.10381	1.12551	1.14752	1.16986
5	1.10408	1.13141	1.15927	1.18769	1.21665
6	1.12616	1.15969	1.19405	1.22926	1.26532
7	1.14869	1.18869	1.22987	1.27228	1.31593
8	1.17166	1.21840	1.26677	1.31681	1.36857
9	1.19509	1.24886	1.30477	1.36290	1.42331
10	1.21899	1.28009	1.34392	1.41060	1.48024
11	1.24337	1.31209	1.38423	1.45997	1.53945
12	1.26824	1.34489	1.42576	1.51107	1.60103
13	1.29361	1.37851	1.46853	1.56396	1.66507
14	1.31948	1.41297	1.51259	1.61870	1.73168
15	1.34587	1.44830	1.55797	1.67535	1.80094
16	1.37279	1.48451	1.60471	1.73399	1.87298
17	1.40024	1.52162	1.65285	1.79468	1.94790
18	1.42825	1.55966	1.70243	1.85749	2.02582
19	1.45681	1.59865	1.75351	1.92250	2.10685
20	1.48595	1.63862	1.80611	1.98979	2.19112
YEAR	4½ PER CENT	5 PER CENT	5½ PER CENT	6 PER CENT	7 PER CENT
1	1.04500	1.05000	1.05500	1.06000	1.07000
2	1.09203	1.10250	1.11303	1.12360	1.14490
3	1.14117	1.15763	1.17424	1.19102	1.22504
4	1.19252	1.21551	1.23882	1.26248	1.31080
5	1.24618	1.27628	1.30696	1.33823	1.40255
6	1.30226	1.34010	1.37884	1.41852	1.50073
7	1.36086	1.40710	1.45468	1.50363	1.60578
8	1.42210	1.47746	1.53469	1.59385	1.71819
9	1.48610	1.55133	1.61909	1.68948	1.83846
10	1.55297	1.62889	1.70814	1.79085	1.96715
11	1.62285	1.71034	1.80209	1.89830	2.10485
12	1.69588	1.79586	1.90121	2.01220	2.25219
13	1.77220	1.88565	2.00577	2.13293	2.40985
14	1.85194	1.97993	2.11609	2.26090	2.57853
15	1.93528	2.07893	2.23248	2.39656	2.75903
16	2.02237	2.18287	2.35526	2.54035	2.95216
17	2.11338	2.29202	2.48480	2.69277	3.15882
18	2.20848	2.40662	2.62147	2.85434	3.37993
19	2.30786	2.52695	2.76565	3.02560	3.61653
20	2.41171	2.65330	2.91776	3.20714	3.86968

EXERCISE 170

1. Find the compound interest at 6% for 5 yr. on \$287.
2. Find the compound interest at $3\frac{1}{2}\%$ for 12 yr. 6 mo. on \$686.70.
3. Find the amount of \$960 for 9 yr. at 5%, interest compounded semiannually.
4. Find the amount of \$1216 for 5 yr. at 4%, interest compounded semiannually.
5. What principal will in 14 yr. at $5\frac{1}{2}\%$ compound interest amount to \$1880.78?
6. At what rate compound interest will \$2200 yield \$4749.40 interest in 17 yr.?
7. In what time will \$1800 at 5% compound interest amount to \$3000?
8. In what time will \$962.44 yield \$1080.72 interest at 6% compound interest?
9. What principal will in 20 yr. at 4% compound interest yield \$2500 interest?
10. At what rate compound interest will \$462.50 yield \$277.98 interest in 12 yr.?
11. What principal will in 10 yr. at 6% amount to \$3612.22, interest being compounded semiannually?
12. In what time at 5% will \$1250 amount to \$2000, interest being compounded semiannually?
13. At what rate per annum will \$500 amount to \$779.83 in 9 yr., interest being compounded semiannually?
14. At what rate compound interest will \$1500 amount to \$2229.15 in 9 yr.?
15. At what rate compound interest will \$2500 amount to \$3620.75 in 15 yr.?

282. Savings Banks. Certain banks where people keep their savings, receiving a low rate of interest, are carefully guarded by the laws of most of the states. These banks are called *savings banks*.

283. Interest in Savings Banks. Usually savings banks pay interest compounded semiannually. The following is a specimen account at a bank paying 2% every six months (4% a year, payable semiannually), on the smallest balance on deposit during the period.

DATE		DEPOSITS		INTEREST		PAYMENTS		BALANCE	
1915									
July	1	\$600	50					\$600	50
July	20	75						675	50
Sept.	6					\$120		555	50
Dec.	7	60						615	50
Dec.	20					65		550	50
1916									
Jan.	1			\$11				561	50
May	9	200						761	50
July	1			11	22			772	72

Here the smallest balance during the first interest period was \$550.50. Interest is computed on the dollars only, the cents being neglected. At 2% the interest on \$550 is \$11. In the second period the smallest balance is \$561.50, the interest being \$11.22.

Some banks allow interest from the first of each month ; others from the first of each quarter ; others, as above, from the first of each half year. The interest is computed on the smallest balance on hand.

If the above interest period were 3 months and the interest were credited every 6 months, the interest would be :

First quarter, 1% of \$555 = \$5.55.

Second quarter, 1% of \$550 = 5.50. Credit \$11.05.

Third quarter, 1% of \$561 = 5.61.

Fourth quarter, 1% of \$561 = 5.61. Credit \$11.22.

Therefore, the difference in interest would be only five cents

EXERCISE 171

In the following, find the balance due on the last date, interest 2% every six months on the smallest balance on deposit during six months preceding Jan. 1 and July 1:

1. Deposits: Jan. 1, 1914, \$750; Feb. 3, \$425; June 1, \$37.50. Payments: Mar. 7, \$230; May 6, \$26.75. Balance, July 1, 1914.

2. Deposits: Jan. 1, 1914, \$675.50; Mar. 2, \$923.75; Sept. 5, \$327.60. Payments: Feb. 7, \$327.40; Oct. 10, \$750. Balance, Jan. 1, 1915.

3. Deposits: Dec. 31, 1915, \$500; Feb. 2, 1916, \$300; Mar. 16, \$250; Aug. 1, \$50; Oct. 5, \$125. Payments: Mar. 6, 1916, \$325; May 11, \$275; Sept. 12, \$25; Nov. 15, \$100. Balance, Jan. 1, 1917.

In the following, find the balance due on the last date, interest being 1% every three months on the smallest balance on deposit during the three months preceding Jan. 1, Apr. 1, July 1, and Oct. 1, and all interest being payable on Jan. 1 and July 1:

4. Deposits: Mar. 1, 1915, \$200; June 3, \$675; July 1, \$350. Payments: Mar. 8, \$50; May 17, \$60; July 8, \$50. Balance, Jan. 1, 1916.

5. Deposits: Apr. 1, 1915, \$375; May 25, \$460; June 14, \$380; Aug. 8, \$750. Payments: May 3, \$75; Sept. 6, \$50; Nov. 8, \$130; Feb. 1, 1917, \$200. Balance, July 1, 1917.

6. A man on July 1 deposited \$2500 in a savings bank and left it for three years. The rate of interest was 4% a year and the interest was added to the principal semiannually. How much less interest did he receive than he would have received had he loaned the money at 5% simple interest?

EXERCISE 172

REVIEW PROBLEMS

1. Find the amount of \$500, compounded semiannually, at the rate of 4% a year, for 5 years.

2. A man deposits \$750 in a savings bank on July 1, 1916. His further deposits are as follows: Aug. 1, 1916, \$300; Sept. 15, 1916, \$250; Dec. 31, 1916, \$1000; June 15, 1917, \$500. He draws out as follows: Oct. 12, 1916, \$400; June 20, 1917, \$375; Aug. 2, 1917, \$200. The savings bank pays 1% every three months on the smallest balance for the preceding quarter, on Jan. 1, Apr. 1, July 1, and Oct. 1. Find the balance on Jan. 1, 1918.

3. Make out a deposit slip for the following deposits, naming some bank in your vicinity: bills, \$575; gold, \$35; silver, \$72.25; checks, \$48.25, \$72.50, \$61.30, \$227.52, and \$86.95. If your balance was \$326.42 before, what was it after the deposit?

4. A merchant's balance on the first of the month was \$1725.60. He has since deposited \$37.50, \$296.75, \$21.30, \$572.32, and \$217.65. He has given checks for \$128.92, \$27.36, \$18.92, \$78.76, and \$273.96. How much is his present balance?

5. Make out a 90-day note for \$1250, dated to-day, payable to R. P. Foster, or order, at some bank in your vicinity. Discount it at 6%. If with the proceeds you buy 10 shares of stock at \$108.50 a share, how much money will you have left?

6. Which is the greater, the proceeds of a note for \$750 for one year, drawing interest at 6%, and discounted on the day of its date at 5%, or the amount of a note for \$700 for one year at 6%? How much greater?

EXERCISE 173

PROBLEMS WITHOUT NUMBERS

1. How is the interest due on the first of the year on a savings-bank account ascertained?

2. Which pays the better interest, if the money is left undisturbed for five years, a 4% promissory note or a savings-bank deposit at 4%? Why?

3. How do you fill out a deposit slip? After entering the items what operation do you perform? How do you make sure of the result?

4. If you know a man's balance in a bank a week ago and his deposits and checks since, how do you proceed to find his balance now?

5. How do you find the discount on a promissory note? How do you find the proceeds?

6. If a note drawing a certain rate of interest is discounted on the day it is made, at the same rate, are the proceeds greater than the face, or equal to it, or less? Why is this?

7. How can a manufacturer discount a claim against a purchaser, the claim not being yet due? How is the discount found?

8. If you know the proceeds and rate of discount, how do you find the face of a note?

9. If you know the face of a note and the proceeds, how do you find the discount? the rate of discount?

10. If a dealer buys some produce and agrees to pay for it in 90 days, with interest at 6%, and the seller takes a note and at once discounts it at 5%, will he receive more than the selling price, or the same amount, or less? Why?

CHAPTER XVII

RATIO AND PROPORTION

284. Ratio. The relation between two like numbers, as expressed by the fraction which has the first number for the numerator and the second for the denominator, is called the *ratio* of the first number to the second.

The ratio of 2 to 3, or $\frac{2}{3}$, is commonly written 2 : 3.

It is better to use the fractional form at first, and then to use the other form, 2 : 3.

285. Antecedent and Consequent. The first term of a ratio is called the *antecedent*, and the second term the *consequent*.

For example, $\frac{5 \text{ ft.}}{6 \text{ ft.}} = \frac{5}{6} = \frac{\text{numerator}}{\text{denominator}} = \frac{\text{antecedent}}{\text{consequent}}$.

286. Ratios always Abstract. The two terms of a ratio always being like numbers, as in 2 : 3, \$4 : \$5, 6 ft. : 7 ft., the quotient is always abstract.

Therefore, *a ratio is always abstract, and its terms may be written as abstract numbers.*

Instead of writing $\frac{2 \text{ ft.}}{3 \text{ ft.}}$ we may therefore write simply $\frac{2}{3}$.

287. Reduction of Ratios. Since a ratio may be expressed as a fraction, therefore

If the terms of a ratio are both multiplied by or both divided by the same number, the value of the ratio is not altered.

Thus, if both terms of the ratio $2\frac{1}{2} : 3\frac{1}{3}$ are multiplied by 6, the resulting ratio is 15 : 20, and these two ratios are equal.

Again, since $\frac{1\frac{5}{6}}{\frac{3}{4}} = \frac{5}{3}$, the simplest expression for $2\frac{1}{2} : 3\frac{1}{3}$ is 3 : 4.

288. Separating a Number in a Given Ratio. Required to separate \$63 in the ratio of 3 to 4.

Since there must be \$3 in the first part to every \$4 in the second, there must be \$7 every time two such amounts are taken out. Therefore the first part must contain $\frac{3}{7}$ of the total and the second $\frac{4}{7}$.

But $\frac{3}{7}$ of \$63 is \$27, and $\frac{4}{7}$ of \$63 is \$36.

These are the results, because $\$27 : \$36 = \frac{3}{4} = \frac{3}{7} = \frac{4}{7}$.

EXERCISE 174

Express the following ratios in simplest form :

- | | | | | |
|----------------------|------------------------------|--|--------------------------------|-------------|
| 1. $\frac{34}{51}$. | 3. $\frac{46}{115}$. | 5. $\frac{\frac{4}{5}}{6}$. | 7. $\frac{13}{3\frac{1}{4}}$. | 9. 58:145. |
| 2. $\frac{57}{76}$. | 4. $\frac{2}{\frac{3}{4}}$. | 6. $\frac{\frac{1}{2}}{\frac{7}{8}}$. | 8. $\frac{2\frac{1}{2}}{10}$. | 10. 93:124. |

Find the missing term, x , in each of the following ratios :

- | | | | |
|-------------------------|-----------------------------------|------------------------------------|-----------------------------|
| 11. $\frac{x}{4} = 2$. | 13. $\frac{x}{9} = \frac{1}{3}$. | 15. $\frac{x}{10} = \frac{3}{5}$. | 17. $x : 9 = 2$. |
| 12. $\frac{6}{x} = 2$. | 14. $\frac{5}{x} = \frac{1}{4}$. | 16. $\frac{7}{x} = \frac{1}{5}$. | 18. $7 : x = \frac{1}{2}$. |

Separate the following numbers in the given ratios :

- | | | |
|---------------|---------------------------------------|------------------------------|
| 19. 28, 3:4. | 22. $31\frac{1}{2}, 2\frac{1}{2}:5$. | 25. \$78, 3:10. |
| 20. 45, 4:5. | 23. \$72, 1:8. | 26. 341, 15:16. |
| 21. 260, 5:8. | 24. \$96, 3:5. | 27. 492, $5\frac{1}{2}:15$. |

28. Two farmers pay \$30 together for some threshing, one having 350 bu. and the other 400 bu. of wheat. What is the share of each?

29. A man leaves an estate of \$13,470, giving \$1 to his widow for every \$2 to his children. How much did the children receive?

EXERCISE 175

PRACTICAL USES OF RATIO

1. The ratio of water to solid matter in apples is $423:77$. How many pounds of water in 250 lb. of apples?
2. In a sack of rough rice, containing 162 lb., the ratio of hulls to cleaned rice is $1:4$. How many pounds of cleaned rice in a sack?
3. A pineapple grower in Florida received in one year \$15 for plants to every \$7.50 for fruit. If his total receipts were \$3375, how much did he receive for the fruit?
4. In fertilizing land for growing peanuts a Virginia farmer used 30 lb. of cotton-seed meal to 13 lb. of cotton-hull ashes. How many pounds of each in 215 lb. of fertilizer?
5. Two partners invest respectively \$2750 and \$3500. They divide their profits, \$3750, in the ratio of their investments. What is the share of each?
6. A recipe for fertilizer for oats requires $12\frac{1}{2}$ lb. of nitrate of soda to 15 lb. of other ingredients. How many pounds of nitrate of soda in $412\frac{1}{2}$ lb. of fertilizer?
7. A recipe for grafting wax calls for 4 parts of resin to $1\frac{1}{2}$ parts of beeswax and 1 part of linseed oil. How many pounds of resin in 26 lb. of the wax?
8. Two rice planters expended \$2240 on an irrigating plant. One owned 80 acres of rice and the other 120 acres. If they paid in the ratio of their acreage, what was the share of each?
9. A New Jersey gardener estimated that the ratio of the cost of planting and caring for asparagus, to the cost of cutting and bunching, was $1:2$. If the cost of both for his crop was \$84, what was the cost of each?

289. Proportion. An expression of equality between two ratios is called a *proportion*.

Thus $\frac{2}{3} = \frac{10}{15}$ and $2 : 3 = 10 : 15$ are proportions, and are read "2 is to 3 as 10 is to 15," or "the ratio of 2 to 3 equals the ratio of 10 to 15."

290. Terms of a Proportion. The first and last numbers of a proportion are called the *extremes*, and the two middle numbers are called the *means*. The extremes and means together are called the *terms* of the proportion.

Thus in the proportion $2 : 3 = 10 : 15$, the extremes are 2 and 15, and the means are 3 and 10.

291. Test of a Proportion. In the proportion $\frac{2}{3} = \frac{10}{15}$ we shall still have an equality if we multiply these equals by the same number. If we multiply by 3×15 we have

$$\frac{3 \times 15 \times 2}{3} = \frac{3 \times 15 \times 10}{15}, \text{ or } 15 \times 2 = 3 \times 10.$$

Therefore, *in any proportion the product of the extremes equals the product of the means*.

292. Finding a Missing Term in a Proportion. The product of the extremes equals the product of the means (§ 291).

Therefore, *either extreme equals the product of the means divided by the other extreme*;

Either mean equals the product of the extremes divided by the other mean.

Hence if three terms are given, the fourth may be found.

(1) Find the missing term of the proportion $x : 6 = 9 : 27$.

By § 292,
$$x = \frac{9 \times 6}{27} = 2.$$

(2) Find the missing term of the proportion $3 : 7 = x : 35$.

Not using § 292, we see that
$$\frac{3}{7} = \frac{x}{35}.$$

Multiply these equals by 35,
$$\frac{3 \times 35}{7} = 15 = x.$$

EXERCISE 176

Find the missing term, designated by x :

1. $\frac{x}{12} = \frac{24}{16}$
2. $\frac{x}{15} = \frac{21}{35}$
3. $x : 15 = 32 : 6$
4. $x : 8 = 15 : 20$
5. $15 : x = 10 : 8$
6. $55 : x = 22 : 10$
7. $12 : x = 18 : 10$
8. $18 : x = 27 : 45$
9. $12 : 18 = x : 27$
10. $25 : 15 = x : 21$
11. $21 : 49 = x : 63$
12. $18 : 45 = x : 35$
13. $10 : 45 = 14 : x$
14. $24 : 54 = 28 : x$
15. $35 : 49 = 45 : x$
16. $\$5 : \$7 = x : 63$
17. $\$9 : x = 189 : 231$
18. $x : \$6 = 85 : 102$
19. $\$15 : \$17 = 75 : x$
20. $21 \text{ ft.} : 5 \text{ ft.} = 147 : x$
21. $35 \text{ ft.} : x = 315 : 99$
22. $2\frac{1}{2} : 7 = 100 : x$
23. $3\frac{1}{3} : 2\frac{1}{2} = x : 30$
24. $12\frac{1}{2} : 16\frac{2}{3} = x : 80$
25. $12\frac{1}{2} : x = 33\frac{1}{3} : 160$
26. $37.5 : x = 250 : 80$
27. $62.5 : 12.5 = 75 : x$
28. $x : 66\frac{2}{3} = 90 : 600$
29. $x : 87\frac{1}{2} = 33\frac{1}{3} : 2\frac{1}{3}$
30. $x : 1.6 = 3.1416 : 1$

31. Show that 29, 435, 17, and 255 are in proportion.

32. Show that 217, 2821, 311, and 3421 cannot be in proportion.

33. What number has the same ratio to 21 that 16 has to 35?

34. If the ratio of 7 to 16 equals the ratio of 63 to some number, what is that number?

35. Show that 401 and 5287 are the extremes and 311 and 6817 are the means of a proportion. Write the proportion in three different ways.

293. Directly Proportional. If one ratio equals another, the terms of the first are said to be *directly proportional* to those of the second.

Since 2 : 3 and 4 : 6 are equal ratios, 2 and 3 are said to be directly proportional to 4 and 6.

In general, the following are among the cases of directly proportional quantities :

Cost of goods is directly proportional to the quantity.

Weight of substances is directly proportional to the volume.

Income from labor is directly proportional to time given.

Distance traveled is directly proportional to time.

294. Inverse Ratio. The result of interchanging the terms of a ratio is called the *inverse ratio* of the given ratio.

Thus 3 : 2 is the inverse of 2 : 3, just as $\frac{3}{2}$ is the inverse of $\frac{2}{3}$.

295. Inversely Proportional. If one ratio equals the inverse of another, the terms of the first are said to be *inversely proportional* to those of the second.

Since 2 : 3 equals the inverse of 6 : 4, 2 and 3 are said to be inversely proportional to 6 and 4 ; that is, $2 : 3 = 4 : 6$.

As an example of inverse proportion, the time required for some work is inversely proportional to the number of laborers. That is, if we take twice as many workmen, it will take half as long for the work.

296. Commercial Applications. Consider the following problem : If 5 T. of hay cost \$87.50, what will 21 T. cost ?

Since the ratio of cost is evidently equal to the ratio of quantity, we have

$$\$x : \$87.50 = 21 \text{ T.} : 5 \text{ T.}$$

Since we cannot multiply dollars by tons, we write the proportion

$$\frac{x}{87.50} = \frac{21}{5} (\$ 286).$$

Multiplying these equals by 87.50,

$$x = \frac{21 \times 87.50}{5} = 367.50, \text{ the number of dollars.}$$

We may write the proportion so that the unknown number will come in any term we wish, the result being the same. Teachers formerly preferred to put it in the fourth term, and this may be done if desired.

If 8 men can do a piece of work in 9 days, how long will it take 12 men to do it ?

Since it takes correspondingly less time as we increase the number of workmen, the ratio of the number of days equals the inverse ratio of the number of men.

Therefore $x \text{ da.} : 9 \text{ da.} = 8 \text{ men} : 12 \text{ men},$
or $x \text{ da.} : 9 \text{ da.} = 8 : 12 (\S 286).$

Therefore $x \text{ da.} = \frac{8 \times 9 \text{ da.}}{12} = 6 \text{ da.}$

In proportion the words "at the same rate" are always understood.

EXERCISE 177

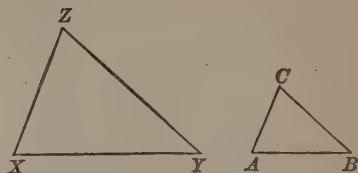
1. If 75 bu. of wheat cost \$72, what will 260 bu. cost ?
2. If 36 yd. of cretonne cost \$9.72, what will 56 yd. cost ?
3. If 42 yd. of ribbon cost \$9.24, what will 75 yd. cost ?
4. If 18 yd. of braid cost \$2.25, what will 62 yd. cost ?
5. If $2\frac{1}{2}$ M feet of lumber cost \$70, what will 7 M feet cost ?
6. If a man earns \$16.50 in 6 da., how much will he earn in 25 da. ?
7. A manufacturer used 71 yd. of braid on 4 sailor suits. How much would he need for 15 such suits ?
8. A merchant bought 640 yd. of muslin for \$36.80. How much did 360 yd. cost ?
9. For 10 aprons a manufacturer needs 15 yd. of goods. How many yards will he need for 75 aprons ?
10. How long should it take 16 men to do some work that 52 men can do in 12 da. ?
11. If 8 men require 75 da. to do a piece of work, how many men should it take to do it in 40 da. ?
12. How far can 36 tons of freight be carried for the money paid for carrying 54 tons 144 miles ?

297. Similar Figures. Figures that are of exactly the same shape are called *similar figures*.

For example, these two triangles are similar.

The corresponding sides of similar figures are proportional.

In these figures, $AB:BC = XY:YZ$. Also, $ZX:XY = CA:AB$.

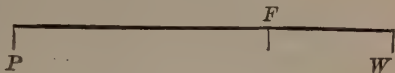


This principle can be used in measuring the height of trees or buildings, as shown in Exs. 3-6 below. It is the principle involved in all map drawing.

EXERCISE 178

1. If $\frac{3}{4}$ in. on a map represents a distance of 375 mi., how many miles will $1\frac{1}{4}$ in. represent?
2. If a distance of 325 mi. is represented on a map by $1\frac{1}{8}$ in., how many inches will represent a distance of 340 mi.?
3. If a tree casts a shadow 50 ft. long when a post 4 ft. high casts a shadow 5 ft. long, how high is the tree?
4. If a spire casts a shadow 200 ft. long when a post $4\frac{1}{2}$ ft. high casts a shadow 5 ft. long, how high is the spire?
5. A class wishing to find the height of the school building, one of the boys stood a yard measure upright and another lay on the ground where he could just see the top of the building in line with the top of the stick. The distance from his eye to the foot of the stick was 4 ft. 6 in., and to the building 45 ft. How high was the building?
6. A man whose eye is 5 ft. 6 in. above the ground sights over the top of a 12-foot pole and just sees the top of a tree. If he is 7 ft. from the pole and 63 ft. from the tree, how high is the tree?

298. The Lever. If we have a lever as shown in this illustration, where F represents the fulcrum on which it rests, we may lift a weight placed at W by applying power enough at P . It has been found that



Power and weight are inversely proportional to their distances from the fulcrum.

Thus, if $WF = 8$ in., $PF = 16$ in., and weight = 7 lb., we have

$$\text{Power} : 7 \text{ lb.} = 8 \text{ in.} : 16 \text{ in.}$$

Therefore
$$\text{Power} = \frac{8 \times 7 \text{ lb.}}{16} = 3\frac{1}{2} \text{ lb.}$$

That is, it would take only $3\frac{1}{2}$ lb. at P to lift 7 lb. at W .

Other simple forms of the lever may be considered if desired.

EXERCISE 179

Given the following distances from the fulcrum, find the power necessary to raise the weight specified:

PF	WF	Weight	PF	WF	Weight
1. 2 ft.	6 in.	17 oz.	6. 6 ft.	1 ft.	360 lb.
2. 13 in.	5 in.	23 oz.	7. 5 ft.	6 in.	375 lb.
3. 17 in.	8 in.	12 lb.	8. 4 ft.	8 in.	900 lb.
4. 32 in.	9 in.	25 lb.	9. 5 ft.	4 in.	750 lb.
5. 48 in.	8 in.	100 lb.	10. $5\frac{1}{2}$ ft.	6 in.	1 T.

Given the following distances from the fulcrum, find the greatest weight that can be raised by the power specified:

PF	WF	Power	PF	WF	Power
11. 3 ft.	6 in.	10 lb.	14. $4\frac{1}{2}$ ft.	6 in.	25 lb.
12. 4 ft.	4 in.	30 lb.	15. $5\frac{1}{2}$ ft.	8 in.	125 lb.
13. 5 ft.	8 in.	100 lb.	16. $6\frac{1}{2}$ ft.	8 in.	150 lb.

EXERCISE 180

MISCELLANEOUS PROBLEMS

1. Two men rent a pasture for \$115. One puts in 9 cattle and the other 14. How much rent should each pay?

In all miscellaneous and review problems the equation and the unknown quantity should be used whenever convenient.

For example, we might let $9x$ represent the number of dollars the first pays, and $14x$ the number of dollars the second pays. Then

$$9x + 14x = 115,$$

and

$$x = 5;$$

whence

$$9x = 45$$

and

$$14x = 70.$$

2. If the ratio of a man's income to his expenses is 7 : 6, and his expenses are \$1800 a year, how much is his income?

3. Taking the ratio of the diagonal to the side as 7 : 5, how long is the diagonal of a square 35 ft. on a side?

4. Taking the ratio of the circumference of a circle to the diameter as 22 : 7, how long is the circumference of a circle whose diameter is 4 ft.?

5. If the ratio of cream to the rest of the milk of a certain quality is 1 : 5, how many pounds of cream in 375 lb. of milk of that quality?

6. If the ratio of butter fat to the rest of the milk of a certain quality is 1 : 24, how many pounds of butter fat in 350 lb. of milk of that quality?

7. If the ratio of solid matter to the rest of the milk of a certain quality is 33 : 217, how many pounds of solid matter in 750 lb. of milk of that quality?

8. Two shippers charter a canal boat for \$84 to carry some freight to a certain city. One ships 75 tons and the other 135 tons. How much should each pay?

9. The ratio of copper sulphate and unslaked lime to water in Bordeaux spraying mixture is 1 : 39. How much copper sulphate and unslaked lime in 160 lb. of the mixture ? How many pounds of each in 240 lb. of the mixture ?

10. If a shipment of 5100 lb. of cattle, live weight, sold for \$225.42, what would 3500 lb. sell for at the same rate ?

11. If a Louisiana farmer paid \$75 for 3 T. 1500 lb. of cottonseed meal for fodder, how much would he have to pay for 5000 lb. ?

12. If in 225 lb. of milk there are 8.1 lb. of butter fat, how many pounds of milk will be required to produce 27 lb. of butter fat ?

13. A farmer had 26 acres planted to potatoes. The crop from 7 acres amounted to 1260 bu. At the same rate how many bushels did he receive from the whole field ?

14. To irrigate a farm at the rate of $\frac{1}{2}$ in. in depth every day requires the flow of 210 gal. an hour through a certain ditch. What flow would be necessary to irrigate it at the rate of 0.3 in. a day ?

15. A manufacturer built a cylindrical tank 32 ft. high. What must be the height of one of the same diameter that it may have a capacity $\frac{5}{8}$ as great ?

16. If a certain kind of bird eats wild berries and also insects that harm fruit trees, and if out of 14 oz. of food eaten by this bird only $1\frac{2}{3}$ oz. is wild berries, what per cent of the bird's food is insects that would harm fruit trees ?

17. A Monroe County farmer sprayed an apple tree, and the crop numbered 1656 sound apples and 186 diseased ones. From a tree not sprayed he obtained 142 sound apples and 1278 diseased ones. Are these numbers inversely proportional or not ?

18. If a farmer needs 108 lb. of seed for a 2-acre lot, how much will he need for $26\frac{1}{2}$ acres?

19. If the cost of 35 ft. of sidewalk is \$31.50, what will 75 ft. cost, the width being the same?

20. If 2.8 bbl. of lime are required for 75 sq. yd. of plastering, how many barrels will be needed for 275 sq. yd.?

21. A stretch of railway track runs 437 ft. on the same grade, $6\frac{1}{2}$ in. per 100 ft. What is the total difference in level?

22. A storage warehouse stored 675 bbl. of apples for 90 days at the rate of \$6 per 100 bbl. for 30 da. How much was the charge?

23. If a contractor agrees to build a sidewalk 4 ft. 6 in. wide in front of a lot for \$36, how much should he charge if it were 5 ft. wide?

24. A man builds a coal bin 6 ft. wide and finds that it holds $11\frac{1}{2}$ tons. How much would it hold if it had been made a foot wider?

25. An ice dealer can gather 2900 tons of ice from a pond when the ice has frozen 12 in. thick. How many tons can he gather when the ice is 15 in. thick?

26. An excavation has been half completed, 35 men having been employed for 16 days. How long should it take to complete the other half if 5 more men are added?

27. Gun metal is composed of 1 part of tin to $5\frac{1}{2}$ parts of copper by weight. How many pounds of tin must be added to 210 lb. 6 oz. of copper to make gun metal?

28. How many pounds of tin are there in 232 lb. 6 oz. of gun metal such as described in Ex. 27?

29. On January 1 a gas meter registers 42,500 cu. ft., and on January 8 it registers 43,400 cu. ft. If this rate is exactly maintained, how much will it register on January 29?

299. Proportional Parts. Just as we have learned (§ 288) how to separate a number into two parts having a given ratio, so we may separate a number into several parts proportional to given numbers.

For example, separate \$1230 into parts proportional to 2, 3, and 5.

The whole quantity may be represented by $2 + 3 + 5$, or 10.

Therefore the respective parts are $\frac{1}{5}$, $\frac{3}{10}$, and $\frac{1}{2}$ of \$1230. These parts are \$246, \$369, and \$615.

Separate 527 ft. into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

Multiply the fractions by 30, the L.C.M. of their denominators. The results are 15, 10, and 6. These numbers may therefore represent the parts, and their sum, 31, will represent the whole.

Therefore the respective parts are $\frac{15}{31}$, $\frac{10}{31}$, and $\frac{6}{31}$ of 527 ft., or 255 ft., 170 ft., and 102 ft.

Two farmers rent a pasture for \$2.55. The first puts in 4 cattle for 2 weeks, and the second 3 cattle for 3 weeks. How much should each farmer pay?

Since 4 cattle in 2 weeks eat as much as 8 cattle in 1 week, and 3 cattle in 3 weeks eat as much as 9 cattle in 1 week, one should pay $\frac{8}{17}$ of \$2.55, or \$1.20, and the other should pay $\frac{9}{17}$ of \$2.55, or \$1.35.

EXERCISE 181

1. Separate 297 into parts proportional to 7, 9, and 11.
2. Separate 253 into parts proportional to $\frac{1}{6}$, $\frac{1}{2}$, and $\frac{1}{4}$.
3. Divide \$2646 between two partners in the ratio of 4 to 5.
4. Divide \$3744 among three partners so that their shares shall be proportional to 3, 4, and 5.
5. Four men invested \$2700, \$3300, \$3500, and \$4000 respectively in a joint business. They gained \$6750. What was each one's share of the profit?

6. Three farmers together pay \$225 for irrigation taxes. The first owns 225 acres, the second 190 acres, and the third 335 acres. What is the tax of each?

7. Three men deliver milk at a creamery. The first delivers 776 lb., the second 938 lb., and the third 870 lb. They receive \$38.76. What is the share of each?

8. Three men pay \$848 for some water power. The first uses 28 horse power, the second 36 horse power, and the third 42 horse power. How much should each pay?

9. Three men rent a yacht for the summer for \$540. The first uses it for 4 weeks, the second 5 weeks, and the third $4\frac{1}{2}$ weeks. How much should each pay?

10. Two farmers pasture cattle in a field, the first putting in 24 head for 6 weeks, and the second 36 head for 4 weeks. The bill being \$43.20, what is the share of each?

11. Two hotel proprietors arrange to transport their employees from Florida to New York at the end of the season for \$2793. The first has 65 employees and the second 82. How much should each proprietor pay?

12. Four boys arrange a three weeks' camping trip. The first stays 18 days; the second comes late and stays 12 days; the third is in camp the last two weeks; the fourth stays all through. The total expense is \$32.50. What is the share of each?

13. A contractor employs 16 men 15 days on one job, and 14 men 12 days on another job. The total expense is \$1020. How much should be charged to each job?

14. A manufacturer ships 1200 lb. of goods 300 miles, and 900 lb. 400 miles. If he pays according to weight and distance, and the total bill is \$24, how much does each shipment cost?



EXERCISE 182

REVIEW PROBLEMS

1. These boys find that this post is 3 ft. 6 in. high and casts a shadow 3 ft. 2 in. long. They find that the shadow of the tree is 34 ft. 10 in. long at the same time. Required the height of the tree.

2. If a building casts a shadow 75 ft. long when a post 6 ft. 2 in. high casts a shadow 12 ft. 6 in. long, how high is the building?

3. If the railway fare for a journey of 75 mi. is \$1.50, what will be the fare for 275 mi.?

4. Two boys weighing respectively 100 lb. and 80 lb. sit 9 ft. apart on the ends of a plank. Not counting the weight of the plank, how far from the heavier boy must the fulcrum be placed so that they will just balance?

5. How much pressure will you have to exert on the handles of a pair of shears just 3 in. from the fulcrum (screw or bolt), in order to exert a pressure of 5 lb. at a point 5 in. from the fulcrum?

EXERCISE 183

PROBLEMS WITHOUT NUMBERS

1. If you double the consequent of a ratio, what else must you do so as not to change the value of the ratio?
2. How do you proceed to separate a number into parts having a given ratio?
3. How do you ascertain whether four quantities are in proportion?
4. Given the extremes and one mean of a proportion, how do you find the other mean?
5. Given the means and one extreme of a proportion, how do you find the other extreme?
6. If you know all of the terms of a proportion except the first, how is that found?
7. If you know all of the terms of a proportion except the antecedent of the second ratio, how is that found?
8. If you know the weight of two volumes of a substance, and one of the volumes, how do you state the proportion?
9. Knowing the number of men required to do a piece of work in a given number of days, how do you find how many men are required to do it in another given number of days?
10. How may you find the height of your school building by means of shadows and the height of a pole?
11. How may you proceed to find the height of a tree by sighting over the top of a pole of known length?
12. How may you tell where to place the fulcrum under a lever so as to lift a given weight by pressing your full weight on one end of the lever?
13. How do you proceed to separate a number into parts proportional to four given numbers?

CHAPTER XVIII

POWERS AND ROOTS

300. Power. The result of taking a number any number of times as a factor is called a *power* of the number.

301. Square. The result of taking a number twice as a factor is called the *second power* or *square* of the number.

Thus the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

302. Square Root. One of the two equal factors of a number is called the *square root* of the number.

Thus the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,
are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Some numbers are *perfect squares*. We also speak of the square root of 5 as $2.236 +$ because the square of 2.236 is nearly 5.

303. Symbols. The square of 5 is written 5^2 ; the square root of 5 is written $\sqrt{5}$. The symbol $\sqrt{}$ is called a *radical sign*. The small figure 2 in 5^2 is called an *exponent*.

304. Square Roots and Areas. If this square is 4 units on a side, the area is 16 square units. Therefore, considering all the numbers as abstract,



The side of a square equals the square root of its area.

305. Square Root by Factoring. The square root of a perfect square is sometimes found by factoring.

For example, since $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = (2 \times 2 \times 3) \times (2 \times 2 \times 3)$, therefore $\sqrt{144} = 2 \times 2 \times 3 = 12$.

306. Table of Squares. The squares of numbers to 25 should be memorized, as follows :

$2^2 = 4$	$8^2 = 64$	$14^2 = 196$	$20^2 = 400$
$3^2 = 9$	$9^2 = 81$	$15^2 = 225$	$21^2 = 441$
$4^2 = 16$	$10^2 = 100$	$16^2 = 256$	$22^2 = 484$
$5^2 = 25$	$11^2 = 121$	$17^2 = 289$	$23^2 = 529$
$6^2 = 36$	$12^2 = 144$	$18^2 = 324$	$24^2 = 576$
$7^2 = 49$	$13^2 = 169$	$19^2 = 361$	$25^2 = 625$

We see at once that $\sqrt{484} = 22$, without taking the trouble to factor as in § 305. In the case of perfect squares above 625, however, the factoring method of finding square roots is often helpful.

EXERCISE 184

State orally the squares of :

1. 9.	4. 13.	7. 25.	10. 11.	13. 18.	16. 15.
2. 12.	5. 17.	8. 19.	11. 40.	14. 23.	17. 24.
3. 20.	6. 21.	9. 16.	12. 22.	15. 14.	18. 30.

State orally the square roots of :

19. 64.	22. 1.	25. 4.	28. 9.	31. 16.	34. 196.
20. 25.	23. 81.	26. 121.	29. 49.	32. 144.	35. 324.
21. 225.	24. 36.	27. 169.	30. 100.	33. 256.	36. 576.

State orally the areas of squares with sides as follows :

37. 20 in.	40. 17 ft.	43. 60 yd.	46. 12 in.
38. 40 in.	41. 19 ft.	44. 70 yd.	47. 16 in.
39. 50 in.	42. 23 ft.	45. 80 yd.	48. 24 in.

State orally the sides of squares whose areas are :

49. 121 sq. ft.	52. 324 sq. in.	55. 625 sq. yd.
50. 289 sq. ft.	53. 169 sq. in.	56. 441 sq. yd.
51. 484 sq. ft.	54. 361 sq. in.	57. 576 sq. yd.

Find the areas of squares whose sides are as follows :

- | | | | |
|-------------|-------------|--------------|------------------------|
| 58. 1.7 ft. | 61. 2.9 in. | 64. 0.62 yd. | 67. $\frac{4}{5}$ mi. |
| 59. 1.9 ft. | 62. 3.7 in. | 65. 1.27 yd. | 68. $\frac{7}{8}$ in. |
| 60. 2.3 ft. | 63. 4.3 in. | 66. 4.93 yd. | 69. $\frac{9}{32}$ ft. |

By factoring, find the square roots of the following :

- | | | | | |
|-----------|-----------|-----------|-----------|-----------|
| 70. 784. | 73. 3600. | 76. 1936. | 79. 900. | 82. 1225. |
| 71. 4356. | 74. 3969. | 77. 2025. | 80. 729. | 83. 1089. |
| 72. 4096. | 75. 3025. | 78. 2916. | 81. 1024. | 84. 1296. |

Find the sides of squares whose areas are as follows :

- | | | |
|------------------|------------------|--------------------|
| 85. 1.44 sq. in. | 88. 2500 sq. ft. | 91. 0.09 sq. mi. |
| 86. 1.21 sq. in. | 89. 3600 sq. yd. | 92. 0.81 sq. ft. |
| 87. 5.76 sq. in. | 90. 6400 sq. rd. | 93. 0.0144 sq. ft. |

Find the perimeters of squares whose areas are as follows :

- | | | |
|------------------|-------------------|--------------------|
| 94. 576 sq. in. | 96. 23.04 sq. in. | 98. 65.61 sq. ft. |
| 95. 5184 sq. in. | 97. 40.96 sq. in. | 99. 110.25 sq. ft. |

100. A square field contains 1936 sq. rd. Find the length of each side.

101. A square field contains 40 acres. Express the length of the field in rods.

102. A square lot has an area of 169 sq. rd. How many rods of fence will be needed to inclose it ?

103. What is the area of the cross section of a square pillar that is $11\frac{1}{2}$ in. on a side ?

104. The area of the cross section of a square pillar is 2.89 sq. ft. What is the width of the pillar ?

105. The area of the cross section of a square rod is 4.84 sq. in. What is the width of the rod ? What is the perimeter of the rod ?

307. Square of the Sum of Two Numbers. Since $47 = 40 + 7$, the square of 47 may be obtained as follows :

$$\begin{array}{r}
 40 + 7 \\
 40 + 7 \\
 \hline
 (40 \times 7) + 7^2 \\
 40^2 + (40 \times 7) \\
 \hline
 40^2 + 2 \times (40 \times 7) + 7^2 \\
 = 1600 + 2 \times 280 + 49 \\
 = 1600 + 560 + 49 \\
 = 2209.
 \end{array}$$

280	49
1600	280
40	7
t	$+ u$

This relationship is seen in the accompanying figure, where the side of the square is $40 + 7$.

Every number consisting of two or more figures may be regarded as composed of tens and units.

Therefore, *the square of a number contains the square of the tens, plus twice the product of the tens and units, plus the square of the units.*

This is the most important principle in square root, and should be clearly understood, both from the multiplication and from the illustration, before proceeding.

Expressed as the square of $t + u$, we have $t^2 + 2t \times u + u^2$.

308. Separating into Periods. The first step in extracting the square root of a number is to separate the figures of the number into groups of two figures each, called *periods*.

Since $1 = 1^2$, $100 = 10^2$, $10,000 = 100^2$, and so on, it is evident that the square root of any number between 1 and 100 lies between 1 and 10; of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any integral number expressed by *one or two* figures is a number of *one* figure; expressed by *three or four* figures is a number of *two* figures; and so on.

If, therefore, an integral number is separated into periods of two figures each, from the right to the left, the number of figures in the square root will be equal to the number of the periods of figures. The last period at the left may have two figures or one figure; for example, 22 09, and 7 89 04 81.

309. Extracting the Square Root. The process of extracting the square root of a number not readily factored will now be considered.

For example, required the square root of 2209.

Separating into periods (§ 308), we see that there will be two integral places in the root.

The first period, 22, contains the square of the tens' number of the root. And since the greatest square in 22 is 16, therefore $\sqrt{16}$, or 4, is the tens' figure of the root.

22 09 (47
16

80 6 09

87 6 09

Subtracting the square of the tens, the remainder contains twice the tens $\times$ the units, plus the square of the units (§ 307). Therefore if we divide by twice the tens (that is, by 80, which is 2×4 tens), we shall find approximately the units. Dividing 609 by 80 (or 60 by 8), we have 7 as the units' figure.

Since twice the tens $\times$ the units, plus the square of the units, is equal to (twice the tens + the units) $\times$ the units, i.e. $2 \times 40 \times 7 + 7^2 = (2 \times 40 + 7) \times 7$, therefore we add 7 to 80 and multiply the sum by 7. The product is 609, thus completing the square of 47.

To check the work, $47^2 = 2209$.

EXERCISE 185

Extract the square root of:

- | | | | |
|---------|----------|----------|-----------|
| 1. 841. | 4. 1369. | 7. 3249. | 10. 8281. |
| 2. 961. | 5. 1681. | 8. 3721. | 11. 9801. |
| 3. 529. | 6. 1521. | 9. 1849. | 12. 6241. |

13. A square lot has an area of 6889 sq. ft. How many feet has it on a side?

14. A square piece of linoleum has an area of 37 sq. ft. 1 sq. in. How many inches has it on a side?

15. The bottom of a square box has an area of 484 sq. in. How long is it?

16. A square lot has an area of 7569 sq. ft. How many feet has it on a side?

310. Explanation by Diagram. It sometimes makes the reasoning in square root more clear if we consider a diagram.



FIG. 1



FIG. 2



FIG. 3

We have seen (§ 307) a figure showing the square on the lines $40 + 7$. In the explanation in § 309 we first took away from the whole square (as in Fig. 3) the square of the tens, or 1600, which is the square of 40 (represented by Fig. 1).

There then remained the two rectangles and the small square as seen in Fig. 3, the combined area being 609 square units. But these rectangles are as long as Fig. 1, and therefore 40 units long. Dividing their area, plus the area of the small square, by their length, we have approximately their width. (We would get their exact width if it were not for the small square.) This gives the units.

If we now add twice the tens to the units, we shall have the length of the two rectangles and the small square ($80 + 7 = 87$). Multiplying this by the units, the width of the rectangles, we have the area of the two rectangles and the small square ($87 \times 7 = 609$).

We see from the figures that the square is now entirely used; that is, the square of tens (Fig. 1) + twice the product of tens and units (making Fig. 2) + the square of the units (making Fig. 3) uses all the area. Therefore the square root is 47.

311. Square Root of Larger Numbers. The method of § 309 will apply to numbers of more than two periods, by considering *the part of the root already found as so many tens* with respect to the next figure. For example, required the square root of 7,890,481.

7 89 04 81 (2809	
4	
40	3 89
48	3 84
5600	5 04 81
5609	5 04 81

When the third period, 04, is brought down, and the divisor, 56, formed, the next figure of the root is 0, because 560 is not contained in 504. Therefore 0 is placed both in the root and in the divisor, and the next two figures, 81, are brought down.

312. Square Roots of Decimals. If the square root of a number has decimal places, the number itself will have twice as many.

Thus, if 0.25 is the square root of some number, the number will be 0.25^2 , or 0.25×0.25 , or 0.0625. Hence, if a given number contains a decimal, we separate it into periods of two figures each, beginning at the decimal point and proceeding toward the left for the integral part, and toward the right for the decimal. The last period of the decimal must have two figures, a zero being annexed if necessary.

Extract the square root of 52.2729.

We see at once that the root will have one integral place. Furthermore, if it is a perfect square it can have only two decimal places, since the square of thousandths would be millionths.

$$\begin{array}{r}
 52.27\ 29\ (7.23 \\
 49 \\
 \hline
 140\ \ 3\ 27 \\
 142\ \ 2\ 84 \\
 \hline
 1440\ \ \ 43\ 29 \\
 1443\ \ \ 43\ 29 \\
 \hline
 \end{array}$$

EXERCISE 186

Extract the square root of:

- | | | | |
|-------------|--------------|---------------|---------------|
| 1. 190,969. | 6. 804,609. | 11. 1036.84. | 16. 3.9204. |
| 2. 743,044. | 7. 194,481. | 12. 82.2649. | 17. 462.25. |
| 3. 401,956. | 8. 173,056. | 13. 0.063001. | 18. 0.003969. |
| 4. 758,641. | 9. 174,724. | 14. 1.5129. | 19. 0.182329. |
| 5. 117,649. | 10. 509,796. | 15. 2.6244. | 20. 0.054756. |

Find the side of a square whose area is:

- | | |
|--------------------|---------------------|
| 21. 12,321 sq. ft. | 22. 8046.09 sq. in. |
|--------------------|---------------------|

23. What is the perimeter of a square whose area is 1944.81 sq. in.?

24. The total area of the sides of a certain cube is found to be 355.74 sq. in. What is the length of an edge? What is the volume of the cube?

313. Approximate Square Roots. If a number is not a perfect square, zeros may be annexed and an approximate value of the root found.

For example, required to extract to three places of decimals the square root of 19.

19.00 00 00 (4.3588+	
16	
80	3 00
83	2 49
860	51 00
865	43 25
8700	7 75 00
8708	6 96 64
87160	78 3600

In this example we proceed in the usual way, annexing pairs of zeros for each decimal place in the root. We carry the work to four decimal places, so as to find the nearest approximation for three places. The result is, therefore, 4.359—; that is, it is nearer 4.359 than 4.358.

314. Summary of Square Root. We may summarize the process of square root as follows:

Separate the number into periods of two figures each, beginning at the decimal point.

Find the greatest square in the left-hand period and write its root for the first figure of the required root.

Square this root, subtract the result from the left-hand period, and to the remainder annex the next period for a dividend.

For a partial divisor double the root already found, considered as tens, and divide the dividend by it. The quotient (or the quotient diminished slightly if necessary) will be the next figure of the root.

To this partial divisor add this next figure of the root for a complete divisor. Multiply the complete divisor by this next figure of the root, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.

Proceed in this manner until all the periods have been thus annexed. The result will be the square root required.

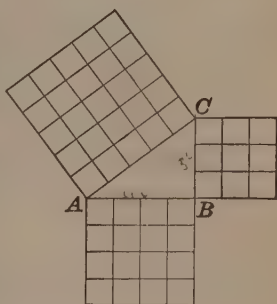
315. Square Roots of Common Fractions. To find the square root of a common fraction we may extract the square roots of the numerator and denominator.

If the denominator is not a perfect square, it is better to multiply both terms of the fraction by such a number as shall make it such, or else to reduce to a decimal and then extract the square root. For example, instead of saying $\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{1.414 +}$, which requires us to divide by a long divisor, it is better to say $\sqrt{\frac{1}{2}} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2} = \frac{1}{2}\sqrt{2} = \frac{1}{2}$ of $1.414 + = 0.707 +$, or else to extract the square root of 0.5.

316. Right Triangle. A triangle that contains a right angle is called a *right triangle*. It is sometimes called a *right-angled triangle*.

317. Hypotenuse. In a right triangle the side opposite the right angle is called the *hypotenuse*.

In the triangle ABC the hypotenuse is AC , the base is AB , and the perpendicular (or altitude) is BC .



318. Square on the Hypotenuse. By counting the small squares in this figure we see that the square on AC equals the sum of the squares on AB and BC . It is proved in geometry that this is true for all right triangles.

Therefore, *in a right triangle the square of the hypotenuse equals the sum of the squares of the other two sides.*

The hypotenuse equals the square root of the sum of the squares of the other two sides.

The base or the perpendicular equals the square root of the difference of the squares of the hypotenuse and the other side.

EXERCISE 187

Extract the square root of :

- | | | | | |
|-------------------|-------------------|--------------------|--------------------|---------------------|
| 1. $\sqrt{144}$. | 3. $\sqrt{361}$. | 5. $\sqrt{529}$. | 7. $\sqrt{1881}$. | 9. $\sqrt{2209}$. |
| 2. $\sqrt{121}$. | 4. $\sqrt{441}$. | 6. $\sqrt{1681}$. | 8. $\sqrt{1849}$. | 10. $\sqrt{3025}$. |

Extract the square root, to two decimal places :

- | | | | | |
|--------|---------|---------|----------|---------------------|
| 11. 2. | 13. 11. | 15. 30. | 17. 125. | 19. $\frac{5}{8}$. |
| 12. 7. | 14. 15. | 16. 50. | 18. 650. | 20. $\sqrt{12}$. |

Reduce to fractions with denominators perfect squares and extract the square root, to two decimal places :

- | | | | | |
|---------------------|---------------------|---------------------|---------------------|---------------------|
| 21. $\frac{1}{3}$. | 22. $\frac{2}{5}$. | 23. $\frac{3}{4}$. | 24. $\frac{5}{6}$. | 25. $\frac{7}{8}$. |
|---------------------|---------------------|---------------------|---------------------|---------------------|

Reduce to decimals and extract the square root, to two decimal places :

- | | | | | |
|---------------------|---------------------|---------------------|-------------------|-----------------------|
| 26. $\frac{5}{8}$. | 27. $\frac{4}{7}$. | 28. $\frac{3}{5}$. | 29. $\sqrt{12}$. | 30. $\frac{11}{12}$. |
|---------------------|---------------------|---------------------|-------------------|-----------------------|

Find the hypotenuse, given the other sides as follows :

- | | | |
|--------------------|--------------------|--------------------|
| 31. 39 ft., 52 ft. | 32. 21 ft., 72 ft. | 33. 51 ft., 68 ft. |
|--------------------|--------------------|--------------------|

Also as follows, carrying the results to three decimals :

- | | | |
|--------------------|--------------------|--------------------|
| 34. 82 ft., 35 ft. | 35. 31 ft., 23 ft. | 36. 27 ft., 43 ft. |
|--------------------|--------------------|--------------------|

Given the hypotenuse and one side, find the other side :

- | | | |
|-------------------|--------------------|--------------------|
| 37. 10 ft., 6 ft. | 38. 17 in., 15 in. | 39. 26 in., 10 in. |
|-------------------|--------------------|--------------------|

Also as follows, carrying the results to three decimals :

- | | | |
|-------------------|--------------------|--------------------|
| 40. 15 ft., 6 ft. | 41. 18 ft., 12 ft. | 42. 23 in., 12 in. |
|-------------------|--------------------|--------------------|

Find the diagonal of the square whose side is :

- | | | | | |
|----------|----------|----------|----------|----------|
| 43. 20". | 44. 32". | 45. 45". | 46. 70". | 47. 75". |
|----------|----------|----------|----------|----------|

319. Area of a Triangle in Terms of the Sides. It is proved in geometry that

To find the area of any triangle given the sides, multiply half the sum of the sides in succession by the three remainders obtained by subtracting each side separately from the half sum of the sides, and take the square root of this product.

For example, find the area of a triangle whose sides are respectively 5 in., 6 in., and 7 in.

The half sum of the sides is $\frac{1}{2}$ of $(5 + 6 + 7)$, or 9. Then $9 - 5 = 4$, $9 - 6 = 3$, and $9 - 7 = 2$. Therefore the area $= \sqrt{9 \times 4 \times 3 \times 2} = \sqrt{216} = 14.696 +$. That is, the area is 14.696 sq. in.

320. Similar Figures. It is also proved in geometry that

The areas of similar figures are to each other as the squares of their corresponding dimensions.

The corresponding dimensions of similar figures are to each other as the square roots of their areas.

That is, if a side of one square is 5 times as long as a side of another, the area is 25 times as much. If one circle has 16 times the area of another, the radius is 4 times as long.

EXERCISE 188

Find the areas (to two decimal places) of the triangles whose sides are:

- | | | |
|----------------|------------------|-------------------|
| 1. 5", 7", 8". | 3. 4", 3", 6". | 5. 12", 13", 15". |
| 2. 6", 8", 9". | 4. 9", 10", 11". | 6. 17", 21", 25". |

7. A triangle has an area of 275 sq. ft. What is the area of a similar triangle whose base is only half as long?

8. A rectangular lot has a front of 160 ft. and is worth \$1000. Find the value, at the same rate per square foot, of a similar lot which has twice the front and twice the depth.

321. Finding the Radius. Since the area of a circle equals 3.1416 times the square of the radius, therefore

The radius of a circle equals the square root of the quotient of the area divided by 3.1416.

For example, if the area is 78.54 sq. in., the number of inches in the radius is $\sqrt{\frac{78.54}{3.1416}} = \sqrt{25} = 5$. Therefore the radius is 5 in.

In Exercise 189 carry the square roots to two decimal places.

EXERCISE 189

Find the area of the circle whose radius is :

1. 17". 2. 29". 3. 37". 4. 5.27". 5. 6.07'.

Find the area of the circle whose diameter is :

6. 38". 7. 46". 8. 29". 9. 7.54". 10. 9.72'.

Find the radius of the circle whose area is :

11. 314.16 sq. in. 12. 113.0976 sq. in. 13. 250 sq. in.

14. What must be the diameter of a water pipe in order that the area of a cross section shall be 2 sq. in. ?

15. What must be the diameter of a water main in order that the area of a cross section shall be 3 sq. ft. ?

16. What must be the diameter of the piston head of an engine in order that the area may be 100 sq. in. ?

17. A cylindrical water tank is 20 ft. high and will contain 28,000 cu. ft. of water. What is the diameter ?

18. A horse tethered by a rope can graze over 1570.8 sq. ft. of ground. How long is the rope ? If the rope was twice as long, over how much ground could he graze ?

19. A tinsmith wishes to make some cylindrical gallon cans. They are to be 10 in. high. What must be the area of the base ? What radius must he use to draw the circle ?

EXERCISE 190

REVIEW

1. Find the square root of 119,716.
2. Find the square root of 170,569.
3. Find to three decimal places the square root of 6.
4. Find to three decimal places the square root of 30.
5. Determine the number of rods around a square field, the diagonal of which is 380 rd.
6. The diagonal of a square is 15 ft. Find to two decimal places the length of one side.
7. The side of a square is 15 ft. Find to two decimal places the length of the diagonal.
8. The area of a square is 37 sq. ft. Find to two decimal places the length of one side.
9. Separate 15,876 into prime factors, and from these factors determine the square root of the number.
10. The area of a circle is 201.0624 sq. in. Find the length of the radius.
11. The area of a circle is 153.9384 sq. ft. Find the length of the radius.
12. The foot of a 37-foot ladder is 12 ft. from the wall of a building against which the top rests. How high does the ladder reach on the wall?
13. How far from the wall of a house must the foot of a 50-foot ladder be placed in order that the top of the ladder may touch a window sill 48 ft. from the ground?
14. A rope stretched from the top of a 28-foot pole just reaches the ground 21 ft. from the foot of the pole. Assuming the rope to be straight, how long is it?

EXERCISE 191

PROBLEMS WITHOUT NUMBERS

1. How do you find the square of a number? the cube?
2. If you know the side of a square, how do you find the area?
3. If you know the area of a square, how do you find the side?
4. How do you find the square root of a number by factoring?
5. To what is the square of the sum of two numbers equal? How would you show it?
6. What is the object of separating a number into periods in extracting square root?
7. How do you proceed to extract the square root of an integer that is a perfect square?
8. How do you proceed to extract the square root of a number that contains decimal places?
9. How do you proceed to extract the square root of a common fraction?
10. How do you find the hypotenuse of a right triangle when the other sides are given?
11. How do you find the base of a right triangle, given the hypotenuse and the perpendicular?
12. How do you find the area of a triangle when the three sides are given?
13. What is the relation between the corresponding dimensions of similar figures?
14. Given the area of a circle, how do you find the radius?
15. How then do you find the circumference?

CHAPTER XIX

PRACTICAL MEASUREMENTS

322. Importance of the Rectangle. In measuring flat surfaces we usually have to deal with rectangles. A floor, a wall, a board, a desk top, and similar objects are usually rectangular. We studied the rectangle in Chapter VII, and we shall now find the areas of certain other rectangles that are found in practical life.

The tables of denominate numbers, given in Chapter VII, should be reviewed as a preparation for this work. The origin of several of the common measures is given in the Appendix on page 509.

EXERCISE 192

1. A schoolroom is 48 ft. 6 in. long and 28 ft. wide. How many square feet in the floor?

2. A sidewalk is 126 ft. 8 in. long and 5 ft. wide. How many square feet in the walk?

3. A ceiling 26 ft. 8 in. long and 18 ft. wide is to be plastered. How many square feet of plastering are needed? How many square yards?

Find the number of square yards in rectangles of these dimensions :

4. 27 ft. 6 in., 6 ft.

9. 16 ft. 4 in., 4 ft. 6 in.

5. 33 ft. 8 in., 21 ft.

10. 18 ft. 6 in., 6 ft. 8 in.

6. 37 ft. 9 in., 16 ft.

11. 25 ft. 4 in., 3 ft. 9 in.

7. 42 ft. 4 in., 18 ft.

12. 26 ft. 8 in., 16 ft. 4 in.

8. 75 ft. 9 in., 48 ft.

13. 36 ft. 9 in., 24 ft. 8 in.

323. Papering Rooms. The standard width of wall paper is 18 in., and single rolls are 8 yd. long, double rolls being 16 yd. long.

Imported papers vary from this standard, both in width and in length of roll. The frieze or border is usually sold by the yard.

It is not easy to estimate exactly the amount of paper needed for a room, on account of the difficulty of matching patterns, particularly when there are doors and windows with spaces above. In general, however,

Measure the height of wall space to be covered, find how many strips can be cut from a roll, and allow two strips to every yard in the perimeter of the room.

A fraction of a roll is to be counted as a whole roll. By a roll is meant a single roll, unless the contrary is stated.

Paper hangers often use this rule for rooms of ordinary height, those in which the distance from the baseboard to the border does not exceed 8 ft.:

Find the number of feet in the perimeter of the room, leaving out the widths of the doors and the windows, and allow a single roll for every $3\frac{1}{2}$ feet.

If there were no loss in matching, and we allowed 8 ft. to each strip, 1 roll (24 ft.) would make 3 strips, and would cover $4\frac{1}{2}$ ft. in width. But on account of loss in matching and spaces over doors and windows, this is a fairly approximate basis for an estimate.

For example, how many double rolls of paper will be required for a room of ordinary height, $16' \times 18'$, with one door and three windows, each 4 ft. wide?

$$\begin{array}{rcl} \text{Perimeter of room} & = 2 \times (16' + 18') & = 68' \\ \text{Width of door and windows} & = 4 \times 4' & = 16' \\ \text{Deducting door and windows} & & \underline{52'} \end{array}$$

Since 1 single roll is allowed to every $3\frac{1}{2}$ ft., 1 double roll will be allowed to every 7 ft. Since $\frac{52}{7} = 7\frac{3}{7}$, therefore 8 double rolls are needed.

In Exs. 1-6 of the following exercise use this paper-hangers' rule.

EXERCISE 193

1. How many rolls of paper will be required for a room $14' \times 15'$, with 1 door and 2 windows, each $3' 6''$ wide?

2. How many rolls of paper will be required for a room $14' \times 17'$, with 2 doors and 2 windows, each $3' 6''$ wide?

3. How many double rolls of paper will be required for a room $16' 3'' \times 18' 8''$, with 1 door and 3 windows, each $3' 6''$ wide?

4. How many double rolls of paper will be required for a room $15' 8'' \times 20' 4''$, with 2 doors and 3 windows, each $3' 8''$ wide?

5. How much will it cost to paper a room $13' \times 15'$, having 1 door and 1 window, each $3' 6''$ wide, the paper costing 35¢ a roll put on?

6. How much will it cost to paper a room $14' 4'' \times 15' 6''$, having 1 door and 2 windows, each $4'$ wide, the paper costing \$1.50 a double roll put on?

7. Find the cost of the border for each of the six rooms mentioned in the above examples, at 18¢ a yard, making no allowance for doors and windows.

8. Find the cost of plain paper for a room 16 ft. long by 15 ft. wide, using strips of 8 ft., at \$1.20 a double roll.

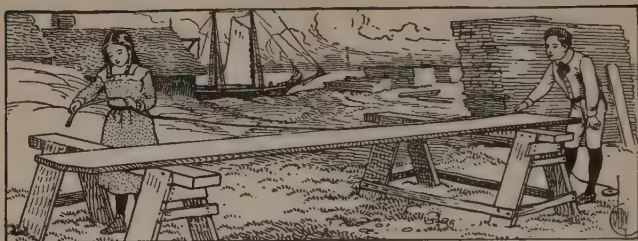
In Exs. 8 and 9, compute the exact cost by the first method given in § 323.

9. Find the cost of plain paper for a room 24 ft. long by 16 ft. 6 in. wide, using strips of 8 ft., at \$1.30 a double roll.

10. Find the cost of plain paper for the ceiling of a hall 48 ft. long and 24 ft. wide, at \$1.40 a double roll.

11. Find the cost of plain paper for a ceiling 12 ft. long and 9 ft. wide, at \$0.65 a roll.

The cost of papering rooms at home should be considered.



324. Board Measure. Boards one inch or less in thickness are sold by the square foot.

Boards more than one inch in thickness, and all squared lumber, are sold by the number of square feet of boards one inch in thickness to which they are equal.

Thus, a board 1 ft. long, 1 ft. wide, and 1 in. thick contains 1 *board foot*, or 1 ft. board measure. A board 12 ft. long, 1 ft. wide, and 1 in. or less in thickness contains 12 ft. board measure; but if the board is $1\frac{1}{4}$ in. thick, it contains $1\frac{1}{4} \times 12$ ft., or 15 ft.

Therefore, to find the board measure of lumber an inch or less in thickness, take the number of square feet on one side.

To find the board measure of lumber more than an inch in thickness, express the length and width in feet, and the thickness in inches. The product of these three numbers will be the number of feet, board measure.

In practice, the width of a board, unless sawed to order, is reckoned only to the next smaller half inch. Thus, a width of $11\frac{3}{8}$ in. is reckoned 11 in.; a width of $13\frac{5}{8}$ or $13\frac{3}{4}$ in. is reckoned $13\frac{1}{2}$ in.

If a board tapers, the average width is used, which the marker finds by taking the width at a point he considers about the average width.

How many feet in a $1\frac{7}{8}$ -inch plank 16 ft. long and 14 in. wide?

$$14 \text{ in.} = 1\frac{1}{2} \text{ ft.}$$

$$1\frac{7}{8} \times 1\frac{1}{2} \times 16 \text{ ft.} = 35 \text{ ft., board measure.}$$

In all such cases employ cancellation.

EXERCISE 194

Find the number of feet, board measure, in inch boards of length and width as follows:

- | | | |
|------------------|-------------------|-------------------------------|
| 1. 16 ft., 4 in. | 4. 16 ft., 9 in. | 7. 18 ft., $9\frac{1}{2}$ in. |
| 2. 14 ft., 6 in. | 5. 18 ft., 8 in. | 8. 16 ft., $6\frac{1}{2}$ in. |
| 3. 12 ft., 8 in. | 6. 16 ft., 10 in. | 9. 14 ft., $8\frac{1}{2}$ in. |

Find the number of feet, board measure, in the following:

10. 8 planks 2 in. thick, 10 ft. long, 12 in. wide.
11. 12 planks 3 in. thick, 14 ft. long, 10 in. wide.
12. 16 planks $2\frac{1}{2}$ in. thick, 16 ft. long, 10 in. wide.
13. 16 planks $1\frac{3}{4}$ in. thick, 10 ft. long, 9 in. wide.
14. 2 sticks of timber 18 ft. long, 1 ft. wide, 8 in. thick.
15. 5 sticks of timber 16 ft. long, 10 in. wide, 9 in. thick.
16. 25 joists 3" by 4" and 16 ft. long.
17. 30 joists 2" by 4" and 14 ft. long.
18. 50 joists 3" by 8" and 16 ft. long.
19. 8 beams 10" by 12" and 12 ft. long.
20. 30 boards $\frac{3}{4}$ in. thick, 8 in. wide, and 12 ft. long.
21. 50 boards $\frac{7}{8}$ in. thick, 10 in. wide, and 16 ft. long.
22. 80 boards 1 in. thick, 8 in. wide, and 14 ft. long.
23. 115 planks 2 in. thick, 10 in. wide, and 18 ft. long.
24. What is the cost of 8 spruce joists, $2" \times 6"$, 12 ft. long, at \$24 per M (that is, per 1000 ft. board measure)?
25. What is the cost of 24 spruce joists, $2" \times 8"$, 16 ft. long, at \$22.50 per M?
26. What is the cost of 6 spruce beams, $8" \times 10"$, 18 ft. long, at \$27 per M?

325. Flooring. Flooring is measured by the 100 sq. ft., or by the 1000 sq. ft.

An area of 100 sq. ft. is called a *square*. The usual estimate of a contractor is by the square, although for small jobs a carpenter is generally employed by the day.

Carpenters usually allow $1\frac{1}{4}$ feet, board measure, for every square foot of flooring to be covered, there being some loss on account of the tongue-and-groove matching of the boards. If the boards are less than 3 in. in width, carpenters usually allow $1\frac{1}{8}$ feet for every square foot.

For example, how many feet of $2\frac{1}{4}$ -inch flooring will be required for a room $14' \times 15'$?

The number of square feet is 14×15 . Allowing $1\frac{1}{8}$ ft. of flooring to every square foot of floor, we have

$$14 \times 15 \times \frac{9}{8} \text{ ft.} = 280 \text{ ft.}$$

EXERCISE 195

Find the number of feet of $2\frac{1}{4}$ -inch flooring material needed for rooms of the following dimensions:

- | | | |
|-----------------------|-----------------------|-----------------------|
| 1. $14' \times 18'$. | 4. $14' \times 20'$. | 7. $16' \times 24'$. |
| 2. $12' \times 14'$. | 5. $16' \times 20'$. | 8. $18' \times 20'$. |
| 3. $12' \times 16'$. | 6. $16' \times 22'$. | 9. $18' \times 24'$. |

Find the number of feet of $2\frac{1}{2}$ -inch flooring material needed for rooms of the following dimensions:

- | | | |
|------------------------|------------------------|------------------------|
| 10. $15' \times 16'$. | 12. $15' \times 20'$. | 14. $18' \times 22'$. |
| 11. $13' \times 16'$. | 13. $17' \times 20'$. | 15. $20' \times 24'$. |

16. What is the cost of $2\frac{1}{2}$ -inch birch flooring for a room $15' \times 18'$, at \$6.50 per 100 sq. ft. of floor surface?

17. What is the cost of laying a floor with $2\frac{1}{4}$ -inch maple boards, the room being $16' \times 24'$, the labor costing \$10.75, and the lumber costing \$48 per M?

326. Lathing. Laths are put up in bundles of 100, each 4 ft. long, and a bundle is estimated to cover 6 sq. yd.

In estimating the number of bundles required, deduct the area of all openings.

For example, how many bundles of laths will be required for the ceiling of a room $14' \times 17'$?

$$14 \times 17 \text{ sq. ft.} = 238 \text{ sq. ft.} = 26\frac{2}{3} \text{ sq. yd.}$$

$$26\frac{2}{3} \text{ sq. yd.} \div 6 \text{ sq. yd.} = 4\frac{1}{2}.$$

Therefore 5 bundles will be required.

EXERCISE 196

1. How many bundles of laths will be required for the walls and ceiling of a room $14' \times 16'$, the ceiling being 8 ft. above the baseboard, allowing 74 sq. ft. for openings?

2. How many bundles of laths will be required for the walls and ceiling of a room $16' \times 18'$, the ceiling being 8 ft. above the baseboard, allowing 80 sq. ft. for openings?

3. If laths are worth 50¢ per bundle and it costs 20¢ per bundle to put them on, how much will it cost to cover 80 sq. yd. of surface?

4. At 72¢ a bundle put on, how much will it cost to lath the walls and ceiling of a room $16' \times 20'$, the ceiling being 8 ft. above the baseboard, allowing 77 sq. ft. for openings?

5. At 69¢ a bundle put on, how much will it cost to lath the walls and ceiling of a room $16' \times 24'$, the ceiling being 8 ft. 6 in. above the baseboard, allowing 65 sq. ft. for openings?

6. At 66¢ a bundle put on, how much will it cost to lath the walls and ceiling of a schoolroom $30' \times 40'$, the ceiling being 14 ft. above the baseboard, allowing 172 sq. ft. for openings?

327. Shingling. Shingles are put up in bunches of 250, each shingle being 16 in. long and averaging 4 in. wide, so that a shingle laid $4\frac{1}{2}$ in. to the weather will cover 18 sq. in., and 8 shingles will cover 1 sq. ft., and 1000 shingles will cover 125 sq. ft.

Many carpenters allow 1000 shingles to 120 sq. ft., thus providing for waste. Others allow 1000 shingles to 100 sq. ft.

EXERCISE 197

Allowing 1000 shingles to 120 sq. ft., find the number of bunches required to cover roofs of the following areas :

- | | | |
|-----------------|-----------------|-----------------|
| 1. 1600 sq. ft. | 3. 1440 sq. ft. | 5. 1880 sq. ft. |
| 2. 1750 sq. ft. | 4. 1860 sq. ft. | 6. 1820 sq. ft. |

Allowing 1000 shingles to 110 sq. ft., find the number of bunches required to cover roofs of the following areas :

- | | | |
|-----------------|-----------------|-----------------|
| 7. 1680 sq. ft. | 8. 1620 sq. ft. | 9. 2160 sq. ft. |
|-----------------|-----------------|-----------------|

10. At \$1 a bunch, how much will the shingles cost for a space of 2000 sq. ft., allowing 1000 shingles to 125 sq. ft. ?

11. At \$1.10 a bunch, how much will the shingles cost to cover 1960 sq. ft., allowing 1000 shingles to 120 sq. ft. ?

12. If 4 bunches of shingles cover 125 sq. ft., and cost \$1.20 a bunch, and if we allow $1\frac{1}{4}$ lb. of cut nails to the bunch, at 4¢ a pound, and pay a man 30¢ an hour, and he uses a bunch of shingles an hour, how much will it cost to cover the two sides of a roof each 32' by 40' ?

13. Allowing 4 bunches of shingles at \$1.15 a bunch to every 115 sq. ft., 7 lb. of nails at $3\frac{1}{2}$ ¢ a pound to each 1000 shingles, and 4 hr. of labor at 30¢ an hour to 1000 shingles, how much will it cost to shingle a barn 60 ft. long, each side of the roof being 25 ft. wide ?

328. Paving. Paving and the laying of concrete walks and floors is estimated by the square foot or the square yard.

EXERCISE 198

Find the cost of paving:

1. A walk $5' \times 75'$, at 15¢ per square foot.
2. A walk $5' 6'' \times 90'$, at 22¢ per square foot.
3. A walk $5' 8'' \times 96'$, at 24¢ per square foot.
4. A walk $6' \times 66' 6''$, at 25¢ per square foot.
5. A walk $6' 6'' \times 42' 8''$, at 30¢ per square foot.
6. A walk $6' 8'' \times 68' 9''$, at 35¢ per square foot.

7. A walk $6' \times 32'$, using bricks laid on edge, the upper surface then being $2'' \times 8''$, the bricks being worth \$8 per M, the labor costing \$18.50, and the cost of preparing the bed for the bricks being \$5.

Find the number of square yards to be covered in street pavements of the following dimensions:

- | | |
|-------------------------|-----------------------------|
| 8. $38' \times 350'$. | 12. $36' 6'' \times 250'$. |
| 9. $41' \times 625'$. | 13. $38' 8'' \times 325'$. |
| 10. $37' \times 420'$. | 14. $41' 4'' \times 425'$. |
| 11. $42' \times 375'$. | 15. $42' 2'' \times 365'$. |

16. How much will it cost to pave a street 42 ft. wide and 625 ft. long, at \$11.40 per square yard?

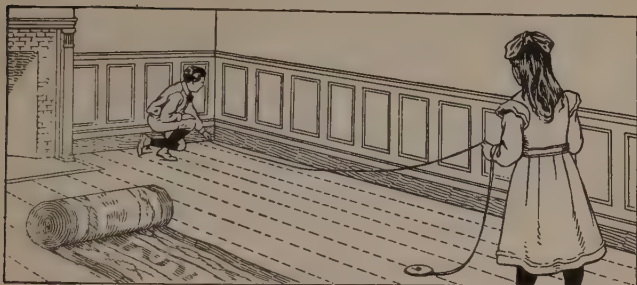
17. A city wishes to put a 6-foot sidewalk around the outside of a public square 175 ft. long and 125 ft. wide. How much will it cost, at \$2.25 per square yard?

18. A man wishes to put a 5-foot concrete sidewalk on two sides of his corner lot. The lot is 75 ft. long and 35 ft. wide. How much will it cost, at 75¢ per square yard?

EXERCISE 199

MISCELLANEOUS PROBLEMS

1. A room has nearly 500 sq. ft. of wall to be papered. Estimate the cost of the paper at 50¢ a roll.
2. A plain wall 14 ft. long and 9 ft. high above the baseboard is to be papered. How many rolls should you allow?
3. A room 15 ft. by 20 ft. is 9 ft. high above the baseboard. Deducting 30 sq. ft. for openings, how many rolls of paper should you allow for the walls?
4. The walls of a room are 8 ft. 6 in. above the baseboard, and the total length of the four walls, allowing for openings, is 72 ft. Allowing 6 in. on each strip for matching, what will the paper cost at 40¢ a roll?
5. How many rolls of plain paper will it take to cover the four sides of a room 20 ft. by 25 ft., and 12 ft. high, the paper being 24 in. wide, allowing $1\frac{1}{2}$ rolls for openings?
6. The walls of a room are to be papered with plain paper worth 30¢ a roll. The walls are 9 ft. high above the baseboard, and their total length, allowing for openings, is 60 ft. At the usual estimate, what will the paper cost?
7. How many square yards of plaster in the walls and ceiling of a room 15 ft. by 24 ft., and 10 ft. high, deducting 16 sq. yd. for openings and baseboard?
8. What will it cost to plaster a schoolroom 32 ft. long, 18 ft. wide, and 12 ft. high, at 32¢ a square yard, deducting 50% of the area of the walls for blackboards, baseboard, and openings?
9. There are 742 sq. yd. of ceiling in a certain school building. If the trustees let a contract for putting up pressed-steel ceilings at 12¢ a square foot, how much will the ceilings cost?



10. This room is $15' \times 18'$. At $65¢$ a yard, how much will it cost for matting 1 yd. wide to cover the floor, no allowances being made?

Measurements of this kind should be made by the class.

11. Find the cost of painting the sides, ends, and tops of 10 freight cars, $8' \times 34' \times 7'$, at $15¢$ a square yard.

12. At \$1.25 a yard, how much will it cost to carpet a room $12' \times 15'$ with plain carpet $\frac{3}{4}$ yd. wide?

13. At \$1.25 a yard, how much will it cost to cover the floor of a room $15' 9'' \times 17' 6''$ with 27-inch carpet, allowing 3 yd. for matching patterns?

14. Find the cost of painting the walls and ceiling of a room $14' \times 16' \times 10\frac{1}{2}'$, at $18¢$ a square yard, making no allowance for openings.

15. Find the cost of carpeting a room $27' \times 30'$ with carpet $\frac{3}{4}$ yd. wide, 3 yd. being allowed for matching, the carpet costing \$2.25 a yard.

16. Find the cost of carpeting a room $31' 6'' \times 37' 6''$ with carpet $\frac{3}{4}$ yd. wide, 2 yd. being allowed for matching, and the carpet costing \$2.45 a yard.

17. A room is 15 ft. by 16 ft. In which direction is it the more economical to lay a plain carpet that is 27 in. wide? Compute the number of yards of carpet to be bought.

18. How many square yards of plaster in the walls and ceiling of a hall 60 ft. long, 40 ft. wide, and 28 ft. high, deducting 36 sq. yd. for openings and baseboard ?

19. If $3\frac{1}{2}$ yd. of material 32 in. wide are required for a shirt waist, how much material 42 in. wide will be required, if it cuts as economically ?

20. If material 32 in. wide costs \$1.50 a yard, what should be the cost of material of the same quality 42 in. wide, at the same rate per square yard ?

21. How many yards of material 30 in. wide will be needed for a dozen aprons, each 29 in. long, allowing $4\frac{1}{2}$ in. for a hem and $2\frac{1}{2}$ in. for the waistband and finishing ? How much will the material cost at 22¢ a yard ?

22. A corridor is 33 ft. long, 4 ft. 6 in. wide, and 9 ft. high (above the baseboard). How many square feet of plaster are needed for the two sides and top, not allowing for openings ?

23. The wind pressure in a hurricane has been known to be as great as 49.2 lb. per square foot. In such a storm how many tons pressure on the side of a large office building 104 ft. long and 308 ft. high ?

24. Find the cost, at 19¢ a square yard, of painting the walls and ceiling of a room 15 ft. by $17\frac{1}{2}$ ft., and 10 ft. high, making no allowance for openings.

25. A painter, estimating the cost of a job, finds there are 925 sq. yd. of surface to be covered. His bid is at the rate of 16¢ per square yard. What is the amount of his bid ?

26. A painter, estimating the cost of a job, finds there are 1240 sq. yd. of surface to be covered. His bid is at the rate of $18\frac{1}{2}$ ¢ per square yard. What is the amount of his bid ?

27. How many square yards of plaster in the walls and ceiling of a room $16' \times 20'$, and 9' high, deducting 14 sq. yd. for openings and baseboard ?

329. Parallelogram and Triangle. As found on pages 106 and 107, the areas of the parallelogram and triangle may be stated briefly as follows:

Parallelogram: $\text{Area} = \text{base} \times \text{altitude}.$

Triangle: $\text{Area} = \frac{1}{2} \times \text{base} \times \text{altitude}.$

The proof by simple paper cutting should be reviewed before undertaking the more advanced applications here given.

EXERCISE 200

Find the areas of parallelograms with bases and altitudes as given :

- | | |
|-----------------------------|--------------------------------------|
| 1. 3 ft. 6 in., 4 ft. 1 in. | 3. 1 rd. $13\frac{1}{2}$ ft., 25 ft. |
| 2. 2 yd. 2 ft., 3 yd. 1 ft. | 4. 1 yd. 4 in., 16 in. |

Find the bases of parallelograms with areas and altitudes as given :

- | | |
|-------------------------------|-------------------------------------|
| 5. 5 sq. yd. 4 sq. ft., 7 ft. | 7. 3 sq. yd. 6 sq. ft., 3 yd. 2 ft. |
| 6. 2 sq. yd. 2 sq. ft., 3 ft. | 8. 1 sq. ft. 8 sq. in., 9 in. |

Find the areas of triangles with bases and altitudes as given :

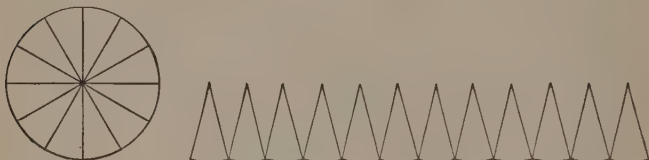
- | | |
|------------------------|-------------------------------------|
| 9. 3 ft. 8 in., 2 ft. | 11. 3 yd. 1 ft., 16 in. |
| 10. 6 ft. 6 in., 7 in. | 12. 2 rd. $3\frac{1}{2}$ ft., 8 in. |

Find the bases of triangles with areas and altitudes as given :

- | | |
|------------------------|------------------------|
| 13. 248 sq. in., 3 ft. | 15. 636 sq. in., 9 in. |
| 14. 265 sq. ft., 1 rd. | 16. 816 sq. ft., 4 yd. |

17. Draw a rectangle 4 in. long and 3 in. high. From the middle point of the top draw lines to the lower corners. Find the area of the rectangle ; of the large triangle ; of each of the two small triangles.

330. Area of a Circle. We may cut a circle as here shown and separate it into a series of figures that are triangles except for their curved bases. If we should call these figures triangles their altitudes would be the radius of the



circle, and the sum of their bases would be the circumference. The area of each triangle (§ 110) equals half the product of the numbers representing the base and altitude. Therefore their total area equals half the product of the numbers representing the circumference and radius.

In geometry this is proved exactly, but this explanation is enough for our purpose.

Thus, if the radius is 2 in. and the circumference (§ 118) is 12.5664 in., the area is $\frac{1}{2}$ of 2×12.5664 sq. in., or 12.5664 sq. in.

The area of a circle equals half the product of the numbers representing the circumference and radius.

331. Area in Terms of Radius. We have found that

	area = $\frac{1}{2} \times$ radius $\times$ circumference (§ 119)
and	circumference = $3.1416 \times 2 \times$ radius (§ 118);
therefore	area = $\frac{1}{2} \times$ radius $\times 3.1416 \times 2 \times$ radius,
or, canceling,	area = $3.1416 \times$ square of radius.

Therefore, *the area of a circle equals 3.1416 times the square of the radius.*

For example, if the radius is 2 in., the area is 3.1416×4 sq. in., or 12.5664 sq. in.

In Exercise 201 use 3.1416 instead of $3\frac{1}{7}$ (§ 118).

EXERCISE 201

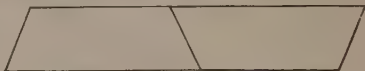
1. A dinner plate has a diameter of 10 in. What area does it cover?
2. A lamp has a base 7 in. in diameter. What area does it cover?
3. What is the area of the cross section of a round steel bar that is 2 in. in diameter?
4. How many square feet in the base of a cylindrical water tank that is 42 ft. in diameter?
5. A horse tethered by a rope 21 ft. long can graze over how many square feet of ground?
6. What is the area of a cross section of a water pipe the radius of which is $1\frac{7}{8}$ in.?
7. What is the area of the greatest cross section of an orange the diameter of which is 4 in.?
8. What is the area of the bottom of a glass jar the diameter of which is 5 in.?
9. A large cylindrical water tank has a diameter of 36 ft. What is the area of the surface of the water?
10. If the pupil of your eye is $\frac{1}{8}$ in. across, what is the area of the opening through which you see?
11. A pipe is 4.25 in. in external diameter. If the iron is 0.375 in. thick, what is the area of an internal cross section of the pipe?
12. If you tie a stone to a string, and swing the stone around so as to make a circle that has a radius of 3 ft., what will be the area of the circle?
13. A water pipe in one of the streets of New York City has a diameter of 3 ft. 4 in. What is the area of the cross section of the pipe?

332. Trapezoid. A four-sided plane figure with two parallel sides is called a *trapezoid*.

333. Area of a Trapezoid. In this figure two equal trapezoids have been placed together, one turned over so that the two make a parallelogram which equals a rectangle whose base is the sum of the two parallel sides.



Trapezoid



Therefore, *the area of a trapezoid equals half that of a rectangle with the same altitude and with a base equal to the sum of the two parallel sides.*

For example, the area of a trapezoid, with altitude 6 in. and parallel sides 10 in. and 15 in., is $\frac{1}{2}$ of $6 \times (10 + 15)$ sq. in., or 75 sq. in.

EXERCISE 202

Find the areas of trapezoids whose altitudes are first given, followed by the two parallel sides:

- | | |
|--------------------|-----------------------|
| 1. 6", 9" and 12". | 7. 13", 19" and 25". |
| 2. 8", 7" and 13". | 8. 14", 18" and 23". |
| 3. 9", 8" and 12". | 9. 19", 25" and 34". |
| 4. 9", 7" and 14". | 10. 25", 37" and 45". |
| 5. 8", 6" and 13". | 11. 29", 38" and 46". |
| 6. 7", 6" and 13". | 12. 44", 43" and 51". |

13. If the area of a trapezoid is 34,500 sq. ft., and the parallel sides are 500 ft. and 650 ft., what is the altitude?

14. If the area of a trapezoid is 44,000 sq. ft., the altitude 160 ft., and one of the parallel sides 450 ft., what is the other parallel side?

15. What is the area of a trapezoid one of whose parallel sides is 172 ft., the altitude being the same length and the other parallel side being twice as long?

Find the area of trapezoids the altitudes of which are given first, followed by the two parallel sides:

16. 6 in., 7 in. and 13 in. 18. 8 in., 3 in. and 12 in.

17. 10 in., 12 in. and 13 in. 19. 12 in., 7 in. and 8 in.



An irregular city block is divided into lots as shown. Find the areas, in square feet, of lots numbered as follows:

20. I. 22. III. 24. V. 26. VII. 28. IX.

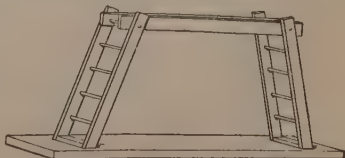
21. II. 23. IV. 25. VI. 27. VIII. 29. X.

In the figure the measurements are made to one decimal place.

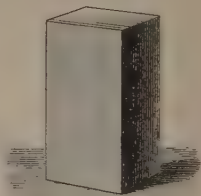
30. This apparatus for a doll's gymnasium is shaped like a trapezoid. The base is 12" long, the top 10", and the height 8". What is the area of the trapezoid?

31. If the base were 10", top 8", and height 6", what would be the area?

32. If the base were 8", the top 10", and the height 7", what would be the area?



334. Surface of a Prism. This figure is a *prism*. The side on which it stands is called the *base*. The top is called the *upper base*. The other sides are called the *lateral faces*. The lateral faces of this kind of prism are all rectangles. The area of all the lateral faces taken together is called the *lateral area*.



A *cube* (§ 114) is a special kind of prism, — one in which all of the faces are squares.

The base of a prism may be a square, any kind of parallelogram, a triangle, a trapezoid, and so on.

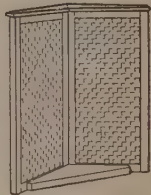
Only prisms the sides of which are rectangles are here considered.

EXERCISE 203

1. A certain prism has a square base 3 in. on a side. The prism is 6 in. high. Find the lateral area.

2. Another prism has for its base a parallelogram of which the length is 4 in. and the width 2 in. The height of the prism is 8 in. Find the lateral area and the total area.

3. A class in manual training made this lamp screen. It is like the two faces of a prism. It is $18\frac{3}{4}$ in. high, and each wing is $9\frac{3}{4}$ in. wide. What is the outside area of the two faces of the screen?



4. The base of this screen is a triangle of which one side is $12\frac{3}{4}$ in. and the altitude drawn to this side is $6\frac{1}{4}$ in. What is the area of the base?

5. The base of a prism is a square $3\frac{1}{2}$ in. on a side. The height is $5\frac{1}{2}$ in. Find the lateral area.

6. The base of a prism is a triangle with sides 3 in., 4 in., and 5 in. The height of the prism is 6 in. and the area of the base is 6 sq. in. Find the area of the entire surface.

7. A prism has a base with six equal sides, each side being $3\frac{1}{2}$ in. long. The height of the prism is 9 in. What is the lateral surface?

8. A beam is 8 in. by 10 in., and its length is 16 ft. It is to be dressed down and we wish to know the lateral area. What is this area?

9. A beam has for its cross section a trapezoid of which the parallel sides are 4 in. and 6 in. and the altitude is 2 in. The other two sides of the trapezoid are 2 in. and 2.8 in. The beam is 6 ft. long. What is the total area, including the two ends?

10. A prism has a base with a perimeter of 9 in. The area of the base is $5\frac{1}{8}$ sq. in. and the height of the prism is $10\frac{1}{2}$ in. What is the total area of the prism?

11. A rectangular water tank is 6 ft. long, 5 ft. wide, and 5 ft. deep, inside measure. It is to be lined with zinc. How many square feet of zinc will be needed, allowing 2 sq. ft. for joining and for turning over the top edge?

12. A pedestal is in the form of a prism. It is $4\frac{1}{2}$ ft. high and the perimeter is 1 ft. 9 in. If we wish to varnish the sides, how many square feet of surface must we cover?

Find the entire surfaces of cubes with edges as follows:

13. 15 in. 15. $3\frac{1}{2}$ in. 17. 2 ft. 1 in.

14. 27 in. 16. $4\frac{3}{4}$ in. 18. 3 ft. 6 in.

Find the lateral areas of prisms, the perimeters of the bases and the altitudes being as follows:

19. $9\frac{1}{8}$ ft., 4 ft. 23. $6\frac{1}{4}$ ft., $7\frac{1}{2}$ ft. 27. $1\frac{3}{8}$ in., $2\frac{3}{4}$ in.

20. $7\frac{2}{3}$ ft., 6 ft. 24. $3\frac{1}{2}$ ft., $4\frac{2}{3}$ ft. 28. $6\frac{1}{4}$ in., $5\frac{3}{5}$ in.

21. $4\frac{1}{2}$ ft., 7 ft. 25. $5\frac{1}{2}$ ft., $3\frac{1}{2}$ ft. 29. $5\frac{1}{4}$ in., $7\frac{3}{8}$ in.

22. $6\frac{2}{3}$ ft., 9 ft. 26. $5\frac{1}{2}$ ft., $6\frac{3}{4}$ ft. 30. $7\frac{1}{8}$ in., $6\frac{7}{8}$ in.

335. Linear Measure from Scale. In practical measurements it is often necessary to draw a figure to scale; that is, to make it a fractional part as large as the original.

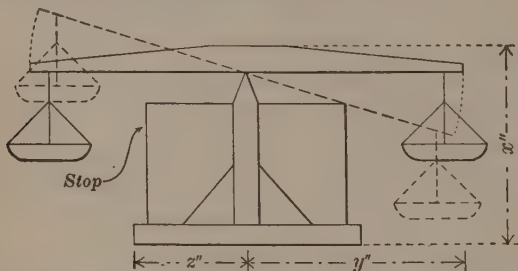
If a line is drawn only $\frac{3}{16}$ as long as the original, the scale is said to be 3 to 16, and this is written like a fraction, thus: $\frac{3}{16}$.

If a line in a drawing is 2 in. long, and the scale is $\frac{1}{50}$, the original line is 50×2 in., or 100 in. long.

EXERCISE 204

1. A New York skyscraper is represented in a drawing as 5 in. high. The drawing is made to the scale of 1 in. to 50 ft. What is the height of the building?

2. The class made some balances. These are the drawings and they are made to the scale $\frac{1}{8}$. Measure the drawings and determine the lengths marked x'' , y'' , and z'' .



3. A school building is represented in a plan as $4\frac{1}{2}$ in. by 8 in. The plan is drawn to the scale of $\frac{1}{40}$ in. to 1 ft. What are the dimensions of the building?

4. A plan for a school flower bed is drawn to the scale of $\frac{1}{10}$ in. to 1 ft. The plan measures $2\frac{1}{2}$ in. by $3\frac{1}{2}$ in. What are the dimensions of the flower bed?

5. In a class in industrial arts a set of balances is made. The drawing shows the arms as $1\frac{1}{8}$ in. long. The drawing is made to the scale $\frac{1}{10}$. What is the length of each arm?

Make drawings of rectangles of the following dimensions :

6. 10 ft. by 15 ft., scale 1 in. to 5 ft.
7. 25 ft. by 35 ft., scale 1 in. to 10 ft.
8. 35 ft. by 45 ft., scale 1 in. to 20 ft.

Make drawings of rectangular pieces of land of the following dimensions, and find the areas in acres :

9. 20 rd. by 35 rd., scale 1 in. to 5 rd.
10. 24 rd. by 36 rd., scale 1 in. to 12 rd. .

11. Make working drawings of the following, writing the scales used : a book cover ; a kite ; a tennis court.

12. Measure the size of the paper upon which you make your drawings. Draw a plan to the scale $\frac{1}{2}$.

13. Measure the size of any other book you are studying. Draw a plan of a page to the scale $\frac{1}{4}$.

14. The cover of a box is $9\frac{3}{8}$ in. long and $6\frac{3}{4}$ in. wide. Draw a plan to the scale $\frac{1}{2}$.

15. A drawing has been made of a box cover. The cover is 14 in. long and the drawing is 3.5 in. long. What scale was used ?

16. A drawing of a floor 24 ft. by 18 ft. is 3.6 in. by 2.7 in. What scale was used ?

17. A drawing is made of a rectangle. The drawing has an area of 6 sq. in. If a scale twice as large were used, what would be the area ? Draw a plan showing both rectangles, so as to show that the answer is correct.

18. A drawing is made of a lamp screen $18\frac{3}{4}$ in. high, and the drawing is $3\frac{1}{2}$ in. high. What scale was used ?

19. A drawing is made of one wing of a lamp screen. The drawing is $1\frac{5}{8}$ in. wide, and it is made to the scale $\frac{1}{4}$. How wide is the wing of the screen ?

20. A farm is plotted as here shown. The scale is 1 in. to 100 rd. Find the number of acres in the farm.

21. A rectangular meadow is plotted to the scale of 1 in. to 50 rd. The plot is 2 in. by 3.2 in. Find the number of acres in the meadow.

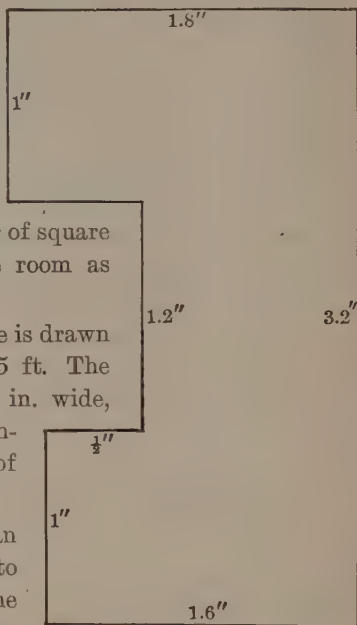
22. A plan for a school-room drawn to the scale of 1 in. to 50 ft. was 1 in. long and 0.7 in. wide. It was then decided to reduce the dimensions of the room 20%. What was the number of square feet of floor space in the room as finally built?

23. A plan for a playhouse is drawn to the scale of 1 in. to 2.5 ft. The plan is 6.4 in. long and 4 in. wide, and the playhouse is rectangular. Compute the area of the playhouse.

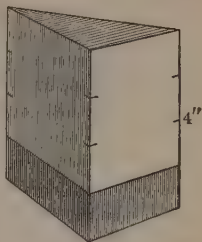
24. The class made a plan for a house. They drew it to the scale $\frac{1}{8}$ in., representing the kitchen as 6 in. long and 4 in. wide. What was the floor space of the proposed kitchen?

25. A plan for a rectangular flower bed is drawn to the scale $\frac{1}{8}$ in. The plan is 3 in. by 4.5 in. How many square feet are there in the area of the bed?

26. In a village park a rectangular space is to be filled. The drawings are made to the scale of 1 in. to 5 ft. They represent the space as 8 in. by 20 in., and the depth as 0.5 in. How many cubic feet of earth are needed to fill the space?



336. Volume of a Prism. In this figure it is evident that upon every square inch of the base there can be placed 1 cu. in., reaching 1 in. high. Therefore, if the prism is 4 in. high, the volume is 4 times the number of cubic inches in the shaded part.



Therefore, *the volume of a prism in cubic units equals the number of square units in the base, multiplied by the number of units in the altitude.*

If v = the volume, b = the base, and h = the height (or altitude), we may say that $v = bh$, two letters written together like this meaning that their values are to be multiplied, one by the other. It is easier to remember that $v = bh$, than to remember the above statement.

For example, find the volume of a prism with base 8 sq. in. and altitude 3 in.

$$3 \times 8 \text{ cu. in.} = 24 \text{ cu. in.}$$

EXERCISE 205

1. A prism has a triangular base containing $12\frac{1}{4}$ sq. in. The height of the prism is $7\frac{1}{2}$ in. What is the volume?

2. A prism has for its base a trapezoid. The parallel sides of the trapezoid are 6 in. and 4 in., and the altitude is 3 in. The height of the prism is 8 in. What is the volume?

3. A class measured a rectangular excavation and found it to be 6 ft. 2 in. wide, 8 ft. 4 in. long and 4 ft. 3 in. deep. How many cubic feet of earth have been removed?

Given one face of a cube, as follows, find the volume of the cube:

4. 36 sq. in.

6. 16.81 sq. in.

8. 17.64 sq. in.

5. 1.44 sq. in.

7. 26.01 sq. in.

9. 10.89 sq. in.

10. A class made this doll's swing. Each large upright piece is $1\frac{1}{4}$ in. by $1\frac{1}{4}$ in. by 12 in. How many cubic inches in each?

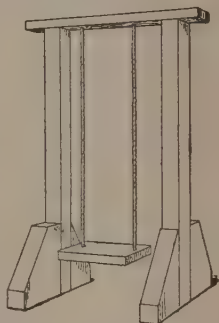
11. The crosspiece at the top is $1\frac{1}{4}$ in. by $\frac{1}{2}$ in. by 7 in. How many cubic inches in this piece?

12. The seat is $3\frac{1}{4}$ in. by $\frac{1}{2}$ in. by $2\frac{1}{4}$ in. How many cubic inches in the seat?

13. The base of a prism is 26.4 sq. ft. and the altitude is 9 ft. 8 in. Find the volume.

14. The base of a prism is a square $8\frac{1}{2}$ in. on a side. The altitude is 25% of the thickness. Find the volume.

15. The base of a prism is a rectangle $3\frac{1}{2}$ in. by $4\frac{3}{4}$ in. The altitude is 50% of the length of the base. Find the volume.



Find the volumes of prisms with these bases and altitudes :

16. 168 sq. ft., 25 ft.

21. 240 sq. ft., 25 ft.

17. 16.8 sq. ft., 2.5 ft.

22. 240 sq. ft., 75 ft.

18. 171 sq. ft., $33\frac{1}{3}$ ft.

23. 472 sq. ft., 50 ft.

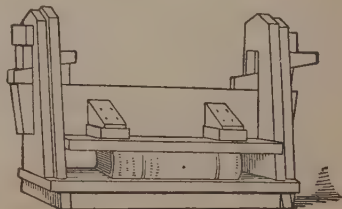
19. 176 sq. ft., $12\frac{1}{2}$ ft.

24. 480 sq. ft., $37\frac{1}{2}$ ft.

20. 174 sq. ft., $16\frac{2}{3}$ ft.

25. 840 sq. ft., $67\frac{1}{2}$ ft.

26. A boy made this book press for binding books. When the upper board is raised to admit a book, the space between it and the lower board is 10 in. by $13\frac{1}{2}$ in. by 6 in. How many cubic inches in this space?



Find the volumes of cubes with edges as follows :

- | | | | |
|-------------|------------------------|------------------------|------------------------------|
| 27. 4.2 in. | 30. $3\frac{3}{4}$ ft. | 33. $4\frac{5}{8}$ in. | 36. 2 ft. 3 in. |
| 28. 5.7 in. | 31. $4\frac{5}{8}$ in. | 34. $7\frac{3}{8}$ in. | 37. 1 ft. 7 in. |
| 29. 6.9 in. | 32. $6\frac{3}{8}$ in. | 35. $5\frac{7}{8}$ in. | 38. 1 ft. $9\frac{1}{2}$ in. |

Find the volumes of prisms with the following bases and altitudes :

- | | |
|------------------------------------|----------------------------|
| 39. 27 sq. in., 8 in. | 49. 28.6 sq. in., 17.5 in. |
| 40. 38 sq. in., $6\frac{1}{2}$ in. | 50. 32.9 sq. in., 18.3 in. |
| 41. 48 sq. in., $7\frac{3}{8}$ in. | 51. 62.7 sq. in., 27.4 in. |
| 42. 21.6 sq. in., 2.7 in. | 52. 43.6 sq. in., 19.8 in. |
| 43. 42.8 sq. in., 8.9 in. | 53. 27.9 sq. in., 16.2 in. |
| 44. 62.75 sq. in., 12.25 in. | 54. 35.4 sq. in., 22.7 in. |
| 45. 49.9 sq. in., 17.36 in. | 55. 73.2 sq. in., 35.8 in. |
| 46. 57.8 sq. in., 19.35 in. | 56. 75.3 sq. in., 27.8 in. |
| 47. 46.7 sq. in., 27.32 in. | 57. 48.9 sq. in., 19.7 in. |
| 48. 58.3 sq. in., 36.27 in. | 58. 56.7 sq. in., 23.4 in. |

59. An oat bin is 12 ft. long and 9 ft. wide. If there are 400 bu. of oats in the bin, what is the depth of the oats ?

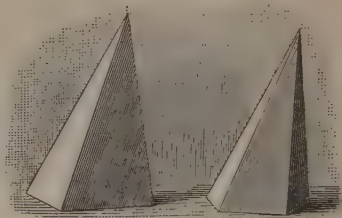
60. A rod has a triangular cross section of $4\frac{2}{3}$ sq. in. The rod is 3 ft. 8 in. long. How many cubic inches does it contain ?

61. An oat bin is 10 ft. long, 8 ft. wide, and 5 ft. deep. Allowing $1\frac{1}{4}$ cu. ft. to a bushel. how many bushels will the bin hold ?

62. A freight car is 8 ft. by 34 ft., and the interior is 7 ft. high. How many cubic feet does it contain ? If it is filled with grain to a height of 5 ft., what is the weight of the grain at 60 lb. to the bushel, allowing $1\frac{1}{4}$ cu. ft. to the bushel ?

337. Pyramid. A solid bounded by a polygon and by triangles that all meet in a point is called a *pyramid*.

The polygon is called the *base*, the triangles taken together are called the *lateral surface*, the point in which they meet is called the *vertex* of the pyramid, and the distance from the vertex to the base is called the *altitude*.



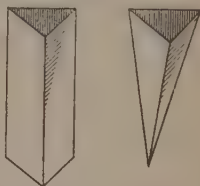
Pyramids

338. Volume of a Pyramid. *The volume of a pyramid equals one third the product of its base and altitude.*

As in § 336, we may indicate this by letters, thus: $v = \frac{1}{3}bh$.

The base and altitude are to be understood as abstract numbers, as usual.

The truth of the statement may be seen by taking a hollow pyramid and a hollow prism of the same base and altitude. The amount of water or sand that the pyramid will contain is exactly one third of the amount the prism will contain.



EXERCISE 206

Find the volumes of pyramids with bases and altitudes as follows:

- | | |
|------------------------|--|
| 1. 32 sq. in., 9 in. | 8. 536 sq. ft., 33 ft. |
| 2. 48 sq. in., 13 in. | 9. $8\frac{1}{4}$ sq. in., $2\frac{1}{2}$ in. |
| 3. 64 sq. in., 15 in. | 10. $9\frac{3}{8}$ sq. in., $3\frac{5}{8}$ in. |
| 4. 96 sq. in., 22 in. | 11. 8.7 sq. in., 3.1 in. |
| 5. 132 sq. in., 26 in. | 12. 12.3 sq. in., 4.7 in. |
| 6. 425 sq. in., 27 in. | 13. 15.75 sq. in., 8.1 in. |
| 7. 640 sq. in., 25 in. | 14. 12.25 sq. ft., 9.2 ft. |

Find the volumes of pyramids with bases and altitudes as follows :

- | | |
|------------------------------|--------------------------------------|
| 15. 751 sq. ft., 37 ft. | 22. 32.1 sq. ft., 1.9 ft. |
| 16. 433 sq. ft., 46 ft. | 23. 428 sq. ft., $17\frac{1}{2}$ ft. |
| 17. 370 sq. in., 19 in. | 24. 363 sq. ft., $12\frac{1}{4}$ ft. |
| 18. 63.9 sq. in., 21.5 in. | 25. 538 sq. in., $15\frac{7}{8}$ in. |
| 19. 45.8 sq. in., 12.8 in. | 26. 480 sq. in., $10\frac{3}{4}$ in. |
| 20. 129.3 sq. in., 3.72 in. | 27. 702.39 sq. in., 16.8 in. |
| 21. 202.11 sq. ft., 8.92 ft. | 28. 187.02 sq. ft., 10.7 ft. |

Find the bases of pyramids with volumes and altitudes as follows :

- | | |
|--|-------------------------------|
| 29. 89.6 cu. in., 12.8 in. | 34. 123.2 cu. in., 17.6 in. |
| 30. 178.2 cu. in., 19.8 in. | 35. 140.4 cu. in., 23.4 in. |
| 31. 114.8 cu. ft., 16.4 ft. | 36. 356.4 cu. ft., 32.4 ft. |
| 32. 137.6 cu. ft., 17.2 ft. | 37. 661.5 cu. ft., 31.5 ft. |
| 33. 189.9 cu. yd., $\overline{21.1}$ yd. | 38. 400 cu. yd., 33 yd. 1 ft. |

39. A stone pyramid has a base 9 ft. square. It is 16 ft. high. At 168 lb. per cubic foot, what does it weigh?

40. The Great Pyramid of Egypt has a square base 764 ft. on a side. It is 480 ft. 9 in. high. Assuming its faces smooth, what is its volume?

41. A church spire is in the form of a prism surmounted by a pyramid. The prism has a base of 1575 sq. ft. and is 82 ft. high. The pyramid has a base of 1486 sq. ft. and is 69 ft. high. How many cubic feet of space does the spire fill?

42. A church spire is in the form of a prism with a square base, surmounted by a pyramid 70 ft. high. The prism is 78 ft. high, and its base is 30 ft. on a side. The base of the pyramid is a square 27 ft. on a side. Find the volume of the spire.

339. Cylinder. We have seen (§ 122) that we may find the volume of a cylinder thus:

Multiply the number of cubic units corresponding to the number of square units of the base by the number of units of altitude.

That is, as in § 336 and § 338, $v = bh$.

For example, if the base has 22 sq. in., and the altitude is 14 in., the volume is 14×22 cu. in., or 308 cu. in.

If the base has a diameter of 6 in., and the altitude is 8 in., then the area of the base is 3.1416×9 sq. in. (§ 331), and the volume is $8 \times 3.1416 \times 9$ cu. in., or 226.1952 cu. in.

We have also seen (§ 123) that we may find the lateral area of a cylinder thus:

Multiply the number of units in the circumference by the number of units in the altitude.

That is, as in § 336 and § 338, if a = the lateral area, c = the circumference, and h = the altitude, we have $a = ch$.

For example, if the circumference is 16 in. and the altitude 9 in., the number of square units of lateral area is 9×16 , or 144. We may express the work thus:

$$9 \times 16 \text{ sq. in.} = 144 \text{ sq. in.}$$

EXERCISE 207

1. If we slit the curved surface of this cylinder from top to bottom and spread it out flat, what will it become? How may we find the area of this surface?

2. What is the area of the curved surface of a cylinder 6 in. high and 8 in. around? 9 in. high and 9 in. around?



3. How many square feet of tin are needed for a pipe 8 ft. long and 6 in. around, allowing 1 sq. ft. for overlapping?

4. How much tin is needed for a pipe 8 ft. 6 in. long and $7\frac{3}{4}$ in. around, allowing 100 sq. in. for overlapping?

5. If a rolling-pin $10\frac{3}{4}$ in. long and 9 in. around is rolled along a table top, making exactly one revolution, over how many square inches has it rolled?

6. How many square feet in the surface of a piece of steam pipe $6\frac{1}{2}$ in. around and 6 ft. long?

7. How many cubic inches of water will it take to fill a pipe the cross section of which is $2\frac{3}{8}$ sq. in. and the length 56 in.?

8. Some boys measured the circumference of a cylindrical water tank. They found it to be 48 ft. What is the radius of the base?

9. What is the area of the base of the tank in Ex. 8?

10. If the tank in Ex. 8 is 30 ft. high, what is its capacity?

Find the volumes of cylinders with altitudes and areas of base as follows:

11. 2.5 in., 23.8 sq. in.

13. 4 ft. $3\frac{1}{2}$ in., $6\frac{7}{8}$ sq. in.

12. 9.8 in., 29.6 sq. in.

14. 5 ft. $7\frac{1}{2}$ in., $9\frac{5}{8}$ sq. in.

Find the lateral areas of cylinders with altitudes and circumferences as follows:

15. 25 in., 80.3 in.

17. $33\frac{1}{3}$ in., $69\frac{1}{2}$ in.

16. 75 in., 40.5 in.

18. $50\frac{3}{4}$ in., 106 in.

19. How many square inches in the curved surface of a wire $1\frac{7}{8}$ in. in circumference and 200 ft. long?

20. A tin cup is $7\frac{7}{8}$ in. in circumference and $3\frac{1}{3}$ in. high, both measures including allowance for soldering. How many square inches of tin are needed for the curved surface?

21. A tin water pipe has a circumference of $9\frac{7}{8}$ in. and a length of 10 ft. 6 in., both measures including allowance for soldering. How many square inches of tin are needed?

Find the volumes of cylinders whose altitudes and areas of base are as follows :

- | | |
|-------------------------|---|
| 22. 23 in., 37 sq. in. | 25. 4 ft. 7 in., $9\frac{1}{2}$ sq. in. |
| 23. 27 in., 46 sq. in. | 26. 4 ft. 3 in., $8\frac{1}{2}$ sq. in. |
| 24. 48 in., 175 sq. in. | 27. 8 ft. 3 in., 5 sq. ft. 24 sq. in. |

Find the altitudes of cylinders whose volumes and areas of base are as follows :

- | | |
|-----------------------------------|--|
| 28. 104 cu. in., 16.25 sq. in. | 30. 127.8 cu. in., 17.75 sq. in. |
| 29. 157.29 cu. in., 37.45 sq. in. | 31. $141.3\frac{3}{4}$ cu. in., 19.5 sq. in. |

Find the areas of base of cylinders whose volumes and altitudes are as follows :

- | | |
|---------------------------|-------------------------------|
| 32. 81 cu. ft., 2.5 ft. | 34. 3931.04 cu. ft., 31.1 ft. |
| 33. 51.6 cu. ft., 3.5 ft. | 35. 9132.42 cu. ft., 41.1 ft. |

Find the volumes of cylinders whose altitudes and radii of base are as follows :

- | | | |
|-----------------------|-----------------------|------------------------|
| 36. 36.4 in., 7.6 in. | 38. 8.75 ft., 3.4 in. | 40. 12.4 ft., 3.1 ft. |
| 37. 27.9 in., 5.7 in. | 39. 9.25 ft., 2.1 ft. | 41. 15.25 ft., 5.4 ft. |

Using $7\frac{1}{2}$ gal. for 1 cu. ft., find the number of gallons in cylindrical tanks of the following diameter and depth of water respectively :

- | | | |
|--|--|------------------------|
| 42. $7\frac{1}{2}$ ft., $3\frac{1}{2}$ ft. | 44. $12\frac{1}{2}$ ft., $10\frac{3}{4}$ ft. | 46. 62.7 ft., 35.4 ft. |
| 43. $9\frac{1}{4}$ ft., $4\frac{1}{2}$ ft. | 45. 23.8 ft., 31.1 ft. | 47. 59.8 ft., 33.3 ft. |

Find the number of gallons of water in wells of the following diameter and depth of water respectively :

- | | | |
|--|--|--|
| 48. $2\frac{1}{2}$ ft., $6\frac{1}{4}$ ft. | 51. $3\frac{1}{8}$ ft., $6\frac{1}{2}$ ft. | 54. $8\frac{1}{8}$ ft., $7\frac{1}{2}$ ft. |
| 49. $3\frac{1}{4}$ ft., $7\frac{1}{2}$ ft. | 52. $5\frac{1}{8}$ ft., $4\frac{3}{8}$ ft. | 55. $9\frac{7}{8}$ ft., $9\frac{3}{8}$ ft. |
| 50. 4 ft., 7 ft. | 53. $4\frac{1}{4}$ ft., 9 ft. | 56. $3\frac{5}{12}$ ft., $9\frac{7}{12}$ ft. |

340. Capacity of Cisterns. The following measurements, used in computing the capacity of cisterns, are here repeated for convenience of reference :

$$1 \text{ gal.} = 231 \text{ cu. in.}$$

$$1 \text{ cu. ft.} = 7.48 \text{ gal.} + = \text{nearly } 7\frac{1}{2} \text{ gal.}$$

$$31\frac{1}{2} \text{ gal.} = 1 \text{ bbl.} = \text{nearly } 4\frac{1}{2} \text{ cu. ft.}$$

On this page use $7\frac{1}{2}$ gal. for 1 cu. ft., $4\frac{1}{2}$ cu. ft. for 1 bbl.

EXERCISE 208

1. How many gallons in a tank containing 462 cu. in. ?
693 cu. in. ? 2310 cu. in. ? 1155 cu. in. ?

2. How many gallons in a tank containing 1 cu. ft. ?
2 cu. ft. ? 10 cu. ft. ? 100 cu. ft. ? 10,000 cu. ft. ?

Find the number of gallons in these rectangular tanks :

3. $4.1 \text{ ft.} \times 3.2 \text{ ft.} \times 1.5 \text{ ft.}$ 5. $9.5 \text{ ft.} \times 7.4 \text{ ft.} \times 3.2 \text{ ft.}$

4. $5.8 \text{ ft.} \times 2.4 \text{ ft.} \times 1.5 \text{ ft.}$ 6. $12.5 \text{ ft.} \times 8.8 \text{ ft.} \times 5.5 \text{ ft.}$

Find the number of barrels in these rectangular tanks :

7. $16.4 \text{ ft.} \times 12.8 \text{ ft.} \times 10.5 \text{ ft.}$ 8. $20.8 \text{ ft.} \times 17.5 \text{ ft.} \times 14.25 \text{ ft.}$

Find the number of gallons in cylindrical tanks of the following diameter and depth of water respectively :

9. 6.4 ft., 12.2 ft. 11. 17.4 ft., 16.5 ft. 13. 36.8 ft., 27.25 ft.

10. 14.5 ft., 13.4 ft. 12. 26.8 ft., 16.25 ft. 14. 52.7 ft., 36.5 ft.

15. A cylindrical water tank is 40 ft. high and the base has a diameter of 38 ft. How many gallons will it hold ?

16. A water main 1 mi. long has a diameter of 28 in. If the main is full, how many gallons does it contain ?

17. A cylindrical water tank is to have a base with a diameter of 45.5 ft. How high must it be to hold 32,175 gal. ?

341. Cone. A solid bounded by a circle and a curved surface ending in a point is called a *cone*.

The circle is called the *base*, the curved surface is called the *lateral surface*, the point in which the lateral surface ends is called the *vertex*, and the distance from the vertex to the base is called the *altitude* of the cone.

In the cones here treated the vertex is directly over the center of the base.

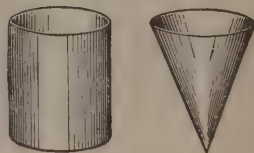
342. Volume of a Cone. *The volume of a cone equals one third the product of its base and altitude.*

For if made hollow, as in § 338, the contents of the cone will fill exactly one third of the cylinder of the same base and altitude.

343. Illustrative Problem. Find the volume of a cone whose base has a radius of 2 in., and whose altitude is 3 in.

$$\text{Area of base} = 3.1416 \times 2^2 \text{ sq. in.}$$

$$\text{Volume} = \frac{1}{3} \times 3 \times 3.1416 \times 2^2 \text{ cu. in.} = 12.5664 \text{ cu. in.}$$



Cone

EXERCISE 209

Find the volumes of cones with bases and altitudes as follows:

1. 6 sq. in., 4 in.

2. 15 sq. in., 7 in.

3. 20 sq. in., 9 in.

4. 34 sq. in., 15 in.

5. 68 sq. in., 21 in.

6. $6\frac{3}{4}$ sq. in., $2\frac{1}{2}$ in.

7. $9\frac{7}{8}$ sq. in., $3\frac{3}{4}$ in.

8. $15\frac{3}{4}$ sq. in., $5\frac{3}{8}$ in.

9. 22.7 sq. in., 8.4 in.

10. 35.75 sq. in., 12.3 in.

11. What is the volume of a cone whose base is a circle with a radius of 3 in., and whose altitude is 8 in.?

12. A funnel in the shape of a cone is closed at the bottom. If the circular top is 3 in. across and the funnel is 4 in. deep (to the vertex), how many cubic inches of water will it hold?

13. A heap of earth is in the form of a cone the base of which is 125 sq. ft. and the altitude of which is 5 ft. How many cubic feet are there in the heap?

14. A cornucopia is conical in shape. The area of the opening is $6\frac{1}{2}$ sq. in. and the altitude is $7\frac{1}{4}$ in. How many cubic inches does it contain?

15. The base of a cone is 27 sq. in., and the volume is 81 cu. in. What is the altitude?

Find the altitudes of cones with bases and volumes as follows:

- | | |
|-------------------------------|---------------------------------|
| 16. 36 sq. in., 48 cu. in. | 20. 9.3 sq. ft., 21.7 cu. ft. |
| 17. 42 sq. in., 98 cu. in. | 21. 12.3 sq. ft., 41 cu. ft. |
| 18. 5.4 sq. in., 10.8 cu. in. | 22. 15.9 sq. yd., 58.3 cu. yd. |
| 19. 6.3 sq. in., 12.6 cu. in. | 23. 33.6 sq. yd., 100.8 cu. yd. |

24. If 40 cu. ft. of earth is heaped up in the form of a cone 4 ft. high, how many square feet will it cover?

Find the bases of cones with altitudes and volumes as follows:

- | | |
|--------------------------|---------------------------|
| 25. 7 ft., 28 cu. ft. | 28. 9 in., 63 cu. in. |
| 26. 9 ft., 45 cu. ft. | 29. 1.1 in., 1.21 cu. in. |
| 27. 1.8 ft., 6.6 cu. ft. | 30. 0.9 yd., 9.9 cu. yd. |

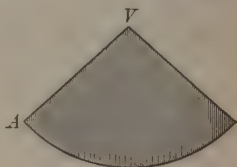
31. A church spire is in the form of a cylinder surmounted by a cone. The base of the cylinder is 1275 sq. ft. and the altitude is 70 ft. The base of the cone is 1150 sq. ft. and the altitude is 62 ft. How many cubic feet of space does the spire fill?

344. Lateral Surface of a Cone. If we should slit the surface of this cone from top to bottom and flatten it out, as here shown, we would have part of a circle. Calling the line VA in the figure the *slant height*,

The lateral area of a right circular cone equals the circumference of the base multiplied by half of the slant height.

As in § 336 and § 338, we may express this in letters, using initials, thus: $a = \frac{1}{2}cs$.

For example, if the circumference of the base of a cone is 4 in. and the slant height is 3 in., the lateral area is $\frac{1}{2}$ of 3×4 sq. in., or 6 sq. in.



EXERCISE 210

1. What is the lateral area of a cone if the circumference of the base is 10 ft. and the slant height is 8 ft.?
2. If the slant height of a cone is 4.2 in. and the circumference of the base is 6.5 in., what is the lateral area?

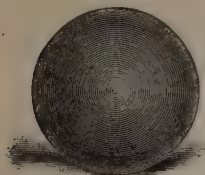
Find the lateral areas of cones with the following circumferences and slant heights:

- | | |
|-------------------|---------------------------------|
| 3. 10 in., 5 in. | 6. 16.5 in., $4\frac{7}{8}$ in. |
| 4. 12 in., 8 in. | 7. 6 ft. 8 in., 1 yd. |
| 5. 20 in., 10 in. | 8. 2 ft. 7 in., 19 in. |

9. What is the entire area of a cone, including the base, the slant height of the cone being 7 ft. and the diameter of the base 7 ft.?

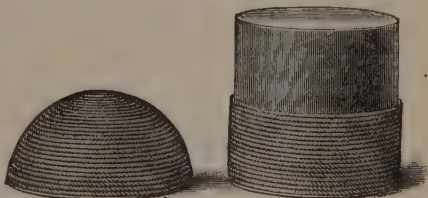
10. A dome is surmounted by a gilded cone the slant height of which is 16 ft. 8 in. and the radius of the base 3 ft. 6 in. How many square feet must be allowed for gilding?

345. Sphere. A solid bounded by a curved surface, all points of which are equally distant from a point within, is called a *sphere*.



The point within from which all points on the surface are equally distant is called the *center*. A straight line from the center to the surface is called a *radius*. A straight line through the center and terminated at each end by the surface is called a *diameter*.

346. Surface of a Sphere. If we wind half of the surface of a sphere with a cord or tape as here shown, and then wind with exactly the same length of the cord or tape the surface of a cylinder having just the same radius, the height of the cylinder equaling the diameter, we find that the cord covers half the curved surface of the cylinder.



Therefore, *the surface of a sphere equals the curved surface of a cylinder of the same radius and height.*

The surface of a sphere also equals 3.1416 times the square of the diameter.

For it is easily seen that the curved surface of a cylinder equals the height times the circumference of the base, and since the height here equals the diameter, therefore

$$\begin{aligned} \text{height} \times \text{circumference} &= \text{diameter} \times 3.1416 \times \text{diameter} \\ &= 3.1416 \times \text{the square of the diameter.} \end{aligned}$$

For practical purposes it is usually sufficient to take $3\frac{1}{7}$ instead of 3.1416 as the multiplier.

If we have given the surface of a sphere, we may find the diameter by dividing the surface by 3.1416, or $3\frac{1}{7}$, and extracting the square root of the quotient.

347. Illustrative Problem. What is the surface of a sphere with a radius of 3 in.?

$$\text{Diameter} = 2 \times 3 \text{ in.} = 6 \text{ in.}$$

$$\text{Area} = 3.1416 \times 6^2 \text{ sq. in.}$$

$$= 3.1416 \times 36 \text{ sq. in.}$$

$$= 113.0976 \text{ sq. in.}$$

EXERCISE 211

Using $3\frac{1}{7}$ as the multiplier, find the areas of the surfaces of spheres with the following diameters:

- | | | | |
|-----------|------------|----------------------|-------------|
| 1. 14 in. | 4. 2.1 in. | 7. $\frac{7}{8}$ in. | 10. 224 in. |
| 2. 35 in. | 5. 2.8 in. | 8. 0.7 in. | 11. 371 in. |
| 3. 42 in. | 6. 4.9 in. | 9. 0.56 in. | 12. 434 in. |

Find the same, using 3.1416 as the multiplier:

- | | | | |
|-----------|-----------|------------------------|-------------|
| 13. 7 in. | 15. 9 in. | 17. $7\frac{1}{2}$ in. | 19. 2.2 in. |
| 14. 6 in. | 16. 4 in. | 18. $6\frac{1}{4}$ in. | 20. 3.5 in. |

Find the same, using 3.1416, the following being the radii:

- | | | | |
|-----------|-----------------------|-----------|-------------|
| 21. 1 in. | 23. $\frac{1}{2}$ in. | 25. 7 in. | 27. 2.5 in. |
| 22. 5 in. | 24. 3 ft. | 26. 9 in. | 28. 3.7 in. |

29. If the earth is considered a sphere of 4000 mi. radius, what is its surface? (Use $3\frac{1}{7}$.)

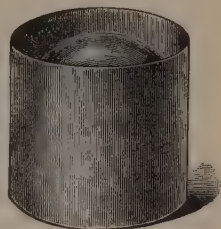
30. How many square inches of flannel will it take to cover a tennis ball 3 in. in diameter? (Use $3\frac{1}{7}$.)

31. If the sun is considered a sphere of 866,500 mi. diameter, what is its surface? (Use $3\frac{1}{7}$.)

32. How many square inches in the surface of a ball 44 in. in diameter? (Use $3\frac{1}{7}$.)

33. What is the diameter of a sphere whose surface is 154 sq. in.? (Use $3\frac{1}{7}$.)

348. Volume of a Sphere. If a sphere is placed in a cylinder of the same diameter and with an altitude equal to the diameter, and the vacant spaces are filled with sand, the sand will be found to fill exactly one third of the cylinder after the sphere is removed.



Therefore, *a sphere is equal to two thirds of a cylinder of the same diameter and altitude.*

The volume of a sphere is $\frac{4}{3} \times 3.1416 \times$ the cube of the radius.

For the volume of the cylinder is the product of the altitude and the base, considered as abstract numbers.

The altitude $= 2 \times$ radius,
and the base $= 3.1416 \times$ square of radius.

Hence volume of cylinder $= 2 \times 3.1416 \times$ cube of radius,
and volume of sphere $= \frac{2}{3}$ of $2 \times 3.1416 \times$ cube of radius
 $= \frac{4}{3} \times 3.1416 \times$ cube of radius.

349. Illustrative Problem. Find the volume of a sphere with a radius of 3 in.

$$\frac{4}{3} \times 3.1416 \times 3^3 \text{ cu. in.} = \frac{4 \times 3.1416 \times 3 \times 3 \times 3}{3} \text{ cu. in.} \\ = 113.0976 \text{ cu. in.}$$

EXERCISE 212

Find the volumes of spheres with radii as follows:

- | | | | |
|----------|-----------|------------|------------------------|
| 1. 2 in. | 4. 15 in. | 7. 2.4 ft. | 10. $2\frac{1}{2}$ in. |
| 2. 5 in. | 5. 23 in. | 8. 4.1 ft. | 11. $3\frac{3}{4}$ in. |
| 3. 7 in. | 6. 42 in. | 9. 3.5 ft. | 12. $4\frac{2}{3}$ in. |

13. Find the weight of a lead ball 2 in. in diameter, lead being 11.35 times as heavy as water, and a cubic foot of water weighing 1000 oz.

14. If a ball has a radius of $2\frac{1}{4}$ in., what is its volume?
15. The sun's diameter is 866,500 mi. Find its volume.
16. If a tennis ball is $2\frac{3}{8}$ in. in diameter, what is its volume?
17. Find the weight of a sphere of marble the diameter of which is 1 ft., marble being 2.7 times as heavy as water.
18. Find the weight of a sphere of glass the diameter of which is 7 in., glass being 2.89 times as heavy as water.
19. If the moon is considered a sphere of radius 1080 mi., what is its volume?
20. A ball is to be put on top of a tower. The diameter of the ball is 2 ft. 6 in. How many cubic inches of space will the ball occupy?
21. Find the volume of a sphere the diameter of which is 6 in.; also of one the diameter of which is 12 in.
22. A ball 8 in. in diameter is placed in a cubical box 8 in. on an edge, inside measure. How many cubic inches of the box is not occupied by the ball?
23. A ball 6 in. in diameter is placed in a cylinder of which the altitude is 6 in. and the diameter of the base is also 6 in. How many cubic inches of the cylinder is not occupied by the ball?
24. Find the volume of a sphere 1 ft. in diameter, of a cylinder of the same diameter and altitude, and of a cone of the same diameter and altitude. The volume of the sphere is how many times the volume of the cone? The volume of the cylinder is how many times the volume of the cone? Compare the volumes of the cylinder and sphere.
25. A sphere and a cylinder are made of iron. They have the same diameter, and the altitude of the cylinder equals the diameter of the sphere. If the sphere weighs 25.8 lb., what does the cylinder weigh?

EXERCISE 213

PROBLEMS WITHOUT NUMBERS

1. If you have determined by careful measurement the outside dimensions of a box that shall exactly fit a given space, how will you determine the dimensions of the various pieces of wood to be used in making the box?

2. By measuring the size of the ice chamber, how can you determine the number of pounds of ice that can be put into an ice box? (Allow 58 lb. to a cubic foot of ice.)

3. By measuring the inside dimensions, how can you determine the number of gallons of water that can be put into a rectangular tank? (Allow 231 cu. in. to a gallon.)

4. What measurements do you need to make in order to determine the area of a rectangle?

State the measurement or measurements necessary in order to determine the following, and tell how it is determined:

- | | |
|-----------------------------|---------------------------|
| 5. Area of a square. | 10. Volume of a cube. |
| 6. Area of a parallelogram. | 11. Volume of a pyramid. |
| 7. Area of a triangle. | 12. Volume of a cone. |
| 8. Area of a trapezoid. | 13. Volume of a cylinder. |
| 9. Area of a circle. | 14. Volume of a sphere. |

15. How would you proceed to find the amount of material necessary to make a cylindrical can of tin, open at the top?

16. How would you proceed to find the number of bricks for a wall, knowing the number of bricks to a cubic foot?

17. How do you find the number of gallons of water that can be put into a tank the capacity of which is known?

18. How do you find the number of barrels of water that can be put into a tank the capacity of which is known?

EXERCISE 214

MISCELLANEOUS PROBLEMS

1. What is the volume of a prism the base of which is 10.5 sq. in. and the altitude $6\frac{1}{2}$ in. ?

2. What is the base of a cylinder the volume of which is 65 cu. in. and the altitude $6\frac{1}{2}$ in. ?

It is often convenient to use the equation, with x or other letters.

Thus, as in § 339,

$$v = bh,$$

or

$$65 = b \times 6\frac{1}{2}.$$

Dividing these equals by $6\frac{1}{2}$,

$$10 = b.$$

3. What is the volume of a pyramid the base of which is 10.5 sq. in. and the altitude $6\frac{1}{2}$ in. ?

4. What is the base of a cone the volume of which is 102 cu. in. and the altitude $8\frac{1}{2}$ in. ?

5. Compute the surface of a dome that is half of a sphere, the radius being 20 ft.

6. What is the edge of a cube of which the entire surface is 26.46 sq. in. ?

7. What is the entire surface of a cube of which the diagonal of the base is $\sqrt{7}$ in. ?

8. What is the volume of a cube of which the diagonal of the base is $\sqrt{11}$ in. ?

9. What is the volume of a cube of which the entire surface is 57.66 sq. in. ?

10. How much will it cost, at \$1.25 a yard, to carpet a room 12 ft. by 15 ft. with plain carpet $\frac{3}{4}$ yd. wide ?

11. How many cubic feet in a cistern that holds 54 bbl. of water, allowing 231 cu. in. to a gallon ?

12. A steel rod 4 ft. 8 in. long has a cross section 2 sq. in. in area. If a cubic foot of water weighs 62.5 lb., and steel is 7.83 times as heavy as water, what does the rod weigh ?

13. If a cubic foot of granite weighs 165 lb., what is the weight of a sphere of granite 1 ft. in diameter?

14. How many gallons of water will fill a tank 14 ft. by 8 ft. by 8 ft., allowing 231 cu. in. to a gallon?

15. A bowl in the form of a hemisphere has a diameter of 4.2 in. How many cubic inches does it contain?

16. If a cubic foot of marble weighs 173 lb., what is the weight of a cylindrical marble column 10 ft. high and 16 in. in diameter?

17. How many loads (cubic yards) of earth must be removed in digging a canal 2 mi. 1740 ft. long, 200 ft. wide, and 16 ft. deep?

18. In a cylindrical jar 4 in. high and 3 in. in diameter, half full of water, a marble 1 in. in diameter is dropped. It causes the water to rise how much?

19. What is the weight of a sphere of steel 7 in. in diameter, steel being 7.8 times as heavy as water, and 1 cu. ft. of water weighing 62.5 lb.?

20. What is the weight of a sphere of marble 3 ft. in circumference, marble being 2.7 times as heavy as water, and 1 cu. ft. of water weighing 62.5 lb.?

21. The surface of a pyramid is made up of four equal triangles 3 ft. on each side. What is the total area? (Carry the result to hundredths.)

22. The diameter of a sphere and the altitude of a cone are equal, and they have equal radii. The volume of the sphere is how many times that of the cone?

23. Taking the radius of the earth as 4000 mi. and the earth as an exact sphere, what is its volume? How many square miles on its surface? (Carry the answers only to the nearest thousand square miles or cubic miles.)

Find the areas of the squares whose sides are as follows :

- | | | | |
|------------------------|------------------------|------------------------|------------------------|
| 24. $\frac{23}{8}$ in. | 26. $1\frac{3}{8}$ in. | 28. $1\frac{7}{8}$ in. | 30. $1\frac{5}{4}$ in. |
| 25. 2.87 ft. | 27. 3.92 ft. | 29. 17.5 rd. | 31. 0.37 in. |

Find the areas of the rectangles whose dimensions are :

- | | |
|---|-------------------------------|
| 32. 3.4 ft., 9.2 ft. | 35. 6.4 in., 7.3 in. |
| 33. $1\frac{1}{4}$ yd., $27\frac{1}{2}$ in. | 36. 4.1 ft., 11.3 in. |
| 34. 4.92 in., 0.97 in. | 37. 14 ft. 3 in., 2 ft. 8 in. |

Find the areas of the triangles whose bases and altitudes are respectively as follows :

- | | |
|-------------------------------------|--------------------------------|
| 38. 4.7 ft., 3.8 ft. | 41. $6\frac{1}{4}$ yd., 27 in. |
| 39. 0.37 in., 0.42 in. | 42. 6.4 in., 4.9 in. |
| 40. $4\frac{1}{2}$ yd., 3 ft. 9 in. | 43. 6.8 ft., 11.9 in. |

Given rectangles with areas and lengths as follows, to find the widths :

- | | |
|------------------------------|------------------------------|
| 44. 306 sq. in., 18 in. | 47. 90.3 sq. in., 43 in. |
| 45. 14.62 sq. in., 4.3 in. | 48. 4.18 sq. in., 2.2 in. |
| 46. 473 sq. in., 3 ft. 7 in. | 49. 850 sq. in., 4 ft. 2 in. |

Given triangles with areas and bases as follows, to find the altitudes :

- | | |
|------------------------------|-------------------------------|
| 50. 864 sq. in., 27 in. | 53. 14.72 sq. ft., 4.6 ft. |
| 51. 101.4 sq. in., 26 in. | 54. 25.62 sq. in., 6.1 in. |
| 52. 0.0704 sq. in., 0.32 in. | 55. 2911 sq. in., 3 ft. 5 in. |

Find the sides of the squares with areas as follows, carrying the results to two decimal places :

- | | | |
|------------------|------------------|------------------|
| 56. 2 A. | 59. 171 sq. ft. | 62. 385 sq. ft. |
| 57. 153 sq. in. | 60. 352 sq. rd. | 63. 251 sq. yd. |
| 58. 7.52 sq. in. | 61. 6.04 sq. yd. | 64. 8.51 sq. in. |

Find the areas of the circles whose radii are :

- | | | |
|---------------|---------------|---------------|
| 65. 14 in. | 68. 15.4 in. | 71. 39.9 in. |
| 66. 64.4 ft. | 69. 71.4 ft. | 72. 92.4 yd. |
| 67. 123.9 rd. | 70. 164.5 rd. | 73. 235.9 yd. |

Find the radii of the circles whose areas are :

- | | | |
|-------------------|-------------------|--------------------|
| 74. 308 sq. in. | 77. 770 sq. in. | 80. 1386 sq. in. |
| 75. 1694 sq. ft. | 78. 1848 sq. ft. | 81. 2618 sq. ft. |
| 76. 40.04 sq. yd. | 79. 44.77 sq. yd. | 82. 141.68 sq. yd. |

Find the volumes and surfaces of the spheres whose radii are as follows :

- | | | |
|--------------|--------------|--------------|
| 83. 181 in. | 85. 172 in. | 87. 625 in. |
| 84. 31.8 in. | 86. 27.6 in. | 88. 34.2 in. |

Find the volumes of the pyramids whose bases and altitudes are respectively :

- | | |
|----------------------------|-------------------------------|
| 89. 21.6 sq. ft., 3 ft. | 91. 16.4 sq. in., 17.1 in. |
| 90. 17.4 sq. in., 12.3 in. | 92. 26.2 sq. ft., 5 ft. 1 in. |

Find the volumes of the cylinders whose altitudes and the radii of whose bases are respectively :

- | | |
|----------------------|-----------------------|
| 93. 2.8 ft., 1.7 ft. | 95. 2.7 in., 7.2 in. |
| 94. 6.2 in., 3.4 in. | 96. 14.2 ft., 8.1 ft. |

Find the volumes of the cones whose altitudes and the radii of whose bases are respectively :

- | | |
|-------------------------------|-------------------------------|
| 97. 6.2 ft., 4.1 ft. | 101. 3.9 ft., 2.4 ft. |
| 98. 3.4 in., 4.2 in. | 102. 6.7 in., 7.8 in. |
| 99. 2.8 in., 3.1 in. | 103. 12.8 ft., 6.1 ft. |
| 100. 4 ft. 7 in., 2 ft. 1 in. | 104. 6 ft. 8 in., 3 ft. 2 in. |

EXERCISE 215

PROBLEMS WITHOUT NUMBERS

1. How would you proceed to find the cost of a sidewalk to be laid in front of the schoolhouse?
2. How would the girls in the class proceed to compute the cost of material and labor for making a dress for school wear?
3. Given the length, breadth, and thickness of a rectangular solid, how would you find the volume?
4. Given the same dimensions, how would you find the total area of the faces?
5. Given the volume of a rectangular solid, and two dimensions, how would you find the third dimension?
6. Given the altitude of a cylinder and the area of the base, how would you find the volume?
7. Given the volume of a cylinder and the area of the base, how would you find the altitude?
8. Given the altitude of a cylinder and the radius of the base, how would you find the volume?
9. Given the volume of a cylinder and the radius of the base, how would you find the altitude?
10. Given the altitude of a cylinder and the circumference of the base, how would you find the lateral area?
11. Given the altitude of a cylinder and the radius of the base, how would you find the lateral area?
12. Given the diameter of a cylindrical water tank and the depth of the water, how would you find the volume?
13. Given the diameter of a cylindrical water tank and the depth of the water, how would you find the number of gallons of water? (Allow $7\frac{1}{2}$ gal. to 1 cu. ft.)

CHAPTER XX

RAPID CALCULATION AND BUSINESS FORMS

350. Short Methods of Operation. Having considered the four fundamental operations of addition, subtraction, multiplication, and division, with integers, common fractions, and decimal fractions, we shall now see what short methods are really practical.

There are many short methods, but most of them are useful only to those who do a great deal of computing. It is also very common at present for professional computers to use machines for all the operations. Most large banks have adding machines, and there are also machines that subtract, multiply, and divide.

351. Short Methods of Addition. The most helpful suggestion in rapid addition is to add by groups instead of by single figures.

For example, in this column the pupil should see 19 when he glances at the three lowest figures; he should see two eights, or 16, in the next three, and 9 in the two top figures. He should therefore think "19, 35, 44," or some similar adding of groups.

Probably he will at first find it easier to group by twos and think "15, 24, 35, 44."

Professional computers often add two columns at once, but this is not recommended for pupils in school, where absolute accuracy is more important than speed.

When there are two or more columns of figures in the addends, professional computers usually write down the sum of each column. In this way an error may be found more easily when the work is checked by adding in the direction opposite to that of the original addition. The immediate discovery of such errors is the most important means of saving time.

352. Rapid Calculation. It is found that reasonable rapidity leads to accuracy, the slow computer being usually less accurate than a fairly rapid one. To acquire fair rapidity, daily speed exercises are recommended.

Teachers should set a reasonable time limit and hold the class to it. This will fit the pupils for the work under pressure which is bound to come in business life. The pupils should be drilled until they can add at the rate of 75 to 100 figures per minute. It is sometimes advantageous to have the class choose sides for contests in speed and accuracy, adding one to the score for every correct result and subtracting one for every incorrect result.

The pupils should use the group method suggested in § 351, and illustrated by the following examples:

3 } 2 } 9 4 }	3 } 2 } 10 1 }	1 } 10 9 } 1 }	3 } 10 7 } 4 }
5 } 10 5 }	4 } 2 } 6 } 9	1 } 10 1 } 8 }	6 } 10 1 }
2 } 5 } 10 3 }	1 } 7 } 10 3 }	2 } 10 1 } 7 }	3 } 10 1 }
6 } 10 4 }	9 2 } 10 5 }	7 1 } 3 } 10	5 } 3 }
5 } 1 } 10 4 }	3 9 2 }	1 3 } 10 2 }	1 } 10 6 }
<u>49</u>	<u>48</u>	<u>47</u>	<u>57</u>

At first combine only up to 10. Larger groups may be practiced later.

Rapid oral work on the combinations is also valuable. For example, $3 + 7$, $9 + 6$, $5 + 2$, $6 + 7$, and so on. This is particularly valuable when the combinations are written on the blackboard, since it is the eye rather than the ear that needs training in number work.

Add as rapidly as possible, trying always to add by groups of two or three figures. Check each result.

28.	29.	30.	31.	32.	33.	34.
22	17	29	22	39	72	50
57	21	32	31	27	90	49
31	32	48	46	96	32	78
63	48	69	79	43	41	62
47	63	71	58	57	78	37
20	45	83	73	82	62	43
68	71	47	69	98	98	81
<u>12</u>	<u>21</u>	<u>53</u>	<u>41</u>	<u>40</u>	<u>32</u>	<u>99</u>
35.	36.	37.	38.	39.	40.	41.
31	54	56	21	87	20	67
79	26	29	71	62	96	43
68	37	32	88	90	34	48
42	63	38	96	81	81	92
32	91	46	40	48	99	77
78	89	54	64	71	87	81
59	77	31	72	30	43	41
<u>41</u>	<u>63</u>	<u>79</u>	<u>88</u>	<u>21</u>	<u>70</u>	<u>61</u>
42.	43.	44.	45.	46.	47.	48.
27	52	42	57	23	54	88
32	38	32	42	47	68	99
51	27	64	83	62	92	22
10	91	82	91	83	38	87
89	61	91	68	92	41	64
21	31	78	92	51	71	33
76	42	51	73	82	56	82
24	38	60	47	68	44	96
<u>36</u>	<u>29</u>	<u>27</u>	<u>42</u>	<u>52</u>	<u>22</u>	<u>36</u>

Add as rapidly as possible, trying always to add by groups of two or three figures. Check each result.

49.	52.	55.	58.	61.	64.
26	97	29	428	\$34.78	\$121.42
43	60	91	607	62.48	137.60
28	89	72	320	71.92	89.48
91	34	30	556	38.46	125.50
72	42	47	707	50.09	38.75
54	80	66	62	77.05	192.50
81	75	55	491	82.91	65.82
<u>24</u>	<u>66</u>	<u>43</u>	<u>377</u>	<u>4.80</u>	<u>48.75</u>

50.	53.	56.	59.	62.	65.
47	84	22	572	\$75.50	\$263.08
63	60	89	681	26.75	148.75
96	32	72	708	8.93	192.68
34	98	35	320	14.98	635.35
81	77	60	572	6.37	870.08
29	63	81	647	24.42	428.70
88	49	92	998	69.70	642.35
<u>72</u>	<u>92</u>	<u>37</u>	<u>123</u>	<u>8.84</u>	<u>408.77</u>

51.	54.	57.	60.	63.	66.
29	35	29	547	\$18.92	\$221.62
68	42	68	629	17.86	342.98
73	87	73	372	23.48	476.35
40	98	52	588	52.96	192.09
96	63	31	793	78.32	778.38
85	47	48	874	91.46	658.02
77	72	77	645	87.00	523.65
42	31	65	880	9.85	807.91
<u>50</u>	<u>60</u>	<u>75</u>	<u>300</u>	<u>10.00</u>	<u>500.00</u>

Add the following without copying in columns:

67. 48, 62, 68, 42, 54, 56, 65, 45, 61, 49, 50, 27, 45.
68. 26, 84, 34, 76, 59, 51, 37, 73, 82, 28, 68, 42, 76.
69. 29, 81, 30, 70, 42, 68, 28, 32, 39, 71, 61, 49, 53.
70. 39, 61, 82, 18, 61, 39, 53, 47, 42, 58, 37, 63, 72, 28.
71. 34, 26, 59, 36, 82, 96, 54, 30, 48, 72, 81, 96, 34, 23.
72. 37, 56, 48, 65, 62, 34, 59, 53, 22, 78, 61, 82, 50, 96.
73. 97, 42, 20, 86, 82, 31, 57, 75, 68, 37, 64, 48, 53, 90.
74. 22, 30, 88, 34, 57, 46, 55, 98, 99, 72, 66, 54, 60, 77.
75. 19, 55, 91, 73, 80, 28, 68, 64, 54, 48, 37, 82, 32, 46.
76. 24, 36, 95, 81, 27, 64, 79, 83, 40, 17, 39, 19, 20, 86.
77. 37, 42, 81, 93, 87, 49, 80, 17, 45, 36, 81, 92, 77, 60.
78. 30, 20, 75, 85, 90, 27, 96, 19, 48, 32, 57, 86, 41, 32.
79. 68, 30, 43, 77, 26, 32, 86, 34, 82, 95, 96, 87, 99, 80.
80. 99, 58, 57, 26, 89, 36, 48, 47, 79, 15, 38, 37, 69, 28.
81. 12, 56, 89, 77, 52, 23, 19, 67, 36, 34, 40, 78, 45, 90.
82. 28, 40, 84, 62, 16, 17, 28, 95, 39, 39, 56, 51, 47, 73.
83. 50, 94, 98, 65, 31, 64, 61, 42, 76, 53, 72, 75, 87, 83.
84. 23, 67, 90, 63, 81, 78, 34, 72, 89, 54, 45, 45, 34, 56.
85. 16, 50, 83, 46, 27, 61, 94, 57, 38, 86, 35, 49, 72, 68.
86. 33, 40, 28, 96, 81, 73, 54, 62, 91, 43, 70, 92, 60, 55.
87. 0.40, 0.32, 0.26, 1.84, 0.96, 0.53, 1.70, 12.40, 23.55.
88. 27, 23, 50, 36, 64, 18, 32, 56, 24, 20, 74, 26, 87, 13.
89. 15, 25, 17, 23, 36, 41, 82, 39, 28, 32, 62, 81, 73, 48.
90. \$1.75, \$2.50, \$3.25, \$4.50, \$5.20, \$6.80, \$10.50, \$2.
91. \$1.50, \$3.75, \$2.25, \$4.50, \$6.60, \$3.40, \$2, \$5, \$1.
92. \$2.75, \$3.25, \$4.10, \$5.40, \$2.50, \$4.25, \$7, \$3, \$4.
93. \$4.20, \$2.60, \$5.30, \$4.20, \$2.60, \$3.80, \$10.75, \$6.

353. Short Methods of Subtraction. There are no special suggestions to be given for subtraction beyond those set forth in § 32 and § 33.

The one given in § 33 is particularly helpful.
 In a case like $40,000 - 17,652$ we may think of the places of the minuend as 3, 9, 9, 9, 10 and subtract from left to right.

3	9	9	9	10
1	7	6	5	2
2	2	3	4	8

EXERCISE 217

Subtract as rapidly as possible, beginning at the left when this can easily be done:

1.	7.	13.	19.
7000	50,000	\$100.00	\$250.
<u>2763</u>	<u>26,492</u>	<u>75.62</u>	<u>75.65</u>
2.	8.	14.	20.
6000	60,000	\$300.00	\$350.
<u>4296</u>	<u>31,429</u>	<u>128.64</u>	<u>82.50</u>
3.	9.	15.	21.
5000	40,000	\$500.00	\$640.
<u>1297</u>	<u>17,792</u>	<u>275.55</u>	<u>121.75</u>
4.	10.	16.	22.
8000	70,000	\$700.00	\$480.
<u>2776</u>	<u>20,078</u>	<u>428.50</u>	<u>135.25</u>
5.	11.	17.	23.
9000	65,000	\$900.00	\$530.
<u>4326</u>	<u>21,572</u>	<u>230.20</u>	<u>192.25</u>
6.	12.	18.	24.
8000	80,000	\$800.00	\$320.
<u>7080</u>	<u>27,600</u>	<u>206.00</u>	<u>175.75</u>

354. Short Methods of Multiplication. There are several short methods of multiplication that are valuable, particularly because certain multipliers are so often used.

Such multipliers are 10, 100, 1000, and other powers of 10; 5, 25, 125, and $12\frac{1}{2}$; $16\frac{2}{3}$, $33\frac{1}{3}$, and $66\frac{2}{3}$; 9, 11, and 75.

355. Multiplication by Powers of 10 and by Aliquot Parts. The advantage of using aliquot parts in decimals and percentage has already been mentioned (pp. 87, 234), and the same general methods may be applied in multiplication.

The most important aliquot parts of 100 are as follows:

$50 = \frac{1}{2}$ of 100	$33\frac{1}{3} = \frac{1}{3}$ of 100	$20 = \frac{1}{5}$ of 100
$25 = \frac{1}{4}$ of 100	$16\frac{2}{3} = \frac{1}{6}$ of 100	$10 = \frac{1}{10}$ of 100
$12\frac{1}{2} = \frac{1}{8}$ of 100	$8\frac{1}{3} = \frac{1}{12}$ of 100	$5 = \frac{1}{20}$ of 100
$6\frac{1}{4} = \frac{1}{16}$ of 100	$4\frac{1}{6} = \frac{1}{24}$ of 100	$2 = \frac{1}{50}$ of 100

Other important fractional parts of 100, but not aliquot parts, are as follows:

$75 = \frac{3}{4}$ of 100	$62\frac{1}{2} = \frac{5}{8}$ of 100	$66\frac{2}{3} = \frac{2}{3}$ of 100
$37\frac{1}{2} = \frac{3}{8}$ of 100	$87\frac{1}{2} = \frac{7}{8}$ of 100	$83\frac{1}{3} = \frac{5}{6}$ of 100

Pupils should be thoroughly drilled upon these important aliquot or fractional parts. They should know that multiplication of an integer by 25 means the addition of two zeros and a mental division by 4, and that to multiply in the long way by $12\frac{1}{2}$, $16\frac{2}{3}$, $33\frac{1}{3}$, $37\frac{1}{2}$, $62\frac{1}{2}$, $87\frac{1}{2}$, etc., is a waste of time and energy.

We may often simplify multiplication, as shown on page 88, thus:

(1) *To multiply by 10, move the decimal point one place to the right, annexing zero if necessary.*

(2) *To multiply by 100, move the decimal point two places to the right, annexing zeros if necessary.*

(3) *To multiply by 1000, move the decimal point three places to the right, annexing zeros if necessary.*

- (4) *To multiply by 50, multiply by 100 and divide by 2.*
- (5) *To multiply by 25, multiply by 100 and divide by 4.*
- (6) *To multiply by $12\frac{1}{2}$, multiply by 100 and divide by 8.*
- (7) *To multiply by 125, multiply by 1000 and divide by 8.*
- (8) *To multiply by $33\frac{1}{3}$, multiply by 100 and divide by 3.*
- (9) *To multiply by $16\frac{2}{3}$, multiply by 100 and divide by 6.*
- (10) *To multiply by 5, multiply by 10 and divide by 2.*
- (11) *To multiply by 75, multiply by 100 and take $\frac{3}{4}$ of the product.*
- (12) *To multiply by $37\frac{1}{2}$, multiply by 100 and take $\frac{3}{8}$ of the product.*
- (13) *To multiply by $62\frac{1}{2}$, multiply by 100 and take $\frac{5}{8}$ of the product.*
- (14) *To multiply by $87\frac{1}{2}$, multiply by 100 and take $\frac{7}{8}$ of the product.*
- (15) *To multiply by $66\frac{2}{3}$, multiply by 100 and take $\frac{2}{3}$ of the product.*
- (16) *To multiply by 0.1, move the decimal point one place to the left ; by 0.01, move the decimal point two places to the left ; by 0.001, move the decimal point three places to the left, and so on.*
- (17) *To multiply by 0.5, take half of the multiplicand.*
- (18) *To multiply by 0.25, take a fourth of the multiplicand.*
- (19) *To multiply by $0.12\frac{1}{2}$, take an eighth of the multiplicand.*
- (20) *To multiply by $0.33\frac{1}{3}$, take a third of the multiplicand.*
- (21) *To multiply by $0.16\frac{2}{3}$, take a sixth of the multiplicand.*
- (22) *To multiply by 0.75, take three fourths of the multiplicand.*

Other similar rules will easily be found by the student. For example, to multiply by 15, multiply by 10 and add half the product to itself ; and to multiply by 150, first multiply by 10 and then multiply by 15 as stated. The student may suggest short methods of multiplying by 1.25 , 1.5 , $1.12\frac{1}{2}$, $1.33\frac{1}{3}$, and so on.

EXERCISE 218

Multiply :

- | | | |
|--------------------------------------|--------------------------------------|-------------------------------------|
| 1. 75×184 . | 17. 0.75 of 48. | 33. $968 \times 0.87\frac{1}{2}$. |
| 2. 75×916 . | 18. $37\frac{1}{2} \times 424$. | 34. $66\frac{2}{3} \times 7266$. |
| 3. 75×936 . | 19. $37\frac{1}{2} \times 1776$. | 35. $66\frac{2}{3} \times 8436$. |
| 4. $75 \times \$1.48$. | 20. $976 \times \$0.37\frac{1}{2}$. | 36. $66\frac{2}{3} \times 843.6$. |
| 5. $75 \times \$3.20$. | 21. $656 \times \$0.37\frac{1}{2}$. | 37. $66\frac{2}{3} \times 84.36$. |
| 6. $75 \times 24,616$. | 22. $568 \times \$0.62\frac{1}{2}$. | 38. 0.5 of 6432. |
| 7. $448 \times \$0.75$. | 23. $976 \times \$0.62\frac{1}{2}$. | 39. 0.5 of 1338. |
| 8. $832 \times \$0.75$. | 24. $664 \times \$0.87\frac{1}{2}$. | 40. 0.5 of 7952. |
| 9. 0.75 of 2436. | 25. $592 \times \$0.87\frac{1}{2}$. | 41. 2952×0.05 . |
| 10. 3574×0.05 . | 26. $368 \times \$0.12\frac{1}{2}$. | 42. 248×0.05 . |
| 11. 0.25 of 536. | 27. $336 \times \$0.33\frac{1}{3}$. | 43. 24.8×0.5 . |
| 12. 0.25 of 5.36. | 28. $297 \times \$0.33\frac{1}{3}$. | 44. 2.48×0.5 . |
| 13. 0.25 of 0.536. | 29. $372 \times \$0.66\frac{2}{3}$. | 45. 248×7.5 . |
| 14. $704 \times \$0.25$. | 30. $492 \times \$0.66\frac{2}{3}$. | 46. 2.48×7.5 . |
| 15. $172 \times \$0.25$. | 31. $558 \times \$0.16\frac{2}{3}$. | 47. 2.48×0.75 . |
| 16. $776 \times \$0.12\frac{1}{2}$. | 32. $672 \times \$0.16\frac{2}{3}$. | 48. $0.12\frac{1}{2} \times 0.64$. |
49. How much will 75 horses cost at \$160 each ?
50. How much will 75 wagons cost at \$144 each ?
51. How much will 124 horses cost at \$75 each ?
52. How much will 168 horses cost at \$125 each ?
53. How much will 736 brooms cost at 25¢ each ?
54. How much will 392 lamps cost at $62\frac{1}{2}$ ¢ each ?
55. How much will 284 boxes of paper cost at 75¢ each ?
56. How much will 564 yd. of linen cost at 75¢ a yard ?
57. How much will 424 yd. of matting cost at $37\frac{1}{2}$ ¢ a yard ?

Find the cost of the following :

58. 976 yd. of velvet @ \$1.25.
59. 496 yd. of Java cream canvas @ 75¢.
60. 432 yd. of single-thread canvas @ 62½¢.
61. 128 yd. of white narrow canvas @ 37½¢.
62. 136 yd. of basket-weave burlap @ \$1.25.
63. A gross of pearl buttons @ 12½¢ a dozen.
64. What will 712 doz. eggs cost @ 12½¢ a dozen ?
65. What will 248 yd. of silk cost @ \$1.25 a yard ?
66. What will 136 yd. of cloth cost @ 25¢ a yard ?
67. What will 216 yd. of cloth cost @ 12½¢ a yard ?
68. What will \$168.40 worth of goods cost @ 25% off ?
69. What will \$63.72 worth of goods cost @ 33⅓% off ?
70. Find 25% of 33⅓% of 12 ; of 600 ; of 1248 ; of 1596.
71. Find 12½% of 50% of 16 ; of 160 ; of 1600 ; of 1456.
72. Find 5% of 10% of 12½% of 80 ; of 1600 ; of 9680.
73. A dealer buys 144 boxes of soap, each weighing 12½ lb. What is the total weight ?
74. At \$33⅓ a month, how much rent will a man pay for his house in a year ? in 6 mo. ?
75. At \$33⅓ a week, how much will a salesman receive if he works 48 wk. ? if he works 52 wk. ?
76. If a man earns \$16⅔ a week, how much will he earn in 36 wk. ? in 48 wk. ? in 52 wk. ?
77. Multiply 1728 by 1¼ ; 172.8 by 12½ ; 17.28 by 125. How do the results differ ? Write the reason for this.
78. Multiply 1728 by 3⅓ ; 172.8 by 33⅓ ; 17.28 by 333⅓. Compare the results and explain, as in Ex. 77.

356. Multiplication by Numbers near 10, 100, and so on.
 Since $9 = 10 - 1$, we may multiply easily by 9 by annexing a zero, which multiplies by 10, and then subtracting the given multiplicand from the product by 10.

$10 \times 7286 = 72860$	Thus 9×7286 is evidently the same as
$1 \times 7286 = 7286$	$10 \times 7286 - 7286$, as here shown. It is evi-
$9 \times 7286 = \underline{65574}$	dently shorter to subtract 7286 from 72,860
	than to multiply 7286 by 9.

From this reasoning and similar courses of reasoning we have the following short processes:

(1) *To multiply by 9, multiply by 10 and subtract the given multiplicand.*

$$\text{Thus } 9 \times \$374.62 = \$3746.20 - \$374.62 = \$3371.58.$$

(2) *To multiply by 11, multiply by 10 and add the given multiplicand.*

$$\text{Thus } 11 \times \$374.62 = \$3746.20 + \$374.62 = \$4120.82.$$

(3) *To multiply by 99, multiply by 100 and subtract the given multiplicand.*

$$\text{Thus } 99 \times 4368 = 436,800 - 4368 = 432,432.$$

(4) *To multiply by 98, multiply by 100 and subtract twice the given multiplicand.*

$$\text{Thus } 98 \times \$435.75 = \$43,575 - 2 \times \$435.75 = \$43,575 - \$871.50 = \$42,703.50.$$

To multiply by 999, 998, and like numbers, proceed in a manner similar to the above.

EXERCISE 219

Multiply:

- | | | |
|--------------------------|--------------------------|-----------------------------|
| 1. $9 \times \$724.56.$ | 5. $99 \times \$128.72.$ | 9. $999 \times \$43.75.$ |
| 2. $9 \times \$875.93.$ | 6. $99 \times \$426.45.$ | 10. $998 \times \$62.86.$ |
| 3. $11 \times \$682.97.$ | 7. $98 \times \$327.47.$ | 11. $1001 \times \$27.63.$ |
| 4. $11 \times \$493.82.$ | 8. $98 \times \$462.09.$ | 12. $1002 \times \$324.75.$ |

357. Multiplication by Multiples of 11. Instead of actually adding the multiplicand to 10 times itself, in multiplying by 11, we may shorten the process a little as follows:

To multiply 892 by 11 we have :

2 as the units' figure ;

$2 + 9 = 11$, giving 1 as the tens' figure ;

$8 + 9 + 1$ (carried) = 18, giving 8 as the hundreds' figure ;

$8 + 1$ (carried) = 9, giving 9 as the thousands' figure.

9812 = final result.

In a similar manner we may multiply by 22.

To multiply 596 by 22 we have :

$2 \times 6 = 12$, giving 2 as the units' figure ;

$2 \times (9 + 6) + 1$ (carried) = 31, giving 1 as the tens' figure ;

$2 \times (9 + 5) + 3$ (carried) = 31, giving 1 as the hundreds' figure ;

$2 \times 5 + 3$ (carried) = 13.

13,112 = final result.

In a similar manner we may multiply by 33, 44, and so on.

EXERCISE 220

Multiply :

- | | | |
|--------------------------|--------------------------|-----------------------|
| 1. 11×7286 . | 13. 22×123 . | 25. 33×102 . |
| 2. 11×9281 . | 14. 22×436 . | 26. 33×213 . |
| 3. 11×3749 . | 15. 22×521 . | 27. 33×404 . |
| 4. $11 \times 40,728$. | 16. 22×620 . | 28. 33×555 . |
| 5. $11 \times 60,709$. | 17. 22×1208 . | 29. 33×660 . |
| 6. $11 \times 49,998$. | 18. $22 \times 21,112$. | 30. 44×111 . |
| 7. $11 \times 27,246$. | 19. $22 \times 10,100$. | 31. 55×202 . |
| 8. $11 \times 39,007$. | 20. $22 \times 30,303$. | 32. 66×312 . |
| 9. $11 \times 27,070$. | 21. $22 \times 33,333$. | 33. 77×222 . |
| 10. $11 \times 60,060$. | 22. $22 \times 47,407$. | 34. 88×111 . |
| 11. $11 \times 70,770$. | 23. $22 \times 55,055$. | 35. 99×111 . |
| 12. $11 \times 99,999$. | 24. $22 \times 72,727$. | 36. 99×202 . |

358. Goods sold by the Hundred, the Thousand, or the Ton.
It is the custom to sell certain kinds of goods by the hundred, certain kinds by the thousand, and certain kinds by the ton. In such cases it is frequently necessary to find the cost of fractional parts of these amounts.

In quoting prices, C is often used for hundred and M for thousand. At \$5.75 per C, means at \$5.75 per hundred; and at \$10 per M, means at \$10 per thousand.

The following examples will illustrate.

(1) Find the cost of 2675 laths at 44¢ per C.

$ \begin{array}{r} \$0.44 \\ 26\frac{3}{4} \\ \hline 33 \\ 264 \\ 88 \\ \hline \$11.77 \end{array} $	$2675 = 26\frac{3}{4} \text{ C.}$ Since 100 laths cost \$0.44, $26\frac{3}{4}$ C will cost $26\frac{3}{4} \times \\$0.44$, or \$11.77. If the problem had asked the cost of 2638 laths, it would have been easier to multiply 26.38 by .44 than .44 by 26.38, and the result would have been the same (§ 41).
---	---

(2) Find the cost of 3924 feet of boards at \$17.50 per M.

$ \begin{array}{r} \$3.924 \\ 17\frac{1}{2} \\ \hline 1962 \\ 27468 \\ 3924 \\ \hline \$68.670 \end{array} $	$3924 = 3.924 \text{ M.}$ Since 1000 ft. of boards cost \$17.50, 3.924 M ft. will cost $3.924 \times \\$17.50$, or \$68.67. Here it is more convenient in practice to use $17\frac{1}{2}$ as the multiplier, changing 3.924 to \$3.924 (§ 41). In explaining the work, however, use 3.924 as the multiplier as above.
---	---

(3) Find the cost of 8712 pounds of anthracite coal at \$6.50 per ton.

$ \begin{array}{r} \$8.712 \\ 3\frac{1}{4} \\ \hline 2178 \\ 26136 \\ \hline \$28.314 \end{array} $	$8712 = 8.712 \text{ M.}$ Since 2000 lb. of anthracite coal cost \$6.50, therefore 1000 lb. will cost $\frac{1}{2}$ of \$6.50, or \$3.25. Since 1000 lb. of coal cost \$3.25, 8.712 M lb. will cost $8.712 \times \\$3.25 = \\$28.31$. Here again it is more convenient to interchange multiplicand and multiplier, using $3\frac{1}{4}$ instead of 3.25 for the latter, and changing 8.712 to \$8.712.
---	---

EXERCISE 221

1. What will 9275 laths cost at 45¢ per C ?
2. What will 27,625 laths cost at 43¢ per C ?
3. What will 48,250 bricks cost at \$8.10 per M ?
4. What will 4925 paving stones cost at \$9.22 per C ?
5. What will 2735 ft. of boards cost at \$18.75 per M ?
6. What will 3690 ft. of boards cost at \$22.50 per M ?
7. What will 16,250 ft. of boards cost at \$24.75 per M ?
8. What will 7 T. 935 lb. of coal cost at \$6.25 per ton ?
9. At 25¢ per C, what is the freight on 32,500 lb. of flour ?
10. If 7275 lb. of plaster cost \$29.10, what is the price per ton ?
11. What will 8275 tiles for the roof of a building cost at \$9.25 per M ?
12. What is the cost of coal per ton when 27,770 lb. cost \$83.31 ?
13. At \$19.50 per M, how many feet of boards can be bought for \$146.25 ?
14. If a man buys 17,250 lb. of coal at \$5.90 per ton, how much does it cost him ?
15. If a dealer pays \$3.10 per M for shingles and sells them for \$3.80, how much does he gain on 29 M ?
16. A coal dealer retails coal at \$6.50 per ton. If it costs \$3.25 at the mine, \$1.70 for freight, and 80¢ for handling, what is his profit on 250 T. 750 lb. ?
17. A lumber dealer buys 59,500 ft. of lumber at \$16 per M. He sells 48,000 ft. at \$19.50 per M, and the balance at \$19 per M. How much is his whole profit ?

359. Short Methods of Division. There are certain divisors that are used so often that it becomes convenient to know short methods of dividing by them.

The student should consult page 91 for the explanations.

(1) *To divide by 10, move the decimal point one place to the left ; by 100, two places ; by 1000, three places ; and so on.*

(2) *To divide by 5, multiply by 2 and divide by 10.*

(3) *To divide by 25, multiply by 4 and divide by 100.*

(4) *To divide by 125, multiply by 8 and divide by 1000.*

(5) *To divide by $33\frac{1}{3}$, multiply by 3 and divide by 100.*

EXERCISE 222

Divide :

- | | |
|--|---|
| 1. $\$9642 \div 10.$ | 11. $23,075 \div 25.$ |
| 2. $\$2796 \div 100.$ | 12. $58,875 \div 25.$ |
| 3. $\$5275 \div 1000.$ | 13. $77,650 \div 25.$ |
| 4. $\$3420.10 \div 5.$ | 14. $62,650 \div 125.$ |
| 5. $\$2700.10 \div 5.$ | 15. $376,750 \div 125.$ |
| 6. $\$121,234 \div \$5.$ | 16. $812,700 \div 33\frac{1}{3}.$ |
| 7. $22,175 \text{ ft.} \div 5 \text{ ft.}$ | 17. $417,600 \div 33\frac{1}{3}.$ |
| 8. $34,175 \text{ ft.} \div 5 \text{ ft.}$ | 18. $222,222 \div 33\frac{1}{3}.$ |
| 9. $427,445 \div 5.$ | 19. $333,333 \div 33\frac{1}{3}.$ |
| 10. $47,125 \div 25.$ | 20. $27,027 \text{ ft.} \div 33\frac{1}{3} \text{ ft.}$ |

21. A manufacturer sells \$3750 worth of typewriters at \$50 each. How many does he sell ?

22. How long will it take an express train to go 362.5 mi. at the rate of 50 mi. an hour ?

23. How long will it take a freight train to go $166\frac{2}{3}$ mi. at the rate of 25 mi. an hour ?

24. A manufacturer sells \$1810 worth of silk at \$1.25 a yard. How many yards does he sell ?

Divide :

25. $6484 \div 66\frac{2}{3}$. 30. $7280 \div 66\frac{2}{3}$. 35. $9276 \div 66\frac{2}{3}$.

26. $2937 \div 16\frac{2}{3}$. 31. $4831 \div 16\frac{2}{3}$. 36. $9134 \div 16\frac{2}{3}$.

27. $9237 \div 12\frac{1}{2}$. 32. $7396 \div 12\frac{1}{2}$. 37. $8187 \div 12\frac{1}{2}$.

28. $6889 \div 33\frac{1}{3}$. 33. $4959 \div 33\frac{1}{3}$. 38. $8892 \div 33\frac{1}{3}$.

29. $4936 \div 125$. 34. $67,400 \div 125$. 39. $39,888 \div 125$.

40. Divide by 25: 13,428, \$37,642, \$135,621, \$427,693.

41. Divide by 125: 17,200, \$68,300, \$276,400, \$317,500.

42. Divide by $33\frac{1}{3}$: 69,720, \$32,960, \$48,750, \$276,320.

43. At 25¢ a yard, how many yards of cloth can be bought for \$37.25?

44. At $33\frac{1}{3}$ ¢ a yard, how many yards of cloth can be bought for \$98?

45. At $12\frac{1}{2}$ ¢ a gross, how many gross of paper fasteners will \$17.75 buy?

46. Divide 3,742,697 by 25, by long division, and see how many seconds it takes. Do the same by the short method stated on page 416. Write the time with the answer.

47. At $33\frac{1}{3}$ ¢ a hundred, how many photograph cards can be bought for \$1.83?

48. At $66\frac{2}{3}$ ¢ a gross, how many gross of pens can be bought for \$24?

49. At $\frac{3}{8}$ yd. each, how many strips of ribbon can we get from a piece 27 yd. long?

50. When a train is traveling at the rate of 32 mi. in 40 min., what is its rate per hour?

51. When a horse is trotting at the rate of $\frac{1}{2}$ mi. in 3 min., what is his rate per hour?

52. When a man is walking at the rate of 40 rd. in 2 min. 30 sec., what is his rate in miles per hour?

360. Short Methods Applied. In the applications of arithmetic to various trades, industries, and professions, in this and the following chapters, short methods should be employed whenever possible. In particular:

(1) *Always estimate the result in advance, comparing the final result with it.*

(2) *Indicate the operations whenever cancellation is possible, and save time and energy by canceling.*

(3) *Never multiply at length by 125, but multiply by 1000 and divide by 8, and proceed similarly with other aliquot parts.*

(4) *Always check each operation as it is performed. If you wait until the end, an early mistake may lead to a waste of time in doing all the rest of the work over again.*

EXERCISE 223

1. How much will 48 yd. of calico cost at $12\frac{1}{2}\text{¢}$ a yard?
2. At $12\frac{1}{2}\text{¢}$ a yard, how many yards of curtain material can be bought for \$3.60?
3. If 25 yd. of curtain material cost \$3.25, how much does it cost per yard?
4. If we save $6\frac{1}{4}\text{¢}$ a yard by purchasing cotton cloth by the quantity, how much is saved on 80 yd.? on 160 yd.?
5. If we buy calico by the 32-yard piece, we save 2¢ a yard. Bought by the yard the price is $12\frac{1}{2}\text{¢}$. What per cent is saved in buying it by the piece?
6. If $7\frac{1}{2}$ yd. of material 32 in. wide are required for a certain garment, how much material 38 in. wide will be required, if it cuts as economically?
7. If material 28 in. wide costs \$1.40 a yard, what should be the cost of material of the same quality 36 in. wide, at the same rate per square yard?

8. How much will 144 boxes cost at $37\frac{1}{2}\text{¢}$ each?
9. How much will 64 desks cost at \$8.75 each?
10. How much will 16 colts cost at \$87.50 each?
11. How much will 375 mules cost at \$125 each?
12. How much will 22 typewriters cost at \$62.50 each?
13. If 25 tons of coal cost \$187.50, what will be the cost of 99 tons?
14. How much will 288 doz. sewing-machine needles cost at $37\frac{1}{2}\text{¢}$ a dozen?
15. How much will 250 doz. yards of millinery braid cost at $62\frac{1}{2}\text{¢}$ a dozen?
16. There are 44,000 yd. in 25 mi. How many yards are there in one mile?
17. How much will 18 thousand feet of oak flooring cost at \$125 a thousand feet?
18. How many bushels of oats will 81 acres produce, if an average acre produces $33\frac{1}{3}$ bu.?
19. How many bushels of oats will 125 acres produce, if an average acre produces 32 bu.?
20. How many bushels of potatoes will 32 acres produce, if an average acre produces 125 bu.?
21. A tailor paid \$210 for some broadcloth at $\$3.33\frac{1}{3}$ a yard. How many yards did he buy?
22. A dealer sold some overcoats for \$2900, at the rate of $\$33.33\frac{1}{3}$ apiece. How many overcoats did he sell?
23. How much will 16 thousand feet of mixed red and white birch flooring cost at \$50 a thousand feet?
24. A farmer owns 78 acres of land covered with a growth of white pine which he values at $\$166.66\frac{2}{3}$ an acre. At what amount does he value his wood lot?

25. What is the cost of 125 pieces of flannel, averaging 40 yd. each, at 50¢ a yard?

26. What is the cost of 25 boxes of lamp shades, a gross to the box, at $33\frac{1}{3}\text{¢}$ per shade?

27. What is the cost of 75 doz. table doilies at $16\frac{2}{3}\text{¢}$ per doily?

28. What is the cost of 250 gro. of penholders, each penholder costing $8\frac{1}{3}\text{¢}$?

29. If it costs $4\frac{1}{8}\text{¢}$ to manufacture a notebook, what will it cost to manufacture 375 gro. at the same rate?

30. At $37\frac{1}{2}\text{¢}$ per book, what will a dealer have to pay for 125 gro. of cashbooks?

31. At $62\frac{1}{2}\text{¢}$ apiece, what will it cost to manufacture 1250 doz. lamps?

32. At $66\frac{2}{3}\text{¢}$ apiece, what will it cost to manufacture 375 doz. iron washstands?

33. At $83\frac{1}{3}\text{¢}$ apiece, what will it cost to manufacture 175 doz. kitchen chairs?

34. At $37\frac{1}{2}\text{¢}$ apiece, how many mats can be bought for \$86.22?

35. At $62\frac{1}{2}\text{¢}$ a yard, how many yards of linen can be bought for \$212.50?

36. At $16\frac{2}{3}\text{¢}$ apiece, how many notebooks can a dealer buy for \$55?

37. At $87\frac{1}{2}\text{¢}$ apiece, how many children's desks can a dealer buy for \$28?

38. A dealer pays \$24 for some inkstands at $33\frac{1}{3}\text{¢}$ each. How many does he buy?

39. A dealer pays \$36 for some arithmetics at $33\frac{1}{3}\text{¢}$ each, and \$70 for some algebras at $66\frac{2}{3}\text{¢}$ each. How many books does he buy?

EXERCISE 224

PROBLEMS IN INTEREST

1. Find the interest on \$1250 for 2 yr. at 5%.
2. Find the interest on \$640 for $2\frac{1}{2}$ yr. at 5%.
3. Find the principal that will yield \$216 in 3 yr. at 6%.
4. How long will it take \$1500 to yield \$300 interest at 5% ? at 4% ? at $2\frac{1}{2}$ % ? at 6% ?

Find the interest on the following sums, given the rate and time as stated, canceling whenever possible :

- | | |
|----------------------------------|--|
| 5. \$250, 4%, $3\frac{3}{4}$ yr. | 10. \$1250, 6%, $2\frac{2}{3}$ yr. |
| 6. \$320, 4%, $2\frac{1}{4}$ yr. | 11. \$1675, 6%, $3\frac{2}{3}$ yr. |
| 7. \$680, 5%, $2\frac{1}{2}$ yr. | 12. \$2450, 5%, $2\frac{1}{2}$ yr. |
| 8. \$440, 6%, $3\frac{1}{3}$ yr. | 13. \$2750, $4\frac{1}{2}$ %, $3\frac{1}{2}$ yr. |
| 9. \$575, 4%, $2\frac{3}{4}$ yr. | 14. \$41.75, 4%, $6\frac{3}{4}$ yr. |

Find the principal that will yield the given interest at the given rate and time, canceling whenever possible :

- | | |
|--------------------------------------|--|
| 15. \$128, 4%, 5 yr. | 20. \$105, 6%, $3\frac{1}{2}$ yr. |
| 16. \$86.40, 4%, 3 yr. | 21. \$101.25, 6%, $2\frac{1}{4}$ yr. |
| 17. \$32.40, $4\frac{1}{2}$ %, 2 yr. | 22. \$33.75, 4%, $2\frac{1}{4}$ yr. |
| 18. \$52.80, $5\frac{1}{2}$ %, 2 yr. | 23. \$154, $5\frac{1}{2}$ %, $2\frac{1}{3}$ yr. |
| 19. \$105, $3\frac{1}{2}$ %, 6 yr. | 24. \$357.50, $5\frac{1}{2}$ %, $4\frac{1}{3}$ yr. |

Find the time it will take the given principal to yield the given interest at the given rate, canceling whenever possible :

- | | |
|-------------------------|--------------------------------------|
| 25. \$125, \$12.50, 4%. | 28. \$450, \$72.00, 6%. |
| 26. \$275, \$38.50, 6%. | 29. \$580, \$63.80, 4%. |
| 27. \$325, \$48.75, 5%. | 30. \$6000, \$770, $5\frac{1}{2}$ %. |

Find the rate at which the given principal will yield the given interest in the given time, canceling whenever possible :

- | | |
|---------------------------|--|
| 31. \$240, \$28.80, 2 yr. | 36. \$525, \$70, $3\frac{1}{2}$ yr. |
| 32. \$480, \$57.60, 3 yr. | 37. \$640, \$89.60, $3\frac{1}{2}$ yr. |
| 33. \$520, \$104, 4 yr. | 38. \$850, \$76.50, $2\frac{1}{4}$ yr. |
| 34. \$420, \$75.60, 3 yr. | 39. \$1200, \$99, $2\frac{3}{4}$ yr. |
| 35. \$5250, \$700, 4 yr. | 40. \$1500, \$210, $2\frac{1}{3}$ yr. |

Find by the 6% method the interest on the following :

- | | |
|------------------------|--------------------------|
| 41. \$175, 60 da., 6%. | 48. \$1200, 60 da., 5%. |
| 42. \$275, 60 da., 6%. | 49. \$1250, 30 da., 5%. |
| 43. \$350, 30 da., 6%. | 50. \$1600, 33 da., 5%. |
| 44. \$420, 30 da., 6%. | 51. \$1640, 90 da., 4%. |
| 45. \$640, 90 da., 6%. | 52. \$1850, 90 da., 3%. |
| 46. \$720, 90 da., 6%. | 53. \$24.50, 63 da., 5%. |
| 47. \$840, 93 da., 6%. | 54. \$76.80, 93 da., 3%. |

55. Find the difference between the interest on \$275.50 for 4 yr. at 5%, and that on \$275.50 for 5 yr. at 4%.

56. Compare the interest on \$3575 for 2 yr. at 6% with that on \$7150 for 2 yr. at 3%, and that on \$7150 for 1 yr. at 6%.

57. Compare the interest on \$640 for $2\frac{1}{2}$ yr. at 5% with that on \$640 for 5 yr. at $2\frac{1}{2}$ %, and that on \$1280 for $2\frac{1}{2}$ yr. at $2\frac{1}{2}$ %.

58. Using a short method, find the interest on \$1750 for 93 da. at 6%.

59. Using a short method, find the interest on \$1575 for 60 da. at 5% ; for 120 da. at $2\frac{1}{2}$ %.

60. Find the interest on \$640 for 2 yr. 6 mo. 15 da. at 6%, using the shortest method you know.

361. Short Methods applied to Business Forms. A bill that gives in detail a list of items of goods sold is often called an *invoice*. The following is an invoice of goods:

Syracuse, N. Y., Dec. 12, 1916

Mr. James P. Dunbar, Canandaigua, N. Y.

Bought of Randall Bros.

Terms: 10 da.

Importers of Dry Goods

236	14	pc. blue flannel				
		40, 41, 41 ¹ , 41 ³ ,				
		40 ² , 40, 41, 42,				
		41 ³ , 42 ¹ , 41, 42 ² ,				
		41, 40	576	33 ¹ / ₃	192	
427	8	pc. silk				
		40, 39, 41 ² , 40 ³ ,				
		39 ¹ , 41, 42 ² , 40	324	\$1 ²⁵ / ₂	405	597

In the above invoice, 41¹ means 41 $\frac{1}{4}$, 41² means 41 $\frac{1}{2}$, or 41 $\frac{1}{2}$, 41³ means 41 $\frac{3}{4}$. This is the customary way of indicating the number of yards in pieces of goods. The numbers 236 and 427 refer to the catalogue or price list in which the goods are described. The 14 and 8 indicate the number of pieces (pc.) of goods. The total number of yards of the first is 576, and of the second 324.

There is no essential difference between a bill and an invoice. The former term is more often used in small retail transactions.

EXERCISE 225

Make out bills, or invoices, for the following :

- 12 $\frac{1}{2}$ doz. colored shirts @ \$14.80, 6 $\frac{1}{4}$ doz. white shirts @ \$15.84, 3 $\frac{1}{2}$ doz. silk ties @ \$11.40, $\frac{2}{3}$ doz. ties @ \$9.75.
- 16 pc. ribbon @ \$1.25, 18 pc. @ \$1.33 $\frac{1}{3}$, 33 pc. @ \$1.66 $\frac{2}{3}$, 24 pc. @ \$1.12 $\frac{1}{2}$, 32 pc. @ \$1.37 $\frac{1}{2}$, 48 pc. @ \$1.75.

3. 2 carloads of coal, 21,700 lb., 24,200 lb., @ \$4.66 $\frac{2}{3}$ per ton; 3 carloads coal, 22,700 lb., 21,900 lb., 20,400 lb., @ \$4.75 per ton. Terms: 3% for cash.

4. 1 gro. cans sardines @ \$3.33 $\frac{1}{3}$ per doz., 8 doz. cans shrimps @ \$1.75 (per doz., as usual in such cases), 9 doz. tins herrings @ \$2.33 $\frac{1}{3}$, 6 $\frac{1}{4}$ doz. cans lobster @ \$2.88, 32 cans mackerel @ 16 $\frac{2}{3}$ ¢. Terms: 4% for cash.

5. 8 doz. packages codfish @ \$1.75, 9 doz. cans salmon @ \$2.33 $\frac{1}{3}$, 15 doz. cans caviar @ \$3.33 $\frac{1}{3}$, 18 doz. cans Yarmouth bloaters @ \$2.33 $\frac{1}{3}$, 24 doz. cans tongue @ \$8.75. Terms: 3 $\frac{1}{2}$ % for cash.

6. 8 doz. jars meat extract @ \$3.66 $\frac{2}{3}$, 16 doz. cans chicken @ \$3.12 $\frac{1}{2}$, 24 doz. cans beef @ \$2.33 $\frac{1}{3}$, 18 doz. cans soup @ \$3.33 $\frac{1}{3}$, 9 doz. cans clam chowder @ \$3.66 $\frac{2}{3}$, 16 doz. cans clam juice @ \$1.06 $\frac{1}{4}$. Terms: 3% for cash.

7. 16 doz. tins crackers @ \$4.25, 18 doz. tins wafers @ \$2.33 $\frac{1}{3}$, 25 doz. tins water crackers @ \$2.80, 9 doz. cartons chocolate rolls @ \$2.33 $\frac{1}{3}$, 24 doz. tins ginger wafers @ \$3.87 $\frac{1}{2}$. Terms: 4% for cash.

8. 30 M (30,000) laths @ \$4.25, 2800 ft. lumber @ \$75 per M, 3400 ft. lumber @ \$50 per M, 5600 ft. lumber @ \$66.66 $\frac{2}{3}$ per M, 6400 ft. lumber @ \$65.50 per M, 7500 ft. lumber @ \$64 per M.

9. 5 armchairs @ \$6.40, 37 kitchen chairs @ \$1.12 $\frac{1}{2}$, 8 kitchen tables @ \$2.25, 3 bedroom sets @ \$33.33 $\frac{1}{3}$, 16 dining tables @ \$12.50, 9 sideboards @ \$16.66 $\frac{2}{3}$, 16 rockers @ \$4.87 $\frac{1}{2}$. Terms: 12% for cash.

10. 20 pc. linen containing 39, 40², 38³, 42, 41¹, 40², 40, 39, 40¹, 41², 43, 39³, 40¹, 42¹, 40², 40¹, 41³, 42, 39¹, 40 yd., @ 66 $\frac{2}{3}$ ¢; 16 pc. silk containing 41¹, 42¹, 40, 39², 42, 40³, 38, 41¹, 43, 41³, 39, 40², 40¹, 41¹, 40, 39² yd., @ \$1.33 $\frac{1}{3}$. Terms: 4% for cash.

362. Statements. A written record showing the totals of a purchaser's debits and credits on various dates, with a statement of the balance due, is called a *statement*.

The following specimen of a statement shows that the items are not included, but only the totals. Itemized bills were sent for each of the totals. The first item shows that on January 8 Mr. Robertson bought merchandise (mdse.) to the amount of \$107.65.

Rochester, N. Y., Feb. 7, 1917

Mr. James Robertson, Lockport, N. Y.

In account with R. B. Thomas & Co.

Jan.	8	To mdse.	107	65		
	10	" "	68	90		
	17	" "	23	45	200	
	12	By cash	48	50		
	24	" "	132	40	180	90
					19	10

EXERCISE 226

Make out statements for the following, inserting names and dates:

1. To mdse., \$27.45, \$326.40, \$192.83, \$298.75, \$92.36, \$42.80. By cash, \$50, \$175, \$240, \$175, \$35.75.

2. To mdse., \$36.90, \$48.75, \$127.30, \$75.50, \$92.75, \$182.50, \$146.38. By cash, \$75, \$125, \$50, \$200.

3. To mdse., \$43.50, \$286.70, \$127.50, \$132.75, \$49.78, \$62.87, \$143.73, \$118.96, \$227.42. By cash, \$75, \$275.

4. To mdse., \$281.62, \$148.96, \$137.50, \$29.75, \$111.22, \$321.75. By cash, \$200, \$150, \$125, \$35.75.

363. Account. A statement of related income and expenses is called an *account*.

1914				1914			
Aug. 3	Balance	125	50	Aug. 3	Rent	75	
4	T. B. Sullivan	26	75	5	Grocery bill	18	42
5	R. S. Peters	87	50	10	Balance	146	33
		239	75			239	75
Aug. 10	Balance	146	33				

This illustrates a cash account. The receipts are placed on the left or *debit* side, and the payments on the right or *credit* side. The balance is found by subtracting the expenses (\$75 + \$18.42) from the receipts (\$239.75). This balance is then carried over to the left side for the beginning of the new week.

364. Account Sales. A statement rendered by a commission merchant to the one who employs him, showing the items of a transaction, is called an *account sales*.

New York, Aug. 8, 1914

Sale of Merchandise for Account of

R. P. Sanders, Ithaca, N.Y.

By HUNTLEY and ROBERTS, 4062 E. Canal St.

Aug. 11	300 bbl. King apples	\$3. ²⁰	960		
	75 bbl. Greenings	2. ⁹⁰	217	50	
					1177 50
	CHARGES				
	Drayage		6	75	
	Commission, 2%		23	55	30 30
	Remitted herewith				1147 20

This shows that Huntley and Roberts have sold \$1177.50 worth of apples for R. P. Sanders, charging drayage and commission and remitting the balance.

EXERCISE 227

Make out accounts, inserting dates, names, and items, and find the balance, given the following :

1. Receipts: Balance, \$127.30, \$75, \$46.50, \$21.75.
Payments: \$50, \$26.30, \$14.90.

2. Receipts: Balance, \$1275.60, \$128.75, \$62.50, \$27.80.
Payments: \$125, \$37.60, \$29.75, \$48.25.

3. Receipts: Balance, \$265.35, \$28.90, \$47.60, \$85. Pay-
ments: \$275, \$10, \$3.60, \$1.75, \$1.35, \$2.80.

4. Receipts: Balance, \$1526.30, \$27.42, \$428.75, \$125.75.
Payments: \$250, \$175, \$26.75, \$135.50, \$12.75.

5. Receipts: Balance, \$1275.50, \$248.75, \$432.50, \$18.75.
Payments: \$360, \$245, \$31.80, \$129.75, \$15.50.

Make out account sales, inserting dates and names, and find the balance remitted :

6. Sold: 400 bbl. Eagle flour @ \$5.90, 350 bbl. Acme flour @ \$6.20. Charges: Freight, \$17.20; cartage, \$16.50; commission, 4%.

7. Sold: 125 crates eggs @ \$6, 2500 lb. cheese @ 12½¢. Charges: Freight and drayage, \$21.75; commission, 4%.

8. Sold: 300 bbl. King apples @ \$3.25, 400 bbl. greenings @ \$2.80, 250 bbl. Baldwins @ \$3.10. Charges: Freight, \$31.50; drayage, \$18.25; commission, 5%.

9. Sold: 6000 lb. creamery butter @ 26¢, 3000 lb. cheese @ 13¢, 200 crates eggs @ \$6.75. Charges: Freight and drayage, \$32.75; commission, 5%.

10. Sold: 1200 bbl. Sun flour @ \$4.80, 600 bbl. Rose flour @ \$5.40. Charges: Freight and drayage, \$42.60; storage @ 5¢ per barrel; commission, 5%.

365. Settlement of Accounts. Find the balance due on the following account, Sept. 10, 1914, computing interest at 6% on each item from its date to the day of settlement, and reckoning the time in days :

1914	DR.	INT.	1914	CR.	INT.
June 29. To Mdse.	\$250	\$3.04	July 3. By Cash.	\$200	\$2.30
July 13. To Mdse.	400	3.93	July 17. By Cash.	125	1.15
July 27. To Mdse.	500	3.75	July 31. By Cash.	350	2.39
			Sept. 10. By bal. acct.	475	
Settled Sept. 10, '14.			Sept. 10. By bal. int.		4.88
	\$1150	\$10.72		\$1150	\$10.72

Hence, the cash balance is \$475 + \$4.88, or \$479.88.

When the balance of account and the balance of interest fall on *opposite* sides, the cash balance is their *difference*.

EXERCISE 228

Find the cash balance, Sept. 10, 1914, of the following accounts, reckoning interest at 6% :

1.

1914	Dr.	1914	Cr.
May 13. To Mdse.	\$250.00	May 27. By Cash.	\$200.00
May 28. To Mdse.	610.00	June 22. By Cash.	500.00
June 17. To Mdse.	300.00	June 29. By Cash.	400.00

2.

1914	Dr.	1914	Cr.
Mar. 7. To Mdse.	\$350.00	Apr. 3. By Cash.	\$150.00
Apr. 10. To Mdse.	98.50	May 2. By Cash.	150.00
May 25. To Mdse.	300.00	June 4. By Cash.	200.00

3.

1914	Dr.	1914	Cr.
May 8. To Mdse.	\$250.00	June 22. By Cash.	\$200.00
June 5. To Mdse.	670.00	July 20. By Cash.	500.00
July 3. To Mdse.	200.00	Aug. 19. By Cash.	300.00

CHAPTER XXI

STOCKS AND BONDS

366. Corporations. A company composed of a number of persons authorized by law to do business of a certain nature is called a *corporation*.

A *municipal corporation* is a corporation organized for local government, such as a state, a county, a city, or a town.

A corporation organized to transact private business is called a *stock company*.

This is an age of stock companies. Almost all extensive business operations are carried on by such companies. Money is invested more frequently through or in stock companies than in any other way, and it is one of the necessities of business training that we know the general features of such corporations.

367. Capital Stock. The amount contributed by the members of a corporation to carry on the business is called the *capital stock*, *capital*, or simply *stock* of the company.

One of the equal parts into which the stock of a company is divided is called a *share* of stock.

The most common amount for a share of stock is \$100, although shares of \$10, \$25, \$50, and other amounts are issued.

368. Stock Certificates. A statement issued by a corporation specifying the number of shares of stock owned by the holder is called a *stock certificate*.

This certificate is usually signed by the president and either the secretary or the treasurer. Certificates of bank stock are usually signed by the president and the cashier.

A person who owns one or more shares of stock is called a *stockholder*.

369. Organization of Corporations. The stockholders of a company select a few of their number to manage the business. These members are called *directors*. The directors elect the president, secretary, treasurer, and other officers.

370. Dividends. A sum paid to the stockholders out of the earnings of a company is called a *dividend*.

The directors pay the expenses of running a company from all the earnings (the *gross earnings*), and from the remainder (*net earnings*) they usually lay aside a portion (*undivided profits*) for emergencies, and divide the rest among the stockholders as dividends.

Dividends are a certain per cent of the face value of the stock. Thus a 5% dividend on 80 shares of stock of \$100 each is \$400.

If a company is losing instead of earning money, stockholders are often required to pay *assessments* on their stock to make up the loss, although the company usually borrows money to help in a temporary emergency.

371. Par Value. The face value of stocks is called the *par value*.

The par value of one share (§ 367) is commonly \$100, and should be so taken in all examples in this chapter.

372. Market Value. The price at which stocks sell is called their *market value*. If stocks sell for more than their face value, they are said to be *above par*, or at a *premium*; if for less than their face value, they are said to be *below par*, or at a *discount*.

If a company is prosperous and pays higher dividends, say 10%, than the money would earn in other ways, the stock will be sought by investors, and therefore a share of \$100 may sell for a larger sum than the par value, perhaps \$150 or more. This is because \$10 (10% of \$100) is $6\frac{2}{3}\%$ of \$150, so that if a man pays \$150 for the stock he is still getting $6\frac{2}{3}\%$ on his investment.

On the other hand, if a company is not prosperous and pays low dividends or none at all, then the stock will be below par.

The market value of stocks varies from day to day and often from minute to minute, depending upon the demand and supply.

373. Kinds of Stock. There are two leading kinds of stock, *preferred* and *common*.

Preferred stock is stock that has a fixed rate of dividend that must be paid before any other dividends can be paid.

The dividend on preferred stock is usually 5% to 7%. Investors who wish good security and a definite rate of income from year to year are inclined to buy preferred stock.

Common stock is stock that is entitled to share in the net earnings of a company after the dividends on the preferred stock have been paid.

If the net earnings of a company are very large, common stock may be worth more than preferred stock. For example, if the preferred stock of a company is entitled to 5%, and if the earnings are such as steadily to pay the common stockholders 10%, the common stock will be worth about twice as much as the preferred.

374. Buying Stock. A purchaser usually buys stock through a *broker*, and the broker obtains it through some other broker who has it for sale. The transaction between the brokers is usually carried on in a place called a *stock exchange*.

A stock exchange is merely a large auction room where brokers having stocks to sell may dispose of them to the highest bidder among the brokers who have orders to buy.

Brokers usually charge *brokerage* of $\frac{1}{8}\%$ for buying and $\frac{1}{8}\%$ for selling stock, computed on the par value. If stock is quoted in the newspaper at $96\frac{1}{2}$, it means that a \$100 share will cost the purchaser $\$96\frac{1}{2} + \$\frac{1}{8}$, or $\$96\frac{5}{8}$. It also means that the man who sells the stock receives $\$96\frac{1}{2} - \$\frac{1}{8}$, or $\$96\frac{3}{8}$, since he also has to pay brokerage.

The only fractions used in stock quotations are halves, quarters, and eighths. Fractional parts of a share are not sold. Stock purchased by speculators is usually in 100-share lots, although investors often purchase smaller amounts.

Stocks form a legitimate and common investment of earnings, in spite of the fact that many persons buy them on credit, hoping to make a profit by some sudden rise in value.

EXERCISE 229

Find the cost of the following stocks, adding $\frac{1}{8}\%$ brokerage in each case:

- | | |
|------------------------------------|------------------------------------|
| 1. 100 shares @ $106\frac{3}{8}$. | 6. 75 shares @ 62. |
| 2. 125 shares @ $112\frac{3}{8}$. | 7. 80 shares @ $75\frac{1}{4}$. |
| 3. 140 shares @ $115\frac{1}{2}$. | 8. 65 shares @ $98\frac{1}{2}$. |
| 4. 275 shares @ $127\frac{1}{4}$. | 9. 125 shares @ $99\frac{1}{8}$. |
| 5. 350 shares @ $142\frac{3}{4}$. | 10. 25 shares @ $127\frac{1}{2}$. |

Find the amount received from the sale of the following stocks, deducting $\frac{1}{8}\%$ brokerage in each case:

- | | |
|------------------------------------|-------------------------------------|
| 11. 75 shares @ 82. | 16. 125 shares @ 110. |
| 12. 25 shares @ $79\frac{1}{8}$. | 17. 150 shares @ $112\frac{1}{2}$. |
| 13. 50 shares @ $96\frac{7}{8}$. | 18. 175 shares @ $121\frac{1}{4}$. |
| 14. 120 shares @ $98\frac{1}{2}$. | 19. 250 shares @ $142\frac{7}{8}$. |
| 15. 250 shares @ 125. | 20. 275 shares @ par. |

21. A man bought 50 shares of stock when it was quoted at $96\frac{7}{8}$, and sold it when quoted at par. Allowing, as usual, $\frac{1}{8}\%$ brokerage on each transaction, how much did he gain?

22. A man bought 75 shares of stock when the market price was 120, and sold it when the market price was 115. Allowing the usual brokerage, how much did he lose?

23. A man bought 125 shares of stock when the market price was $102\frac{7}{8}$, and sold it when the market price was still the same. Allowing the usual brokerage, how much did he lose on the transaction?

24. A man has \$3875 to invest. He decides that he will buy a certain stock quoted at $96\frac{3}{8}$. Allowing the usual brokerage, how many shares can he buy, and how much money will he have left?

375. Newspaper Quotations. In the daily newspapers may be found quotations showing the prices at which leading stocks were sold in the last session of the stock exchange.

The great center for trading in railway stocks is the New York Stock Exchange, on Wall Street. Many large cities have stock exchanges in which local stocks are bought and sold. Stocks of manufacturing and industrial corporations are known as *industrial stocks*.

The following are specimens of newspaper quotations:

American Express Co.	208 $\frac{7}{8}$	Iowa Central	18 $\frac{5}{8}$
Atchison, Topeka, & S. F.	101 $\frac{7}{8}$	Louisville & Nashville	149 $\frac{3}{8}$
Baltimore & Ohio	100 $\frac{3}{4}$	Missouri Pacific	42 $\frac{3}{8}$
Central of N. J.	271	N. Y. Central	108 $\frac{1}{8}$
Chicago & Alton	24 $\frac{1}{4}$	Northern Pacific	120 $\frac{7}{8}$
Illinois Central	139 $\frac{7}{8}$	Union Pacific	170 $\frac{1}{8}$

In all cases remember to take into account the brokerage of $\frac{1}{8}\%$.

EXERCISE 230

Using the above quotations, find the cost of:

1. 150 shares of Iowa Central.
2. 125 shares of Illinois Central.
3. 75 shares of Baltimore & Ohio.
4. 110 shares of Chicago & Alton.
5. 25 shares of American Express Co.
6. 175 shares of Louisville & Nashville.
7. 180 shares of Central of New Jersey.
8. 50 shares of Atchison, Topeka, & Santa Fe.

Find the amount received from the sale of:

9. 25 shares of Missouri Pacific.
10. 35 shares of New York Central.
11. 75 shares of Northern Pacific.

Find the gain or loss in buying 25 shares of the following stocks as quoted on page 433, and selling at the price given below :

- | | |
|---|---------------------------------------|
| 12. American Exp., 210. | 18. Iowa Cent., $23\frac{1}{2}$. |
| 13. Atchison, $103\frac{1}{2}$. | 19. Louisv. & N., 151. |
| 14. Balt. & Ohio, $109\frac{7}{8}$. | 20. Missouri Pac., 41. |
| 15. Central of N.J., $265\frac{1}{2}$. | 21. N.Y. Cent., $107\frac{1}{2}$. |
| 16. Chic. & Alton, $28\frac{3}{4}$. | 22. Northern Pac., 125. |
| 17. Ill. Cent., $141\frac{1}{8}$. | 23. Union Pacific, $180\frac{1}{2}$. |

24. A man bought 250 shares of American Express at the price quoted on page 433, and sold 100 shares at 207 and the rest at $209\frac{1}{2}$. Did he gain or lose on the lot, and how much?

25. A man bought 250 shares of Baltimore & Ohio at the quotation in Example 14. He kept it a year, receiving meantime a dividend of 6%, and sold it at 112. Money being worth 5%, did he gain or lose, and how much?

26. A man bought 25 shares of New York Central at 108, 125 shares of Iowa Central at 29, and 50 shares of Illinois Central at 140. He sold them at the quotations given on page 433. Did he gain or lose, and how much?

27. A man bought 150 shares of Atchison at $101\frac{1}{8}$. He borrowed the money to pay for it, at 5% interest. He kept the stock 6 months and sold it at the quotation given on page 433, having meantime received a dividend of 3%. Did he gain or lose, and how much?

28. A man bought 75 shares of American Express, 25 shares of Iowa Central, 50 shares of Chicago & Alton, and 150 shares of New York Central, at the quotations given on page 433, and sold them at the quotations given at the top of this page. Did he gain or lose, and how much?

Find the number of shares of stock that can be bought with the following sums, at the quotations given, allowing the usual brokerage, and state the amount of money left in each case:

29. \$12,000, $119\frac{7}{8}$.

30. \$15,000, $74\frac{7}{8}$.

31. \$12,500, $62\frac{3}{8}$.

32. \$14,700, $73\frac{3}{8}$.

33. \$11,750, $58\frac{5}{8}$.

34. \$10,350, $51\frac{5}{8}$.

35. \$12,750, $63\frac{5}{8}$.

36. \$12,000, $29\frac{3}{4}$.

37. \$15,000, $74\frac{5}{8}$.

38. \$6000, $58\frac{3}{4}$.

39. \$7000, 139.

40. \$8000, $79\frac{5}{8}$.

41. \$9000, $22\frac{1}{4}$.

42. \$4000, $112\frac{1}{8}$.

43. \$9000, $114\frac{1}{8}$.

44. \$7000, $114\frac{7}{8}$.

45. \$6100, $121\frac{5}{8}$.

46. \$27,500, $68\frac{1}{2}$.

47. When New York, New Haven & Hartford R.R. stock is quoted at $136\frac{1}{2}$, how many shares can a man buy with \$6850, allowing the usual brokerage, and how much money will he have left?

48. When Reading R.R. stock is quoted at $143\frac{3}{4}$, how many shares can a man buy with \$10,100, allowing the usual brokerage, and how much money will he have left?

49. When Western Union Telegraph stock is quoted at $79\frac{3}{8}$, how many shares can a man buy with \$12,000, allowing the usual brokerage, and how much money will he have left?

50. A man has \$15,000 to invest. He wishes to put about half of it in a certain manufacturing concern of which the stock is selling at 149, and about half in railroad stock that is selling at $74\frac{3}{8}$. Allowing the usual brokerage, how many shares of each can he buy, and how much money will he have left?

376. Declaring Dividends. If the capital stock of a company is \$150,000, and the net earnings for six months are \$4875, the directors might be justified in declaring a dividend of $3\frac{1}{4}\%$, since \$4875 is $3\frac{1}{4}\%$ of \$150,000. They will probably, however, declare a dividend not exceeding 3% (\$4500) and carry the balance as undivided profits to the surplus fund to meet an emergency.

EXERCISE 231

1. If the capital is \$125,000, and \$3125 is distributed as dividends, what is the rate of dividend?

2. If the capital is \$75,000, and a dividend of $3\frac{1}{2}\%$ is declared, what is the amount of the dividend?

3. If a company declares a dividend of $4\frac{1}{2}\%$, and this amounts to \$4050, what is the capital?

4. If a company declares a dividend of $5\frac{1}{2}\%$, and this amounts to \$4,152,500, what is the capital?

5. If a company declares a dividend of 4% on its capital stock, how much dividend will a man receive who owns 75 shares?

6. A company having a capital of \$75,000 loses \$1875. If an assessment is made to pay this loss, what per cent of his stock must each stockholder be assessed?

7. The net earnings of a company having a capital of \$150,000 amounted to \$7677.56 in 6 months. The company carried \$927.56 to its surplus fund and divided the remainder in dividends. What was the rate of dividend?

8. A railroad company having a capital of \$178,632,000 earns \$98,369,059 in one year. Its operating expenses are \$75,803,333, and it pays interest on its debt and carries to surplus a total of \$11,847,806, dividing the remainder in dividends. What is the rate of dividend for the year?

377. Rate of Income. If stock paying a 6% dividend is bought at $149\frac{7}{8}$, or 150 including the brokerage, the owner receives \$6 on every \$150 invested. He therefore receives $\$6 \div \$150$, or 4% on his investment.

Always add the brokerage to the market price.

EXERCISE 232

Given the rate of dividend and the market price, find the rate of income on the investment:

- | | | |
|----------------------------|--|-----------------------------|
| 1. 5%, $124\frac{7}{8}$. | 7. $4\frac{1}{2}\%$, $112\frac{3}{8}$. | 13. 5%, $109\frac{7}{8}$. |
| 2. 8%, $199\frac{7}{8}$. | 8. 8%, $159\frac{7}{8}$. | 14. 4%, $98\frac{7}{8}$. |
| 3. 7%, $174\frac{7}{8}$. | 9. 7%, $87\frac{3}{8}$. | 15. 6%, $120\frac{3}{8}$. |
| 4. 7%, $139\frac{7}{8}$. | 10. $4\frac{1}{2}\%$, $74\frac{7}{8}$. | 16. 7%, $130\frac{1}{8}$. |
| 5. 9%, $149\frac{7}{8}$. | 11. 4%, $94\frac{7}{8}$. | 17. 10%, 190. |
| 6. 10%, $124\frac{7}{8}$. | 12. $4\frac{1}{2}\%$, $82\frac{3}{8}$. | 18. 15%, $299\frac{7}{8}$. |

19. Which will bring the better income, a 5% promissory note or a 6% stock quoted @ $119\frac{7}{8}$?

20. Which will bring the better income, a 5% promissory note or a 7% stock quoted @ 150? How much better?

21. Which will bring the better income, a 5% promissory note or an 8% stock quoted @ $149\frac{7}{8}$? How much better?

22. Which will bring the better income, a 6% stock quoted @ $139\frac{7}{8}$ or a 5% stock quoted @ $119\frac{7}{8}$? How much better?

23. If an investor receives \$6 on each share of stock he owns, and this is 5% on what he paid for the stock, how much did it cost him? What was the quoted market price?

24. If an investor receives 6% on his investment in a stock that pays 9% dividends, how much did the stock cost him? What was the quoted market price?

378. Bond. A written or printed promise to pay a specified sum at a given time, signed by the maker and often bearing his seal, is called a *bond*.

Bonds are usually secured by *mortgages* on the property of the maker, these being agreements by which the holders of the bonds may sell the property if either the interest or principal is unpaid.

Bonds are called *coupon bonds* if they have small promissory notes (coupons) annexed, which are cut off and presented for payment when the interest is due. They are called *registered bonds* if the owner's name is registered on the books of the company.

379. Illustrative Problems. How much is the income on \$7000 worth of 4% bonds?

The income is 4% of \$7000, or \$280.

Three-year bonds, paying 5%, are quoted at $102\frac{7}{8}$. What rate of income would an investor receive on the cost?

They cost $\$102\frac{7}{8} + \$\frac{1}{8}$, or \$103, for every \$100 face value.

They pay \$5 for every \$100 face value, but at the end of 3 years the investor receives only \$100 instead of \$103, or an average loss of \$1 a year (not counting interest on this amount). Therefore the net income is only \$5 - \$1, or \$4, a year on the \$103 invested.

Then $\$4 \div \$103 = 0.03883 +$, the approximate rate of income.

EXERCISE 233

Find the income on the following bonds:

- | | | |
|---|---------------------------------|---------------------------------|
| 1. 4%, \$9000. | 4. 3%, \$8500. | 7. $4\frac{1}{2}\%$, \$17,000. |
| 2. 5%, \$7000. | 5. 4%, \$6500. | 8. $5\frac{1}{2}\%$, \$12,000. |
| 3. 6%, \$6000. | 6. $3\frac{1}{2}\%$, \$13,500. | 9. $6\frac{1}{2}\%$, \$17,000. |
| 10. 6%, due in 2 years, quoted at $101\frac{7}{8}$. | | |
| 11. 7%, due in 4 years, quoted at $101\frac{7}{8}$. | | |
| 12. 5%, due in 10 years, quoted at $112\frac{3}{8}$. | | |

13. How many \$1000 bonds, quoted at $101\frac{1}{8}$, can be bought for \$2040, allowing the usual brokerage?

14. How many \$1000 bonds, quoted at $102\frac{3}{8}$, can be bought for \$3075, allowing the usual brokerage?

15. How many \$500 bonds, quoted at $103\frac{5}{8}$, can be bought for \$2075, allowing the usual brokerage?

16. How many \$1000 bonds, quoted at $104\frac{1}{8}$, can be bought with \$7500, allowing the usual brokerage, and how much money will be left?

17. How many \$500 bonds, quoted at $105\frac{7}{8}$, can be bought with \$5500, allowing the usual brokerage, and how much money will be left?

18. How many \$100 bonds, quoted at $102\frac{3}{8}$, can be bought with \$2100, allowing the usual brokerage, and how much money will be left?

Find the number of \$1000 bonds that can be bought with the following sums, at the quotations given, allowing the usual brokerage, and state the amount of money left over in each case:

19. \$2200, $108\frac{3}{8}$.

25. \$8750, $105\frac{1}{4}$.

20. \$3400, $109\frac{7}{8}$.

26. \$9000, $98\frac{5}{8}$.

21. \$4050, $101\frac{1}{8}$.

27. \$7900, $87\frac{3}{8}$.

22. \$5075, $101\frac{1}{4}$.

28. \$10,250, 102.

23. \$6100, $101\frac{3}{8}$.

29. \$14,000, $98\frac{7}{8}$.

24. \$7500, $102\frac{7}{8}$.

30. \$77,250, $96\frac{3}{8}$.

31. A man has \$22,500 to invest. He buys 100 shares of stock quoted at $111\frac{1}{8}$ and with the rest he buys as many \$1000 bonds as possible, these being quoted at $112\frac{5}{8}$. Allowing the usual brokerage, how many bonds does he buy, and how much money has he left?

EXERCISE 234

REVIEW PROBLEMS

1. Find the cost of 36 shares of N. Y. Central stock at $130\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

2. What will be the total cost to the purchaser of 96 shares of stock at $126\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

3. What per cent is realized from an investment in $4\frac{1}{2}\%$ stock at $124\frac{7}{8}$, brokerage $\frac{1}{8}\%$?

4. Find the number of shares of stock that can be bought for \$5160.75 @ $122\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

5. What dividend is a stock paying when the rate on the investment is $4\frac{4}{9}\%$, if the stock was bought for $110\frac{1}{8}$?

6. A man bought stock at 97 and sold at 102, brokerage in each case $\frac{1}{8}\%$. Find the net gain per share.

7. What would be the cost of an investment in U.S. 4's (United States 4% bonds) @ $131\frac{1}{4}$, brokerage $\frac{1}{8}\%$, to secure an annual income of \$720?

8. A man sold through his broker 156 shares of stock @ $3\frac{1}{2}\%$ discount, paying a brokerage of $\frac{1}{8}\%$. Find the brokerage and the net proceeds of the sale.

9. How much better rate of income will be received from an investment in 6% stock at $137\frac{3}{8}$ than in 4% stock at $109\frac{7}{8}$, brokerage in each case being $\frac{1}{8}\%$?

10. A man paid \$2550 for some 5% railway stock at $127\frac{3}{8}$, brokerage $\frac{1}{8}\%$. How many shares did he buy, and what was his annual income from the stock?

11. A man bought 50 shares of stock at $120\frac{1}{4}$, and 6 months later a $3\frac{1}{2}\%$ dividend was declared; he then sold the stock at 118, brokerage in each case $\frac{1}{8}\%$. Find his net gain or loss, money being worth 5%.

12. Which gives the better rate of income, 5% stock bought when quoted at $139\frac{7}{8}$ or $3\frac{1}{2}\%$ bonds bought when quoted at $99\frac{7}{8}$?

The time of maturity of the bond not being given, it need not be considered in this problem, and similarly in other cases on this page. In this problem we have to consider $\$5 \div \$140$ compared with $\$3.50 \div \$100$.

13. Which gives the better rate of income, a 6% bond at $119\frac{7}{8}$ or a 5% promissory note, allowing the usual brokerage in purchasing the bond?

We have to consider $\$6 \div \$120$, compared with $\$5$ on $\$100$.

14. If you had some money to invest, which would you prefer, a stock that regularly pays 8%, quoted at $159\frac{7}{8}$, a 5% bond at $109\frac{7}{8}$, or a $5\frac{1}{2}\%$ promissory note, the security being equally good?

Compare $\$8$ return on $\$160$ invested, $\$5$ return on $\$110$ invested, and $\$5.50$ return on $\$100$ invested. The $\$8$ on $\$160$ gives a rate of income of $\frac{5}{16}\%$, or 5%. What per cents do the others give?

15. If you had some money to invest, which would you prefer, a stock that regularly pays 7%, quoted at $149\frac{7}{8}$, a 4% bond at $89\frac{7}{8}$, or a $4\frac{1}{2}\%$ promissory note, the security being equally good?

Find which pays the better per cent, and how much, not considering the brokerage.

16. A 5% bond at 121 or a 4% bond at 97.

17. A 5% bond at 115 or a 4% bond at 92.

18. A 5% bond at 130 or a 6% bond at 156.

19. A $4\frac{1}{2}\%$ bond at par or a 5% bond at 111.

20. A 5% bond at 20 above par or a 4% bond at 4 below par.

This means a 5% bond at 120, or a 4% bond at 96.

EXERCISE 235

PROBLEMS WITHOUT NUMBERS

1. If you know the newspaper quotation on a certain stock, how do you find the cost of a share?
2. If a certain stock is quoted at a specified amount above par, how do you find the cost of a share?
3. About what would have to be the rate of dividend on a railroad stock to have it at par? to have it below par?
4. If you own some railroad stock and should sell it through a broker and buy it back at the same quoted price, how would you compute your loss?
5. If you own some stock and wish to sell it, and know the market price, how much will you receive for the stock?
6. Knowing the capital and the net earnings of a company, and the amount to be carried to surplus, how do you find the rate of dividend?
7. Knowing the quotation on a certain stock, and the rate of dividend, how do you find the rate of income on the amount invested?
8. How do you find the income on a certain number of bonds of a given par value, the rate being known?
9. How do you find the income on a certain number of bonds of a given par value, the time they have to run and the rate being known?
10. If the rate of interest on some bonds is known, and also the amount of income, how do you find the par value of the bonds?
11. What is the difference between preferred stock and common stock? between common stock and a bond? How do you decide which is best for an investment?

CHAPTER XXII

INSURANCE

380. Insurance. An agreement by which, for a lawful consideration, a company agrees to pay an indemnity for loss by fire or other specified cause is called *insurance*.

381. Policy. The written contract of insurance is called a *policy*. The sum agreed to be paid by the terms of an insurance policy is called the *face* of the policy. The consideration, or sum paid for insurance, is called the *premium*.

There are several kinds of insurance. There are companies that insure people's lives (life insurance); that insure against loss by fire (fire insurance), by accidents to ships (marine insurance), by accidents to people (accident insurance), by burglary (burglary insurance), and so on. Those who insure are often called *underwriters*.

382. Fire Insurance. Insurance against loss or damage by fire is called *fire insurance*.

In fire insurance the premium is sometimes stated as a certain sum on \$100, and sometimes as a certain rate per cent.

EXERCISE 236

Find the premiums on these policies at the rates specified:

- | | | |
|-------------------------------|--------------------------------|----------------------------------|
| 1. \$1500, $1\frac{1}{4}\%$. | 7. \$3800, $2\frac{1}{2}\%$. | 13. \$13,500, 2%. |
| 2. \$1500, $1\frac{1}{2}\%$. | 8. \$4500, $1\frac{3}{4}\%$. | 14. \$15,000, $1\frac{1}{4}\%$. |
| 3. \$1750, $1\frac{3}{4}\%$. | 9. \$5000, $1\frac{1}{2}\%$. | 15. \$12,750, $1\frac{3}{4}\%$. |
| 4. \$2250, $2\frac{1}{4}\%$. | 10. \$6500, $2\frac{1}{4}\%$. | 16. \$18,750, $2\frac{3}{4}\%$. |
| 5. \$3500, $1\frac{1}{2}\%$. | 11. \$7500, $2\frac{1}{2}\%$. | 17. \$15,200, 2.1%. |
| 6. \$2600, $2\frac{3}{4}\%$. | 12. \$12,500, 1%. | 18. \$16,500, 1.2%. |

Find the faces of policies, given the premiums and rates :

- | | | |
|---------------|----------------------------|--------------------------------|
| 19. \$65, 1%. | 21. \$19, $\frac{1}{2}$ %. | 23. \$19.75, 1%. |
| 20. \$48, 2%. | 22. \$24, $\frac{3}{4}$ %. | 24. \$41.25, $2\frac{1}{2}$ %. |

Find the rates, given the faces of policies and premiums :

- | | | |
|----------------------|-------------------|----------------------|
| 25. \$1800, \$18. | 27. \$2800, \$56. | 29. \$2100, \$31.50. |
| 26. \$1650, \$41.25. | 28. \$3200, \$56. | 30. \$3500, \$52.50. |

Find the premiums on the following policies, given the premiums per \$100:

- | | | |
|---------------------|---------------------|-----------------------|
| 31. \$2750, \$1.25. | 33. \$4800, \$1.75. | 35. \$15,000, \$2.50. |
| 32. \$3250, \$1.50. | 34. \$5600, \$2.25. | 36. \$27,500, \$2.75. |
37. At \$1.10, what is the premium on a \$3750 policy ?
38. At \$1.30, what is the premium on a \$5500 policy ?
39. A building worth \$4500 is insured for $\frac{2}{3}$ of its value at $1\frac{3}{4}$ %. What is the premium ?
40. The premium for insuring some property at \$1.75 is \$96.25. What is the face of the policy ?
41. Some property worth \$2700 is insured for $\frac{2}{3}$ of its value, the annual premium being \$27. What is the rate ?
42. Merchandise worth \$3600 is insured for 80% of its value at \$1.50 on \$100. What is the premium ?
43. If a 3-year policy for \$2500 costs \$60, what is the rate of premium per year ?
44. A building is insured at 60¢ per \$100, the annual premium being \$14.40. If the face of the policy is 80% of the value of the building, what is the value ?
45. A factory is insured for \$7000 in one company and \$5000 in another. The factory is partly destroyed by fire, the insurance adjusters fixing the damage at \$8000. How much should each company pay ?

383. Life and Accident Insurance. Insurance whereby some person known as the beneficiary receives a certain sum on the death of the insured, or when the insured reaches a certain age, is called *life insurance*.

Insurance whereby the insured receives a certain sum in case of accident, or his beneficiary receives it in case of death by accident, is called *accident insurance*.

The beneficiary may be the estate of the insured, or one or more persons named in the policy.

There are various forms of life insurance, but for the purposes of this arithmetic and of explaining the general principles of the subject, the standard form known as *ordinary life* is sufficient.

The rate in life and accident insurance is always given as the cost of \$1000 worth of insurance, the rate \$27.75 meaning that the annual premium on a policy for \$1000 is \$27.75. This subject is not, therefore, an application of percentage, but is properly considered here in connection with the other insurance problems.

EXERCISE 237

Find the premiums on these policies at the given rates:

- | | |
|---------------------|------------------------|
| 1. \$5000, \$21.75. | 6. \$4500, \$22.35. |
| 2. \$7500, \$22.65. | 7. \$3500, \$24.65. |
| 3. \$3500, \$28.45. | 8. \$12,000, \$25.62. |
| 4. \$4500, \$26.36. | 9. \$14,000, \$26.38. |
| 5. \$7000, \$27.75. | 10. \$15,500, \$29.46. |

11. If a man takes out a policy for \$7500 at \$26.36 per thousand, how much premium will he pay in 20 payments?

12. A man takes out a life insurance policy for \$4500 at \$24.30 per thousand and also an accident policy for \$3500 at \$6.25 per thousand. What do the two premiums together amount to in a year?

EXERCISE 238

REVIEW PROBLEMS

1. What is the face of the policy when the premium at $\frac{3}{4}\%$ is \$21?
2. A house valued at \$3216 is insured for $\frac{5}{8}$ of its value at $\frac{3}{4}\%$. What premium is paid?
3. A house valued at \$2750 is insured for 80% of its value at $\frac{3}{4}\%$. What premium is paid?
4. A man pays \$105 once in 3 years for insuring his house at $\frac{1}{4}\%$ annually. Find the face of the policy.
5. A house is insured for 80% of its value at 1%, the premium being \$36.80. What is the value of the house?
6. A house valued at \$4400 is insured for 80% of its value, the premium being \$44. What is the rate of insurance?
7. A man has a life insurance policy of \$7500 on which he pays an annual premium of \$28.60 per thousand. How much does he pay in premiums in five years?
8. A man has a life insurance policy on which he pays \$201 a year. The premiums are at the rate of \$26.80 per thousand. What is the face of the policy?
9. A man who pays an annual premium of $2\frac{1}{2}\%$ on a life insurance policy of \$1500 dies after 18 payments. How much more did his heirs receive than he paid?
10. A man pays an annual premium of \$5 a thousand on an accident insurance policy. If the total annual premium is \$62.50, what is the face of the policy?
11. A man pays an annual premium of \$7.50 a thousand on an accident insurance policy. The face of the policy is \$15,000. What is the annual premium?

CHAPTER XXIII

TAXATION

384. Tax. Money levied by a government for its support and for public purposes is called a *tax*.

385. Kinds of Taxes. There are two general kinds of taxes, direct and indirect. A *direct* tax is a tax levied on a person, or on the value of his property or business. An *indirect* tax is a tax levied in other ways, as on goods imported into the country, on tobacco, or on alcoholic liquors.

The expenses of the United States government are, in general, met by indirect taxes. The expenses of the several states, counties, cities, towns, and villages are, in general, met by direct taxes.

386. Poll Tax. A tax levied upon a person is called a *poll tax*.

In many places the poll tax is no longer collected.

387. Property Tax. A tax levied upon property is called a *property tax*.

This may be upon land and buildings (real estate), or upon money, securities, and movable property generally (personal property).

388. License. A permit to engage in some specified act or to enjoy some special privilege is called a *license*.

For example, licenses to run automobiles or to own dogs.

389. Assessing Taxes. Taxes are usually assessed against the individual by officers called *assessors*. They decide how much each person's property is worth and the rate of tax.

The *tax rate* is sometimes expressed as a rate per cent, but more often as the number of mills on a dollar.

390. Collecting Taxes. Taxes are collected by an officer known as the *collector*.

The collector sometimes receives a fixed salary and sometimes a commission on the amount of tax collected.

After the tax collector has collected taxes he pays the money over to an officer known as the *treasurer*, and by him it is paid out for public services according to law.

For example, a tax of \$43,600 is to be raised by a town which contains 1200 polls, taxed at \$1 each, and which has taxable property assessed at \$3,200,000. The town receives from the state \$7200 as its share of the railroad tax, and other corporation taxes collected by the state. Find the rate of taxation, and the tax paid by Mr. Brown, if his property is assessed at \$5960, if he pays for 1 poll, and if the collector's commission is 1%.

Total tax		\$43,600
Deduct poll taxes	\$1200	
Amount received from state	<u>7200</u>	
Total to be deducted	\$8400	<u>8,400</u>
Net property tax		\$35,200

Tax on \$1 = $\$35,200 \div 3,200,000 = \$0.011$.

That is, the tax is 11 mills on \$1, or \$11 on \$1000, or 1.1%.

Therefore Mr. Brown will pay as follows:

Property tax, 1.1% of \$5960,	\$65.56
Poll tax	<u>1.</u>
Property and poll tax	\$66.56
Add the collector's commission, 1% of \$66.56,	<u>.67</u>
Total tax, including commission	\$67.23

In practice, in a case like that of the above example, the assessors would probably make the tax rate 12 mills, or 1.2%, in order to provide for contingencies such as the reduction of assessments.

The subject of taxes should be treated as far as possible, in each school, from the standpoint of local conditions. In this connection there may properly be considered general questions of maintenance of the home, including rent, water, taxes, insurance, and interest on any mortgage that may be on a man's property.

391. Tax Table. Assessors usually prepare a table like the following, which is arranged for $5\frac{1}{2}$ mills on a dollar:

TAX TABLE. RATE $5\frac{1}{2}$ MILLS ON \$1

	0	1	2	3	4	5	6	7	8	9
0	0000	0055	0110	0165	0220	0275	0330	0385	0440	0495
1	0550	0605	0660	0715	0770	0825	0880	0935	0990	1045
2	1100	1155	1210	1265	1320	1375	1430	1485	1540	1595
3	1650	1705	1760	1815	1870	1925	1980	2035	2090	2145
4	2200	2255	2310	2365	2420	2475	2530	2585	2640	2695
5	2750	2805	2860	2915	2970	3025	3080	3135	3190	3245
6	3300	3355	3410	3465	3520	3575	3630	3685	3740	3795
7	3850	3905	3960	4015	4070	4125	4180	4235	4290	4345
8	4400	4455	4510	4565	4620	4675	4730	4785	4840	4895
9	4950	5005	5060	5115	5170	5225	5280	5335	5390	5445

Here the first figure of the number of dollars assessed is given at the left, and the second one at the top. Thus, at $5\frac{1}{2}$ mills on \$1, the tax on \$100 is \$0.55; on \$140, \$0.77; on \$2500, \$13.75.

For example, find by the above table the tax of James Johnson, who pays no poll tax and has his property assessed at \$9765, the collector's commission being 1%.

Tax on \$9700 (to the right of 0, and under 7)	\$53.35
Tax on \$65 (to the right of 6, and under 5)	.36 (.3575)
Property tax	\$53.71
Collector's commission, 1% of \$53.71,	.54
Total tax	\$54.25

EXERCISE 239

Using the above tax table, find the tax on the following amounts, adding the collector's commission of 1%:

- | | | | |
|-----------|------------|--------------|---------------|
| 1. \$250. | 4. \$4000. | 7. \$7550. | 10. \$11,500. |
| 2. \$850. | 5. \$3650. | 8. \$9000. | 11. \$11,250. |
| 3. \$900. | 6. \$4550. | 9. \$12,000. | 12. \$39,750. |

Make out a tax table for the tax rate of 9 mills on a dollar. Then find the tax on the following amounts, adding the collector's commission of 1%:

13. \$350.	17. \$5000.	21. \$5550.	25. \$12,500.
14. \$550.	18. \$3550.	22. \$8000.	26. \$26,500.
15. \$700.	19. \$4750.	23. \$13,000.	27. \$48,750.
16. \$3600.	20. \$3750.	24. \$35,000.	28. \$82,350.

29. The tax rate is 11 mills. How much tax must be paid on property assessed at \$4500?

In Exs. 29-36 omit the collector's commission.

30. The tax rate is 13 mills. How much tax must be paid on property assessed at \$12,500?

31. If a man's tax is \$97.50, and the rate is 13 mills on a dollar, what is the assessed valuation of his property?

32. If a man's property is assessed at \$6500, and his tax is \$78, what is the rate of tax?

33. If the assessed valuation of the property in a town is \$486,500, what must be the tax rate to raise \$5838?

34. The tax rate is 14 mills. A man pays a tax of \$54.64, of which \$2 is poll tax. What is the assessed valuation of his property?

35. If the state tax is 1 mill, the county tax $3\frac{1}{2}$ mills, and the town tax 4 mills, how much tax must a man pay on a house assessed at \$5000?

36. A man died, leaving \$17,500, of which \$16,000 was willed to his children and \$1500 to a friend. Under the laws of New York state there is an inheritance tax of 1% to be paid by children on sums in excess of \$5000 (and less than \$50,000), and 5% by those not related on sums in excess of \$1000 (and less than \$50,000). How much tax must be paid on these bequests?

392. Indirect Taxes. Indirect taxes have already been defined in § 385. One of the most important is the tax on goods imported into this country. This tax is paid to the United States government.

393. Duties or Customs. Taxes levied by the government on imported goods are called *duties* or *customs*.

Duties are collected at customhouses situated in large cities or in important seaports or places on the border between different countries, as between the United States and Canada or Mexico.

394. Kinds of Duty. There are two kinds of duty, specific and ad valorem. A *specific duty* is a definite sum levied on each unit by which an article is measured or weighed. An *ad valorem* duty is a certain per cent on the value of the goods.

395. Tariff. A schedule of duties on different articles, such as are imposed by our Congress, is called a *tariff*.

Goods on which duty is not levied are placed on the *free list*.

EXERCISE 240

Find the ad valorem duty on each of the following invoices of goods at the rate specified:

- | | | |
|----------------|------------------|--------------------|
| 1. \$475, 30%. | 6. \$1500, 25%. | 11. \$12,450, 20%. |
| 2. \$625, 35%. | 7. \$1650, 30%. | 12. \$42,375, 30%. |
| 3. \$750, 40%. | 8. \$2700, 35%. | 13. \$29,362, 35%. |
| 4. \$835, 45%. | 9. \$3650, 40%. | 14. \$32,275, 50%. |
| 5. \$975, 60%. | 10. \$7575, 50%. | 15. \$75,295, 60%. |

16. Find the specific duty on a gross of articles at 17¢ each.

17. Find the specific duty on 275 articles at \$1.25 each.

18. The duty paid on a certain invoice of goods was \$4500, at 60% ad valorem. What was the valuation on which the duty was reckoned?

19. Find the specific duty on 756 lb. of a certain material at 55¢ a pound.

20. An importer pays \$952.50 ad valorem duty on goods valued at \$6350. What is the rate of duty?

21. Ad valorem duty is levied on goods valued at \$2750, making the total cost \$3850. What is the rate of duty?

22. If the duty at 45% is \$3723.75, what is the valuation on which the duty is reckoned?

23. If a duty of \$3237.50 ad valorem is levied on goods valued at \$9250, what is the rate of duty?

24. The duty on certain farm products is \$4 a ton. If a dealer imports $82\frac{3}{4}$ tons, what is the duty?

25. Some goods cost the importer \$10,875, including the duty of 45%. How much was the duty?

26. Some goods cost the importer \$3712.50, including the duty of 35%. What was the original cost?

27. The cost of certain goods plus the duty of $33\frac{1}{3}\%$ was \$8800. Required to find the original cost.

28. The duty on certain goods is 22¢ a pound, plus 30% ad valorem. If the goods weigh 275 lb. and are worth \$360, what is the total duty?

29. The duty on a certain product is 60¢ a pound, plus 45% ad valorem. If the product weighs 168 lb. and is worth \$380, what is the total duty?

30. A man imported some goods on which he paid a specific duty of \$385. If the duty is 22¢ a pound, how many pounds did he import?

31. A man imported 475 lb. of goods on which he paid a specific duty of 6¢ a pound and an ad valorem duty of 25%. The total duty was \$216. What was the valuation on which the duty was reckoned?

CHAPTER XXIV

MISCELLANEOUS PROBLEMS

EXERCISE 241

1. If 48 yd. of cloth costing 3s. 6d. a yard are sold for £10 4s., what is the per cent of gain?
2. What will it cost for rails for 5 mi. of double track, if the rails cost \$28 a ton and weigh 100 lb. per yard?
3. A and B together have \$153. If $\frac{3}{4}$ of A's money is equal to $\frac{2}{3}$ of B's, how much has each?
4. Find the weight of a bar of cast iron 6 in. wide, 4 in. thick, and 3 ft. long, allowing 0.26 lb. per cubic inch.
5. A bar of steel 3 in. square weighs 155.52 lb. Allowing 0.288 lb. per cubic inch, how long is the bar?
6. If the pressure of steam in a boiler is 140 lb. to the square inch, find the pressure on a valve $2\frac{7}{8}$ in. in diameter.
7. How much will 275 cu. in. of steel weigh, if 1 cu. ft. weighs 490 lb.?
8. A boiler contains 480 smoke tubes, each with an internal diameter of $2\frac{3}{8}$ in. Find the total cross area of all the tubes for draft.
9. A boat travels 15 mi. down stream in $2\frac{1}{2}$ hr. The boat's rate of travel in still water is $4\frac{1}{2}$ mi. an hour. In what time can the boat return?
10. A man has $\frac{5}{8}$ of his property invested in a farm and $\frac{1}{8}$ of it in railway stock. The remainder, \$1500, is deposited in a savings bank. Find the value of the farm and of the railway stock.

11. The amount of water that flows over Niagara Falls averages 264,000 cu. ft. a second. Find the average number of gallons carried over the falls in a second.

12. The diagonal of a square is 20 ft. Find to two decimal places the length of one side.

13. If $\frac{7}{8}$ in. on a map represents an actual distance of $35\frac{1}{2}$ mi., what distance on the map represents 28.4 mi.?

14. How many loads of gravel averaging 1 cu. yd. will be required to grade $2\frac{3}{4}$ mi. of road, the gravel to be laid 14 ft. wide and 6 in. deep?

15. If a dealer pays \$9737.50 for some wool, including the purchasing agent's commission of $2\frac{1}{2}\%$, how much is the amount of the commission?

16. The sugar contained in the sugar beet is $6\frac{1}{4}\%$ of the weight of the beet. How many pounds of beets will be required to produce $122\frac{1}{2}$ lb. of sugar?

17. How many boards 12 ft. long will be required to build a straight fence 4 boards high about a rectangular field 40 rd. wide containing 20 acres?

18. Find the circumference of a circle whose diameter is 22.6 in., using 3.1416 as the ratio of circumference to diameter; also using $3\frac{1}{7}$; also using the ratio 355:113.

19. If 15 burners, each consuming 6 cu. ft. of gas per hour, are used on an average 4 hr. per day for 365 days, what will be the gas bill at 80¢ per thousand?

20. Flint glass contains by weight 72% sand, 14% soda, 12% lime, and 2% alumina. Find the number of pounds of each of these substances in 150 lb. of flint glass.

21. A bell weighing 1250 lb. is composed of 77% copper, 21% tin, and the rest zinc. How many pounds of each metal does the bell contain?

22. Bronze contains 9 parts of copper by weight to 2 parts of tin. How many pounds of tin in a bronze statue that weighs 2662 lb.?

23. The time from one new moon to the next is 29 da. 12 hr. 44 min. 3 sec. Express this in days and the decimal fraction of a day.

24. Find the difference in the number of acres in two fields, one of which contains 75 sq. rd., and the other of which is 75 rd. square.

25. Find to the nearest pound the weight of the air in a schoolroom $30' \times 16' \times 12'$, if water is 770 times as heavy as air and weighs 1000 oz. per cubic foot.

26. If a certain gun metal is composed of 84% copper and the rest tin, how many pounds of tin will be needed with 58.8 lb. of copper to make the gun metal?

27. A man, computing the cost of 54 articles, multiplied by mistake the cost of each by 45 instead of 54, and obtained \$121.50. What was the cost of the 54 articles?

28. A company with a capital of \$350,000 earned net \$28,250 last year. It paid a 7% dividend and carried the balance to surplus. How much did it carry to surplus?

29. A gardener finds that a bin 8 ft. long, 3 ft. 6 in. wide, and 5 ft. deep holds 112 bu. How many bushels in a bin that is 25% longer, 25% narrower, and has the same depth?

30. Find the cost of a stair carpet at \$1.35 a yard for a straight flight of 19 steps 10 in. wide and 8 in. rise, allowing 15 in. extra for the bottom of the stairs and the same for the top.

31. Water is flowing into a rectangular tank whose base is 6 ft. 8 in. long and 4 ft. 6 in. wide, at the rate of 2 cu. ft. in 3 min. How long will it take the water to fill it to a depth of 6 ft.?

32. Find the cost of 325 boards 16 ft. long, 4 in. wide, and $\frac{7}{8}$ in. thick at \$27 per M.

33. Find the number of dry quarts a cylindrical measure $16\frac{1}{2}$ in. in diameter and 1 ft. high will hold.

34. At what rate will \$1250 yield \$112.50 interest in 2 yr.? at what rate will it yield \$206.25 interest in 3 yr.?

35. What fraction of the year 1912 is the time from January 20 to March 15, including both of these days?

36. On seven successive days at noon the thermometer indicated 66° , 72° , 61° , 68° , 73° , 62° , 65° . What was the average for the week?

37. On seven successive days the barometer indicated 29.22 in., 29.18 in., 29.65 in., 30.02 in., 30.12 in., 30.18 in., and 30.21 in. What was the average for the week?

38. A cubic inch of iron weighs $4\frac{5}{8}$ oz. Find the weight in pounds of an iron bar 1 in. square at the end and 1 yd. long; 1 in. square at the end and 15 ft. long.

39. The driving wheels of a locomotive are 6 ft. 8 in. in diameter. How many revolutions a minute does each wheel make when the locomotive is traveling 48 mi. an hour?

40. When water is heated from 32° to 212° it expands $\frac{1}{24}$ in volume. If a cubic foot of water at the freezing point weighs $62\frac{1}{2}$ lb., what does a cubic foot of boiling water weigh?

41. A man bought a bill of goods in Paris to the value of 2600 fr. He paid 40% ad valorem duty and \$17.65 freight, and sent a draft for the purchase price. What was the total cost in United States money?

42. A pole is broken by the wind 12 ft. above the ground. The two parts hang together, but the upper part is bent over and touches the ground 12 ft. from the base. How long was the pole originally? (Two decimals.)

43. If it takes 1 bbl. of lime and $\frac{5}{8}$ cu. yd. of sand to make mortar for 1000 bricks, and if $22\frac{1}{2}$ bricks are allowed per cubic foot of wall, how much lime and how much sand are needed for a wall $28' \times 6' \times 1' 6''$?

44. The diagonal of one face of a cube is $\sqrt{450}$ in. Find the number of square inches in the entire surface of the cube and the number of cubic inches in the volume.

45. If a horse is tethered by a 40-foot rope, the length of the largest circumference he can walk around is how many times what it would be if he were tethered by a 20-foot rope?

46. In Ex. 45 the horse can graze over how many times as much ground in the first case as in the second?

47. One man can do a piece of work in 8 days and another man can do it in 10 days. How many days will it take the two together to do the work?

48. If X can build a wall in 18 days, Y in 15 days, and Z in 12 days, how many days will it take all three working together to build the wall?

49. If X, Y, and Z working together can dig a ditch in 24 hr., and X and Z working together can dig it in 32 hr., how many hours will it take Y alone to dig the ditch?

50. A water tank can be filled by three pipes, running separately, in 4 hours, 5 hours, and 6 hours respectively. In how many hours will the empty tank be filled by all three running together?

51. If the eye is 8 in. above the surface of a lake it can see about 1 mi.; if it is $2^2 \times 8$ in. above the surface, 2 mi.; if it is $3^2 \times 8$ in. above the surface, 3 mi.; and so on, the height being approximately 8 in. multiplied by the square of the distance in miles. About how far can a man see in one direction, if his eye is 36 ft. above the surface of the sea? (Two decimals.)

52. How many steps of 2 ft. 9 in. each would you have to take a minute in order to walk at the rate of 3 mi. an hour?

53. A machine does some work in 8 hr. If its capacity is increased $14\frac{2}{7}\%$, how long will it take to do the work?

54. Of three partners, the first is entitled to $37\frac{1}{2}\%$ and the second to $43\frac{3}{4}\%$ of the profits. If they make \$4800, what is the share of each of the three?

55. A certain alloy of metal is composed of $17\frac{1}{2}\%$ tin and the rest copper. How many pounds of tin are needed with $742\frac{1}{2}$ lb. of copper to make the alloy?

56. Two cans have a capacity of 5 pt. and 8 pt. respectively. How could you measure out 14 pt. into a third can by using these alone?

57. A dealer marked a set of furniture 25% above cost. He then sold it at 10% less than the marked price, and made a profit of \$5. What did the furniture cost him?

58. A piece of carpet 27 in. wide weighs 4 lb. to the yard. What is the weight of carpet necessary to cover a room 18 ft. wide and 24 ft. long?

59. If a gallon of water weighs $8\frac{1}{4}$ lb. and a cubic foot of water weighs 1000 oz., how many gallons are there in a cubic foot of water?

60. What is the area of the square inscribed in a circle whose radius is $3\sqrt{2}$ in., so that the corners lie on the circumference? of the square circumscribed about the circle so that its sides just touch the circumference?

61. The area of a circle with radius 3 in. is 28.2744 sq. in. Required the area of a circle with radius 6 in.

62. The equatorial diameter of the earth is estimated as 20,926,202 ft., and the polar diameter as 20,854,895 ft. Express each in miles and find the difference.

63. A man computing the cost of 56 articles made the mistake of multiplying by 65 instead of 56, and obtained \$487.50. What was the correct cost of the lot?

64. Two masts of a ship are 52 ft. apart, one being 80 ft. high and the other 70 ft. Supposing them perpendicular to the deck, how far is it from the top of one mast directly to the top of the other? (Two decimal places.)

65. In closing out a line of goods a merchant asked 25% less than the cost of a certain article, but at an auction he succeeded in getting $33\frac{1}{3}\%$ more than this asking price. Did he gain or lose on the cost, and what per cent?

66. Water is flowing into a tank whose base measure is 4' 2" by 3' 6", at the rate of 1 cu. ft. every 90 sec. How long will it take to fill the tank to a depth of 1' 6"?

67. A 12-inch gun can fire a shell weighing 850 lb. every 30 sec. At this rate how many pounds of shell could a battleship fire from four 12-inch guns in 2 min. 30 sec.?

68. A train leaves a city at 10.45 A.M. and reaches another city $127\frac{1}{2}$ mi. distant at 5 minutes past 2 P.M. Allowing 20 minutes for stops, what is the rate per hour?

69. A farmer finds that a bin 8 ft. long, 3 ft. 6 in. wide, and 5 ft. deep holds 112 bu. How many bushels in a bin that is half as long, 6 in. wider, and 1 ft. less in depth?

70. If 12 men can lay the asphalt blocks in a certain length of a city street in 16 da., how many men must be added to the force, to complete the work 4 da. quicker?

71. A village has a water tank 42 ft. in diameter. In 3 hr. 45 min., when no water is being pumped in, the water is lowered 3 ft. How many gallons are drawn out on an average per hour? (Use $7\frac{1}{2}$ gal. to 1 cu. ft.)

72. The rates of the express and local trains are 45.6 mi. an hour and 28.4 mi. an hour respectively. What time is saved by taking the express train for a distance of 161.88 mi.?

73. From a tank full of water $\frac{3}{5}$ of the water was drawn off. The tank then lacked 70 gal. of being half full. What is the capacity of the tank?

74. A tank that holds $166\frac{1}{2}$ gal. is $\frac{2}{3}$ full of water. How long will it take a pipe supplying water at the rate of 3.7 gal. a minute to fill the rest of the tank?

75. A man who owned some shares of stock in a mill sold $\frac{1}{3}$ of his shares to A, $\frac{1}{5}$ to B, and $\frac{1}{6}$ to C. He then had 90 shares left. How many shares had he at first?

76. A boy lost $\frac{1}{6}$ of his kite string in a tree and 30 ft. of it in some telegraph wires. He then had $\frac{3}{4}$ of it left. How long was it at first?

77. How many bricks will be required for the walls of a house 42 ft. long, 28 ft. front, and 18 ft. high, deducting 3 doors 7 ft. 6 in. by 4 ft., and 9 windows 5 ft. by 3 ft., the walls to be 1 ft. thick, allowing 22 bricks to 1 cu. ft.?

78. The sugar contained in the sugar beet is $6\frac{1}{4}\%$ of the weight of the beet. In a recent year we produced in the United States 967,223,000 lb. of sugar beets. How many pounds of sugar did this represent?

79. A man has a salary of \$1750 a year. He pays 15% of his salary for rent, 25% for groceries, 12% for meats, 13% for clothing, and \$112.50 for other expenses. How much does he spend for each of these items, and what per cent of his salary does he save?

80. How many tons of coal can be placed in a bin 16 ft. 6 in. long, 7 ft. wide and 6 ft. deep, allowing 33 cu. ft. to the ton?

CHAPTER XXV
VOCATIONAL PROBLEMS
EXERCISE 242

PROBLEMS IN HOUSEHOLD ARTS

1. A recipe calls for 4 lb. of fruit, 2 lb. of sugar, and 1 pt. of water. If we wish to use 15 lb. of fruit, how much sugar and water must we use?

2. What multiplier must you use in enlarging a recipe that calls for 3 lb. of fruit, when you wish to use 16 lb.?

3. If 2 oz. of gelatin stiffens 8 cups of liquid, how much gelatin will be needed to stiffen 10 cups?

4. In making some preserves a girl used twice as much fruit as sugar, and twice as much sugar as water. Each is what per cent of the total amount?

5. In preserving strawberries a girl uses 12 lb. of berries to 6 lb. of sugar. The weight of sugar is what per cent of the weight of berries? of the total weight of berries and sugar? The weight of berries is what per cent of this total weight?

6. If we use 4 lb. of sugar to 8 lb. of raspberries in making preserves, the weight of the sugar is what per cent of the weight of the berries? of the total weight of sugar and berries? The weight of the berries is what per cent of this total weight?

7. The ice chamber in an ice box is 24 in. by 16 in. by 18 in. Ice weighs 8% less than water, and 1 cu. ft. of water weighs $62\frac{1}{2}$ lb. If ice comes in rectangular pieces weighing 10, 25, 50, 100, 150, and 200 lb., which one of these pieces would come nearest to filling the chamber?

Problem Data. The following price list may be used in solving the problems on page 463 and similar problems.

The data may also be secured by the pupils by inquiry at home or at some grocery. This may be made the basis of practical problems in simple domestic bookkeeping.

Coffee, Maracaibo, 18¢ per pound ; 5 lb. for 85¢.

Java and Mocha, 31¢ per pound ; 5 lb. for \$1.45.

Old Government Java, green, 27¢ per pound ; 5 lb. for \$1.30.

Tea, Formosa Oolong, 50¢ per pound.

English Breakfast, 40¢ per pound.

Ceylon, finest, 80¢ per pound.

Cocoa, half-pound cans, 33¢ ; \$3.75 per dozen cans.

Sugar sirup, half-gallon cans, 50¢ ; 5-gallon can, \$4.

Maple sirup, gallon cans, \$1.45 ; \$16.50 per dozen cans.

Honey, 8-ounce bottles, 40¢ ; \$4.50 per dozen bottles.

Flour, buckwheat, 5¢ per pound ; a bag of 24½ lb., \$1.10.

Self-raising, 3 lb. for 19¢ ; 6 lb. for 35¢.

Wheat, 4¢ per pound ; \$7 per barrel of 196 lb.

Oatmeal, 12¢ per pound ; 5 lb. for 50¢.

Tapioca, 8¢ per pound.

Best imported, 22¢ per package ; \$2.50 per dozen packages.

Breakfast cereal, 14¢ per package ; \$1.60 per dozen packages.

Macaroni, 12¢ per package ; 25 packages for \$2.75.

Crackers, Alberts, 23¢ per tin ; \$2.65 per dozen tins.

Soda, 5¢ per carton ; 55¢ per dozen cartons.

Soups, half-pint cans, 10¢ ; pint cans, 16¢ ; quart cans, 28¢ ; \$3.25 per dozen quart cans.

Bacon, American, 22¢ per pound.

English, 40¢ per pound.

Sliced, in jars, 26¢ per pound ; \$3.10 per dozen jars.

Herrings, 15¢ per can ; \$1.75 per dozen cans.

Asparagus, 50¢ per can ; \$5.75 per dozen cans.

Prunes, 20¢ per pound.

Mustard, 14¢ per ¼ pound ; 24¢ per ½ pound ; 47¢ per pound, in tins.

Olives, 32¢ per bottle ; \$3.75 per dozen bottles.

Olive oil, 65¢ per bottle ; 38¢ per ½ bottle ; \$7.75 per dozen bottles.

Allspice, 15¢ per pound.

Cloves, 40¢ per pound.

Cinnamon, 40¢ per pound.

Nutmegs, 55¢ per pound.

Pepper, 22¢ per pound.

Ginger, 20¢ per pound.

EXERCISE 243

PROBLEMS IN HOUSEHOLD ARTS

1. If a family needs a dozen cans of cocoa, how much is saved by purchasing at the dozen rate?

2. If a family needs 5 gal. of sugar sirup, how much is saved by purchasing a 5-gallon can instead of buying ten half-gallon cans? What per cent is saved?

3. How much would a hotel save by purchasing maple sirup by the dozen gallon cans instead of buying single cans? What per cent would it save?

In such problems the equation method of solution may profitably be followed. Its use should be encouraged throughout the course.

Thus, $x\%$ of \$17.40 = \$0.90.

This equation may now be solved.

4. In which is the per cent of saving greater, in buying honey by the dozen bottles instead of by the bottle, or in buying maple sirup by the dozen cans instead of by the can?

5. How much is buckwheat flour a pound? At this rate, what will $24\frac{1}{2}$ lb. cost? How much is saved in buying a bag?

6. What is the per cent of saving in buying self-raising flour by the 6-pound package instead of by the 3-pound package?

7. By inquiry at home, make out a grocery list for a week, from page 462. Make two pages of a home account, the left-hand page showing the amount received, and the right-hand page showing the amounts spent for groceries.

Similar monthly and yearly lists may be made by the class, as suggested in the New York State Syllabus.

8. Make out a bill for six items of groceries, making the proper extensions and footing. Receipt the bill.

9. Make out a bill for ten items of groceries, as in Ex. 8.

EXERCISE 244

PROBLEMS IN HOUSEHOLD ARTS

1. How much will it cost, at \$1.35 a yard, to carpet a room 18 ft. by 19 ft. 6 in. with plain carpet $\frac{3}{4}$ yd. wide?

2. How much will it cost, at \$1.40 a yard, to cover the floor of a room 20 ft. 3 in. by 28 ft. 6 in. with 27-inch carpet, allowing 2 yd. for matching patterns?

Similar practical problems should be secured by the pupils at home, and brought to the class for solution.

3. How many sheets for single beds can be made from a piece of sheeting $\frac{6}{4}$ wide and 64 yd. long, each sheet being $2\frac{1}{2}$ yd. long with a top hem $3\frac{1}{2}$ in. and a bottom hem 2 in., an allowance of $\frac{1}{4}$ in. being made for each hem?

In buying sheeting the width of the material is given in quarters of a yard. Sheetting $\frac{6}{4}$ wide means 6 quarters of a yard wide. Sheetting $\frac{6}{4}$ wide is used on single beds, $\frac{8}{4}$ wide on three-quarter beds and $\frac{10}{4}$ wide on double beds.

4. A piece of sheeting of $\frac{8}{4}$ width contains 56 yd. How many sheets for three-quarter beds can be made from it, allowing the same measurements as in Ex. 3?

5. A piece of sheeting of $\frac{10}{4}$ width contains 48 yd. How many sheets for double beds can be made from it, allowing the same measurements as in Ex. 3?

The class should study the simple measurements needed in the household, including the measurement of materials used in food and clothing. The cost of furnishing a kitchen, laundry, dining room, living room, and bedroom, with the expense of renovating them, should also be considered. Local prices and conditions make these problems real to the pupils.

Where gas is used, the meter should be read by the class, and the cost of lighting computed. The relative cost of different systems of heating and lighting in the school and home should be considered according to local conditions.

EXERCISE 245

PROBLEMS IN MANUAL ARTS

The following price list may be used in solving the problems on this page, and similar problems :

School workbench	\$8.50	Plane	\$1.70
Jack plane	1.75	Oilstone	1.00
Backsaw	.95	Screwdriver, 4"	.20
Ripsaw	.90	Screwdriver, 12"	.60
Chisel, $\frac{1}{4}$ "	.20	Vise for woodwork	8.25
Chisel, $\frac{1}{2}$ "	.25	Miter box	8.50
Try-square	.25	Miter square	.65
Compasses	.15	Steel square	.75
Block plane	.80	Brace	1.00

1. What is the amount of a 10% discount on a workbench ? on a vise ? on a jack plane ? on a miter box ?

2. What is the cost of 5 quarter-inch chisels ? of 4 block planes ? of a workbench, a jack plane, and 2 backsaws ?

3. What is the cost of 3 quarter-inch chisels, 4 half-inch chisels, a try-square, and 2 pairs of compasses ?

4. What is the cost of two block planes, an oilstone, and 2 miter squares, a discount of 10% being allowed ?

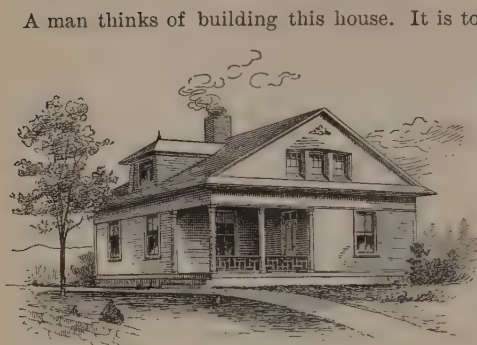
5. What is the cost of 2 steel squares and a four-inch screwdriver, a discount of 10% being allowed ?

6. What is the cost of a twelve-inch screwdriver, 2 vises, 4 ripsaws, and a miter box, a discount of 12% being allowed ?

The work in Manual Arts varies with different schools. There should be a constant correlation between it and arithmetic. Pupils may, as local conditions allow, make sketches for a school building, with simple working drawings. They may work out a cost slip of various articles made, including compensation for labor, the cost of power, the interest on the money invested in the school shop, and the materials. Where there is wood turning, proportion may be applied to the speed of the lathe, taking the speed of the motor as a basis, and similar problems may be devised.

EXERCISE 246

PROBLEMS IN MANUAL ARTS



A man thinks of building this house. It is to be $27\frac{1}{2}' \times 34'$. The walls are to be 14' to the eaves, the ceilings in the first floor 9' and in the second floor 8'. A builder estimates the cost of some of the materials. Part of the computation is given in Exs. 6-8; the rest is left for the

class to work out. "Feet of lumber" means feet board measure (B.M.).

1. If a board 8' long, 1' wide, and 1" thick is needed for a certain purpose, how many feet of lumber are needed?
2. If a board 9' long, 4" wide, and 1" thick is needed for a certain purpose, how many feet of lumber are needed?
3. How many feet in a piece of lumber 9' long, 4" wide, and 2" thick?
4. How many feet of lumber in a piece of scantling 12' long and 2" by 4" in cross section?
5. If a "2 by 4" (that is, 2" by 4") scantling is 10' long, how many feet of lumber does it contain?
6. In building the above house the man needs, with other material, the following. Complete each statement.

Sills: 3 pieces, $6'' \times 8''$, 16' long, 192' B.M.

4 pieces, $6'' \times 8''$, 14' long, (?)' B.M.

6 pieces, $6'' \times 8''$, 12' long, (?)' B.M.

1 piece, $2'' \times 8''$, 14' long, 19' B.M.

1 piece, $2'' \times 9''$, 8' long, (?)' B.M.

7. Joists:	30 pieces, $2'' \times 9''$, 16' long, (?)' B.M.
	18 pieces, $2'' \times 8''$, 12' long, (?)' B.M.
	13 pieces, $2'' \times 8''$, 6' long, (?)' B.M.
	27 pieces, $2'' \times 8''$, 16' long, (?)' B.M.
	27 pieces, $2'' \times 8''$, 12' long, (?)' B.M.
	27 pieces, $2'' \times 6''$, 16' long, (?)' B.M.
8. Studding,	5 pieces, $4'' \times 6''$, 14' long, (?)' B.M.
etc.:	100 pieces, $2'' \times 4''$, 14' long, 933' B.M.
	35 pieces, $2'' \times 4''$, 10' long, 233' B.M.
	25 pieces, $2'' \times 4''$, 18' long, (?)' B.M.
	45 pieces, $2'' \times 5''$, 12' long, (?)' B.M.
	50 pieces, $2'' \times 4''$, 8' long, 267' B.M.
	50 pieces, $2'' \times 4''$, 14' long, 467' B.M.
	40 pieces, $2'' \times 6''$, 18' long, (?)' B.M.
Bridging, etc.:	175' B.M.
Roof sheathing, etc.:	875' B.M.
The total in Exs. 6-8 should be	<u>7847' B.M.</u>

9. How much will the lumber mentioned in Exs. 6-8 cost at \$60 per M?

10. A man needs 2000 ft. B.M. of siding at \$65 per M; 1950 ft. of flooring at \$45; 125 ft. of flooring for the porch at \$65. What will this material cost?

11. A man needs 28 foundation posts at 40¢ each; water tables, cornice, corner boards, etc., \$35; porch columns, \$7; 16 doors at an average cost of \$5.75 each; 16 windows at an average cost of \$5.50 each; hardware, \$50; chimney and plastering, \$200; painting, \$148. What is the total cost of these items?

The class should make out a bill for the general supply of lumber used in manual training, or for some repairs to the building, using actual market prices. Where the school teaches woodworking the class should lay out simple geometric forms in wood; for example, the hexagonal top of a taboret, noting the value in degrees of the angle.

EXERCISE 247

PROBLEMS IN AGRICULTURE

1. If 75 lb. of fertilizer furnish 12 lb. of plant food, what is the per cent of plant food?

2. A farmer paid \$5.40 for a bushel of clover seed, but only 60% was good seed. What was the cost of a bushel of good seed?

3. If a cow's average daily ration is 28 lb. of ensilage, 12 lb. of clover hay, $6\frac{1}{2}$ lb. of bran, and $3\frac{1}{2}$ lb. of corn meal, what per cent of the total is each kind of food?

4. A pound of grafting wax contains 6 oz. of resin, 6 oz. of beeswax, and $1\frac{1}{8}$ oz. of tallow. Each of these substances is what per cent of the grafting wax?

5. A farmer had 40 A. in oats. He estimated the use of the land as worth \$220 a year, and the labor expended as \$300. He paid \$120 for fertilizers and \$18 for seed. Each of these items was what per cent of the total cost?

6. An experiment station found that by delaying the plowing a week in the spring the land lost in moisture at the rate of 9.13 lb. per square foot. How many tons of water were lost to the acre?

7. At 35¢ a rod, what will a ditch cost dug along the two longer sides of an acre of land 8 rd. wide?

8. If the soil is cultivated to the depth of 4 in., how many cubic feet of soil are cultivated to an acre? How much does this weigh, allowing 75 lb. to the cubic foot?

Besides the many problems relating to agriculture that are given in this work, the pupils should prepare others, including the calculation of areas of small plots of land from actual measurement. Many problems have been given in calculating the capacity of tanks, cisterns, watering troughs, silos, storage bins, and barrels, but in rural schools others should be prepared by the class.

EXERCISE 248

PROBLEMS IN AGRICULTURE

1. If a shipment of butter contains 86.6% butter fat, how many pounds of butter represent 433 lb. of butter fat?

2. If milk produces $10\frac{1}{2}\%$ of its weight of cheese, what weight of milk is required to produce 75 lb. of cheese?

3. If a quart of water weighs 2 lb., and a quart of milk weighs 2.064 lb., what is the specific gravity of milk? (That is, milk is how many times as heavy as water?)

4. If a lot of cream contains 25.9% butter fat, what is the weight of butter fat in 12 cans, each containing 81 lb. of cream?

5. How many feet of 2-inch plank will it take for a barn having a floor space of 1700 sq. ft.? How much will it cost at \$50 a thousand?

6. Allowing 4 ft. 3 in. by 9 ft. 6 in. of floor space for a stall, and 4 ft. 6 in. by 3 ft. extra for a manger, how much floor space will be required for 32 cows?

7. If each cow averages 2400 lb. of milk per year, and the milk contains 4.3% butter fat, how many pounds of butter fat will 32 cows produce in a year?

8. If a cow eats $3\frac{1}{4}$ tons of hay a year, worth \$12 a ton, and 900 lb. of ground feed worth 85¢ per hundred, besides \$6 worth of pasturage, what will it cost to keep 32 cows one year?

In rural schools there should be special computations suited to farm practice, such as income from dairy cows, feed rations, and the cost of farm labor. The class should be instructed in keeping the books of a farm, such as inventories, time books, stock accounts, and cash accounts, besides considering bills, notes, and receipts relating to local farming conditions.

EXERCISE 249

FARM BUILDINGS

1. A silo is built with inside measurements as follows : length, 14 ft.; width, 11 ft.; depth, 36 ft. Allowing 40 cu. ft. to a ton of ensilage, how many tons of ensilage will this silo contain ?
2. The concrete wall and floor of the above silo are 22 in. thick. How many cubic feet of concrete are needed ?
3. If a poultry house should contain 10 cu. ft. of space to every pound of live weight of the fowls, and the fowls average 6 lb. each, how many fowls may be put in a house 36 ft. long, 15 ft. wide, and averaging 10 ft. in height ?
4. How many feet of 1-inch lumber will be required to board up the gables of a barn 32 ft. wide, the peak being 12 ft. above the eaves ?
5. At each end of a barn 32 ft. wide is a brick wall 12 ft. high and 18 in. thick. Allowing 22 bricks to a cubic foot, what will the bricks cost at \$7.50 a thousand ?
6. It is desired to build a cylindrical silo 14 ft. in diameter that shall hold 50 tons of ensilage. Allowing 40 cu. ft. to the ton, what must be the height of the silo ? (Take $3\frac{1}{7}$ in finding the area of the base.)
7. Allowing $1\frac{1}{4}$ cu. ft. to the bushel, how many cubic feet must a bin contain to hold 500 bu. ? Give two convenient sets of measures for such a bin.
8. A cylindrical watering trough for a barnyard is to be 8 ft. in diameter and 3 ft. deep inside measure. How many gallons of water will it hold, allowing $7\frac{1}{2}$ gal. to the cubic foot ? (Take $3\frac{1}{7}$ in finding the area of the base.)
9. How many square feet in the inside area of the watering trough of Ex. 8 ?

EXERCISE 250

CREAMERY PROBLEMS

1. A farmer sells 26,250 lb. of milk to a creamery in a certain month. The milk averages 4.2% butter fat. With how many pounds of butter fat does the creamery credit him in that month?

2. If 6 lb. of butter fat are needed in making 7 lb. of butter, what is the value of the butter produced from 1236 lb. of butter fat, the butter being worth 24¢ a pound?

3. A certain dairy sells to a creamery milk averaging 3.75% of butter fat during a certain month. The butter fat weighs 630 lb. How many pounds of milk does the dairy sell in that month?

4. A farmer has two cows, one supplying 986 lb. of milk testing 3.1% butter fat in a certain month, and the other 812 lb. testing 4.2%. If the creamery allows him 27¢ a pound for butter fat, which cow pays him the more for that month, and how much more, the feed costing the same?

5. The books of a creamery show that one of its customers delivers on seven successive days the following number of pounds of cream: 32, 33, 31, 34, 33, 35, 32. This cream averaged 30.2% butter fat, worth 28¢ a pound. What was the value of the butter fat for the week?

6. A creamery received in one day 1250 lb. of milk averaging 16% cream, besides receiving 475 lb. of cream. If all the cream averaged $27\frac{1}{2}\%$ butter fat, how many pounds of butter fat did the creamery produce that day?

7. A certain creamery uses 7500 lb. of milk in a week. The skim milk amounts to 80% of the whole milk and contains 0.6% butter fat. How many pounds of butter fat are lost in the skim milk?

EXERCISE 251

PROBLEMS IN AGRICULTURE

1. In 25 lb. of oats it is found that there are 1.9 lb. of fat-producing matter. How much is there in 30 lb.?

2. In 15 lb. of corn it is found that there is 0.135 lb. of muscle-producing matter. How much is there in 20 lb.?

3. To lessen potato scab the seed potatoes are soaked in a solution of $\frac{1}{2}$ pt. of formalin to 15 gal. of water. What is the per cent of formalin in the mixture?

Local problems of this kind should be proposed in rural schools, practice being given in estimating amounts and cost.

4. Bordeaux mixture, for spraying, contains $1\frac{1}{4}\%$, by weight, of copper sulphate, $1\frac{1}{4}\%$ of unslaked lime, and the rest water. How many pounds of each in a barrel containing 250 lb. of the mixture?

5. In a certain fertilizer for land on which corn is grown, $43\frac{3}{4}\%$ is acid phosphate, $47\frac{1}{2}\%$ is cottonseed meal, and the rest is kainite. How many pounds of each are there to a ton?

6. In a certain fertilizer for land on which cotton is grown, $53\frac{3}{4}\%$ is acid phosphate, and the rest is 32 parts fish scrap to 5 parts muriate of potash. How many pounds of each to a ton?

7. A farmer reads that a good mixture of seed for his meadow is, by weight, as follows: timothy, 40%; redtop, 40%; red clover making up the rest. At 40 lb. of seed to the acre, how many pounds of each should he sow?

8. The following is, by weight, a good mixture of seed for a pasture: Kentucky blue grass, 25%; white clover, $12\frac{1}{2}\%$; perennial rye, $28\frac{1}{8}\%$; red fescue, $9\frac{3}{8}\%$; redtop, 25%. At 32 lb. to the acre, how many pounds of each are used?

9. If 1 qt. of corn will plant 230 hills, what will the corn cost for 6210 hills at 25¢ a quart?

10. If 1 qt. of corn will plant 230 hills, and costs 25¢, how many hills can be planted with \$4 worth of corn?

11. The beans necessary to plant 11,250 hills cost \$26.25, at 35¢ a quart. How many hills will 1 qt. of beans plant?

12. What is the cost of seed for 7000 celery plants at 23¢ an ounce, allowing $\frac{1}{2}$ oz. of seed to 1000 plants?

13. The cost of seed for 2500 cauliflower plants, at \$2.30 an ounce, was \$5.75. How many plants were there to an ounce of seed?

14. When peas for planting cost \$3.50 a bushel, it was found that the seed for planting 20 A. cost \$105. How many bushels of seed were allowed to an acre?

15. When corn for planting 25 acres costs \$12.50, and 8 qt. of seed are allowed to the acre, what is a bushel of seed corn worth?

16. An experiment showed that a corn-fed hog gained 21% of the weight of the corn he ate. At 56 lb. to the bushel, how many bushels of corn must he eat in order to gain $47\frac{1}{5}$ lb.?

17. In feeding 750 lb. of dry cornstalks to cattle it was found that 225 lb. were wasted, but that when the green stalks were put into a silo and then fed to the cattle, only 32 lb. were wasted out of 640 lb. What was the ratio of the per cent of waste in the latter case to that in the former?

18. A farmer mixed a ton of fertilizer for his corn land, using half cottonseed meal @ \$24.60 a ton and the rest acid phosphate and muriate of potash in the ratio of 19 to 1. The acid phosphate cost \$11.50 a ton, and the muriate of potash \$42.50 a ton. What did the fertilizer cost?

EXERCISE 252

PROBLEMS OF THE MANUFACTURER

1. How much will a manufacturer receive for 6 rockers @ \$6.75, 8 dressers @ \$12.50, and 12 bedsteads @ \$15.75, discounted at 8%?

2. A manufacturer sells the following bill of goods: 4 doz. plates @ \$1.38, 8 doz. cups @ \$2.27, $\frac{1}{2}$ doz. covered dishes @ \$9.50, 12 doz. plates @ \$1.45, $\frac{1}{4}$ doz. pitchers @ \$6.40. The bill is discounted at 3% and he gains 20% on the goods. Required the cost of manufacture.

3. Iron expands $\frac{3}{16}$ in. to the foot when heated to a red heat. How much longer will a casting be when heated to a red heat, if it is 3 ft. 7 in. long before it is heated?

4. A manufacturing company has a capital of \$275,000, its annual business is $3\frac{1}{4}$ times its capital, and its annual profits are $7\frac{1}{2}\%$ of its annual business. How much are its annual profits?

5. A manufacturer sells brass lamps to a dealer at such a price that the dealer can sell them at \$6.60 and make a profit of 32%. The manufacturer himself makes a profit of 25%. What is the cost of manufacture per lamp?

6. On which will a manufacturer make the greater per cent of profit,—an article which it costs \$2.90 to make, and which he sells for \$3.33 $\frac{1}{3}$ less 3%, or one costing \$8.70 to make, which he sells for \$12 less $\frac{1}{8}$? How much greater?

7. A manufacturer sold to a retail dealer the following bill of goods: 9 dining sets @ \$62.50, 16 rockers @ \$5.35, 8 sideboards @ \$32.50, 6 card tables @ \$8.75, 4 china closets @ \$17.50. He discounted this bill at 8% and 2%. How much did he receive for the goods?

8. In casting brass hinges the brass shrinks in cooling 1.5625% of its length when melted. How long must be the mold for a hinge that is to be $5\frac{7}{8}$ in. long?

9. The wooden pattern from which an iron casting is made weighs $6\frac{1}{4}\%$ as much as the iron. If the pattern weighs $32\frac{1}{2}$ lb., how much does the casting weigh?

10. A manufacturer has a water tank that is 8 ft. 3 in. wide, 12 ft. 6 in. long, and 9 ft. 8 in. deep. If a cubic foot of water weighs $62\frac{1}{2}$ lb., what is the weight of the water required to fill the tank?

11. A manufacturer pays a salesman a salary of \$1250 a year and a commission of $1\frac{1}{2}\%$ on all sales, and allows him \$1000 a year for expenses. If the salesman sells \$172,500 worth of goods, what does the manufacturer pay him, including salary, commissions, and expenses?

12. An iron founder knows that an iron casting will weigh 17 times as much as the wooden pattern. If he has 5 patterns for the parts of a given piece that he wishes to cast, and these weigh 6 lb. 8 oz., 7 lb. 4 oz., 18 lb. 12 oz., 9 lb. 8 oz., and 12 lb. 4 oz., what will the casting weigh?

13. A manufacturer is building a chimney with a circular flue for his boiler room. He has two horizontal boilers, each containing 96 tubes 2 in. in diameter. His chimney must have the same cross-section area as the sum of all the cross-section areas of the tubes of the boilers. Required the diameter of the flue.

14. In a large dyeing establishment there is a tank 8 ft. long, 7 ft. wide, and 6 ft. deep. If the pressure on the end, when the tank is full of water, is equal to half the weight of the water in a rectangular tank whose base is the area of the end of the tank and whose height is the depth of the tank, how much is this pressure?

15. The following is the pay roll of a manufacturer for a week :

PAY ROLL

For the week ending

*

19 *

No.	NAME	NO. OF HOURS PER DAY						TOTAL TIME	WAGES PER HOUR	TOTAL WAGES	
		M.	T.	W.	T.	F.	S.				
1	J. P. Driscoll	9	9	8	8½	9	9	52½	25¢	\$13	12
2	R. L. Bennett	9	8½	9	9	8	9	*	25¢	*	*
3	P. F. Purvis	9	8¼	8	8½	9	9	*	30¢	*	*
4	B. J. Mead	8	9	9	9	8½	8	*	33⅓¢	*	*
5	R. K. King	8	8	9	9	9	9	*	35¢	*	*
6	M. L. Drake	9	9	8	0	9	9	*	27⅓¢	*	*
		*	*	*	*	*	*	*		*	*

Fill out the spaces marked with an asterisk (*).

Make out pay rolls (inserting names), the numbers, the hours per day, and wages per hour being as follows :

16. No. 1 : 9, 9, 9, 8, 8½, 9, 30¢ ; No. 2 : 8, 8, 8, 9, 9, 8, 27½¢ ; No. 3 : 8, 8½, 9, 9, 9, 9, 28¼¢ ; No. 4 : 8, 8, 8, 8, 8½, 8, 33⅓¢ ; No. 5 : 8, 9, 8, 8, 8, 8½, 32¢.

17. No. 1 : 9, 9, 9, 9, 9, 9, 37½¢ ; No. 2 : 9, 9, 9, 8¾, 8¾, 8¾, 30¢ ; No. 3 : 9, 9, 9, 9, 9, 8¾, 32¢ ; No. 4 : 9, 8, 8¾, 8½, 9, 9, 27¢ ; No. 5 : 9, 9, 9, 8½, 8½, 8, 28¢.

18. No. 1 : 8, 8, 8, 8, 8, 8, 32¼¢ ; No. 2 : 8, 8, 7¾, 7½, 7¾, 7¾, 30¢ ; No. 3 : 8, 8, 8, 7¾, 7¾, 7¾, 28¢ ; No. 4 : 7½, 7¼, 7½, 8, 7¾, 7¾, 32¢ ; No. 5 : 8, 8, 8, 8, 8, 8, 33⅓¢.

19. No. 1 : 8, 7¾, 7¾, 7¾, 7¾, 7½, 28¢ ; No. 2 : 8, 8, 7, 0, 8, 8, 30¢ ; No. 3 : 8, 8, 8, 7¾, 8, 8, 35¢ ; No. 4 : 8, 8, 8, 7½, 8, 7¾, 32⅓¢ ; No. 5 : 8, 8, 8, 8, 7½, 7½, 32⅓¢.

20. No. 1 : 8, 8, 7½, 7, 6¾, 8, 31¢ ; No. 2 : 8, 7, 8, 8, 7¾, 8, 31½¢ ; No. 3 : 7¾, 7½, 7¼, 7, 6¾, 0, 28¢ ; No. 4 : 7, 7, 7½, 7¾, 0, 7, 26¢ ; No. 5 : 8, 8, 8, 8, 8, 8, 36¢.

EXERCISE 253

PROBLEMS OF THE CITY WORKINGMAN

1. A man's salary was decreased $3\frac{1}{2}\%$ and was then \$1785.25. What was the salary before it was decreased?

2. A man's salary was increased $3\frac{1}{2}\%$ and was then \$1914.75. What was the salary before it was increased?

3. If a man works 312 da. a year at \$3.25 a day, and saves \$101.40, what per cent of his income does he spend?

4. A man receiving a salary of \$22 a week worked all but 2 wk. 1 da. in a year of 365 da. He paid \$25 a month rent. How much money had he left for other expenses?

5. A laborer earns \$2.33 $\frac{1}{3}$ a day for 6 da. in the week. If he works for 51 wk. and spends 25% of his wages for rent, how much money is left for other expenses?

6. A man works 6 da. in the week for 48 wk., except on five holidays. He takes two rides on the trolley car each day that he works, at 5¢ a ride. How much does he pay in a year for trolley rides?

7. A boy is offered a salary of \$8 a week for the first year in a store, 25% more the second year, and an increase of 50% the third year. If he works 50 wk. in a year, what is his total salary for the first 3 yr.?

8. A laborer earns 35¢ an hour, and works 8 hr. a day for 5 da. in the week, and 5 hr. on Saturday. He works 50 wk. in the year, and earns during the year \$62.50 for overtime work. After paying his rent he has \$680 left for other expenses. What per cent of his income is paid for rent?

9. A man who has been earning \$75 a month is offered an increase of 20% or a salary of \$20 a week. Allowing 52 wk. to the year, which offers the better income, and how much better, for the year?

EXERCISE 254

PROBLEMS OF THE MACHINIST

1. What is the circumference of a fly wheel that is 7' 6" in diameter?
2. An emery wheel 10 in. in diameter makes 2000 revolutions per minute. What is its grinding speed?
3. A fly wheel 6 ft. in diameter makes 100 revolutions per minute. At what rate per minute does a point on the rim travel?
4. How many revolutions per minute must be made by a circular saw 2 ft. in diameter, in order that the saw may have a cutting speed of 7539.84 ft. per minute?
5. A piston of an engine has an area of 175 sq. in. Required the diameter of the piston.
6. It is required to make a circular plate of the same material, thickness, and weight as a rectangular plate 2' 4" by 3' 1". What is its diameter?
7. If the diameter of a piston is 27 in., and the steam pressure is 120 lb. per square inch, what is the total pressure exerted on the piston?
8. What is the total external surface of the tubes in a boiler 18 ft. long, there being 112 tubes and the tubes being $3\frac{1}{2}$ " in diameter?
9. Two cogwheels are so geared that for every revolution of the larger the other revolves 12 times. The larger wheel has 204 cogs. How many cogs has the smaller?
10. The fly wheel of an engine 8 ft. in diameter makes 250 revolutions per minute. It is directly connected by a belt with a wheel 2 ft. 8 in. in diameter. How many revolutions per minute does the smaller wheel make?

CHAPTER XXVI

ALGEBRAIC NOTATION

396. Algebraic Forms. We have learned that the area of a rectangle equals the product of its base and height. We express this more briefly in *algebraic* form as follows:

$$a = bh.$$



Here we have let a stand for the word *area*, b for the word *base*, and h for the word *height*. When we write bh it indicates the *product* of b and h .

In arithmetic 25 means $20 + 5$, but in algebra bh means $b \times h$.

When two letters are written side by side in algebra, with no sign between them, the product of their numerical values is to be taken.

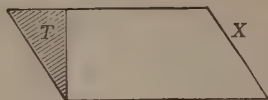
Thus if $b = 4$ and $h = 2$, then $a = bh = 4 \times 2 = 8$. In writing algebraic forms it is not customary to express denominations, like feet or inches. When we speak of the product of two lines, this is a brief way of expressing the product of their *numerical values*.

EXERCISE 255

Given $a = bh$, find the value of a when:

- | | |
|--------------------------------|---|
| 1. $b = 5, h = 3.$ | 6. $b = 52.6, h = 7.9.$ |
| 2. $b = 7, h = 2\frac{1}{2}.$ | 7. $b = 125\frac{3}{4}, h = 26\frac{1}{4}.$ |
| 3. $b = 9, h = 4\frac{1}{3}.$ | 8. $b = 137\frac{1}{4}, h = 52\frac{1}{2}.$ |
| 4. $b = 12, h = 5\frac{3}{4}.$ | 9. $b = 325.75, h = 124.25.$ |
| 5. $b = 27.4, h = 5.4.$ | 10. $b = 438.25, h = 226.75.$ |

397. Parallelogram. We have learned that the area of a parallelogram equals the product of its base and height. We may therefore express this in algebraic form thus: $a = bh$.



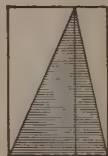
For the triangle T may be cut off and placed at X , the parallelogram then becoming a rectangle.

If $b = 5$ and $h = 3\frac{1}{2}$, then $a = bh = 5 \times 3\frac{1}{2} = 17\frac{1}{2}$. If b and h represent inches, then a represents square inches.

398. Triangle. We have learned that the area of a triangle equals half the product of its base and height. This is expressed in algebraic form thus: $a = \frac{1}{2}bh$.

For the triangle may be cut as here shown so that it is seen to be half of the rectangle of base b and height h .

If $b = 7$ and $h = 10\frac{1}{2}$, then $a = \frac{1}{2}bh = \frac{1}{2}$ of $7 \times 10\frac{1}{2} = 36\frac{3}{4}$.



EXERCISE 256

Given $a = bh$, find the value of a when:

1. $b = 7.2$, $h = 3.4$.

3. $b = 127\frac{3}{4}$, $h = 27.4$.

2. $b = 8.9$, $h = 4.7$.

4. $b = 22.75$, $h = 14\frac{3}{4}$.

Given $a = \frac{1}{2}bh$, find the value of a when:

5. $b = 42$, $h = 25$.

8. $b = 23.8$, $h = 4.75$.

6. $b = 36$, $h = 19$.

9. $b = 41.2$, $h = 19.65$.

7. $b = 56$, $h = 27$.

10. $b = 33.3$, $h = 24.50$.

11. A playground is l feet long and w feet wide. Find the area in square feet. What is the number of square feet when $l = 124$ and $w = 62\frac{1}{2}$?

12. A triangular space at the end of a house, under the roof, is f feet wide and h feet high. What is the area in square feet?

399. Trapezoid. We have learned that the area of a trapezoid equals half the product of the sum of the two parallel sides multiplied by the height.



For an equal trapezoid may be turned over and put down by the side of the first one, as here shown. The whole figure, or twice the trapezoid, then equals a parallelogram whose base is the sum of the parallel sides of the trapezoid. The trapezoid is therefore half this parallelogram.



We indicate this in algebraic form as follows :

$$a = \frac{1}{2}(b + b')h,$$

where b and b' (" b prime ") are the two parallel sides, usually called the *bases*, and h is the height.

The parentheses show that b and b' are to be added before being multiplied by h . The operations indicated within parentheses are always performed first. Thus, if $b = 6$, $b' = 5$, and $h = 4$, we have $a = \frac{1}{2}(b + b')h = \frac{1}{2}(6 + 5) \times 4 = \frac{1}{2} \text{ of } 11 \times 4 = 22$.

EXERCISE 257

Given $a = \frac{1}{2}(b + b')h$, find the value of a when:

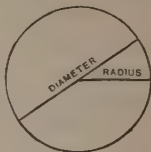
1. $b = 6$, $b' = 4$, $h = 3$.
2. $b = 9$, $b' = 5$, $h = 7$.
3. $b = 11$, $b' = 8$, $h = 6$.
4. $b = 13$, $b' = 7$, $h = 7$.
5. $b = 19$, $b' = 9$, $h = 9$.
6. $b = 21$, $b' = 9.5$, $h = 7.2$.
7. $b = 32$, $b' = 16$, $h = 19$.
8. $b = 28$, $b' = 9.8$, $h = 8.6$.
9. $b = 4.9$, $b' = 3.4$, $h = 1\frac{1}{2}$.
10. $b = 4\frac{3}{4}$, $b' = 3\frac{1}{4}$, $h = 2\frac{1}{2}$.

11. The lower base of a trapezoid is 10 in. and the altitude and upper base are each 5 in. Find the area.

12. A field is in the form of a trapezoid, with bases x and y , and with altitude z . What is the area of the field? How many square rods are there in the field if $x = 30$ rods, $y = 26$ rods, and $z = 24$ rods?

400. Circumference. We have learned that the circumference of a circle equals $3.1416 \times$ the diameter. In mathematics the number 3.1416, which is nearly $3\frac{1}{7}$, is represented by the Greek letter π (pronounced *pī*). We may therefore express this law as follows:

$$c = \pi d,$$



where c stands for circumference, d for diameter, and π for $3\frac{1}{7}$ or 3.1416, nearly.

Since the diameter equals twice the radius, we may write $2r$ for d , and have $c = \pi \times 2r$, or $c = 2\pi r$.

Thus if $d = 7$, and we take $3\frac{1}{7}$ as the value of π , we have

$$c = \pi d = 3\frac{1}{7} \times 7 = 22.$$

If $r = 5$, we have $c = 2\pi r = 2 \times 3\frac{1}{7} \times 5 = 31\frac{1}{2}$.

EXERCISE 258

Given $c = \pi d = 2\pi r$, and taking $\pi = 3\frac{1}{7}$, find c when:

- | | | | |
|---------------|----------------|----------------|---------------------------|
| 1. $d = 14$. | 4. $d = 3.5$. | 7. $r = 7$. | 10. $r = 6.3$. |
| 2. $d = 21$. | 5. $d = 4.9$. | 8. $r = 3.5$. | 11. $r = 9.1$. |
| 3. $d = 28$. | 6. $d = 7.7$. | 9. $r = 5.6$. | 12. $r = 10\frac{1}{2}$. |

Taking $\pi = 3.1416$, find c when:

- | | | | |
|----------------|----------------|----------------|--------------------------|
| 13. $d = 10$. | 15. $d = 50$. | 17. $r = 30$. | 19. $r = 2\frac{1}{2}$. |
| 14. $d = 20$. | 16. $d = 25$. | 18. $r = 40$. | 20. $r = 7.5$. |

21. A workman measures the diameter of a steel shaft and finds it to be $3\frac{7}{8}$ in. What is the circumference?

22. If you describe a circle with a radius of 12 in., how many inches will there be in the circumference?

23. If a globe has a diameter of 14 in., what is the circumference?

401. Area of a Circle. We have learned that

1. The area of a circle equals half the product of the circumference and radius.

2. The area of a circle equals 3.1416 times the square of the radius.

We may express the first of these laws as follows:

$$a = \frac{1}{2} cr.$$

To express the second of these laws, since $c = 2\pi r$ (§ 400) we may put $2\pi r$ for c , and then we have $a = \frac{1}{2}$ of $2\pi rr$, or, writing r^2 ("r square") for rr ,

$$a = \pi r^2.$$

Thus if $r = 10$, we have $a = \pi r^2 = 3.1416 \times 10^2 = 314.16$.

EXERCISE 259

Given $a = \frac{1}{2} cr$, find a when:

- | | |
|-------------------------------|--|
| 1. $r = 5$, $c = 31.416$. | 4. $r = 2\frac{1}{2}$, $c = 15.708$. |
| 2. $r = 10$, $c = 62.832$. | 5. $r = 25$, $c = 157.08$. |
| 3. $r = 20$, $c = 125.664$. | 6. $r = 50$, $c = 314.16$. |

Given $a = \pi r^2$, and taking $\pi = \frac{22}{7}$, find a when:

- | | | | |
|-------------------------|-----------------|-----------------|-----------------|
| 7. $r = 7$. | 9. $r = 14$. | 11. $r = 2.8$. | 13. $r = 4.9$. |
| 8. $r = 3\frac{1}{2}$. | 10. $r = 2.1$. | 12. $r = 35$. | 14. $r = 7.7$. |

Given $a = \pi r^2$, and taking $\pi = 3.1416$, find a when:

- | | | | |
|---------------|----------------|----------------|----------------|
| 15. $r = 5$. | 16. $r = 10$. | 17. $r = 20$. | 18. $r = 50$. |
|---------------|----------------|----------------|----------------|

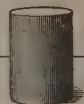
19. How many square feet in the area of a circle whose radius is 2 ft.? (Take $\pi = 3.1416$.)

20. How many square feet in the circular base of a water tank whose radius is 15 ft.? (Take $\pi = 3.1416$; then take $\pi = 3\frac{1}{7}$, and find the difference in the results.)

402. Cylinder. We have found that the volume of a cylinder equals the product of its base and height; that is, $v = bh$, where v stands for volume, b for base, and h for height.

But since $b = \pi r^2$ (§ 401), we may put πr^2 for b , and then we have

$$v = \pi r^2 h.$$



403. Formula. We therefore see that algebra enables us to put a long rule of arithmetic into a brief form. An algebraic form of this kind is called a *formula*.

When we find the value of an algebraic expression by putting numbers in place of the letters, we are said to *substitute* numbers for the letters and to *evaluate* the formula.

EXERCISE 260

Express the following statements in formulas:

1. The lateral area of a cylinder equals the product of the circumference and height. (Use l for lateral area, c for circumference, and h for height.)

2. The lateral area of a cylinder equals 3.1416 times the diameter multiplied by the height.

3. The lateral area of a cylinder equals twice 3.1416 times the radius multiplied by the height. (Use r for radius, and in general in a formula use the initial letter for a word.)

4. The volume of a prism equals the product of the base and height.

5. The volume of a pyramid equals one third the product of the base and height.

6. The volume of a cone equals one third the product of 3.1416 times the square of the radius of the base multiplied by the height.

7. The formula for the lateral surface of a cone in terms of the circumference of the base and k the distance from the vertex to the circumference of the base is $l = \frac{1}{2} kc$. Write this as an ordinary sentence.

8. The formula for the lateral surface of a cone in terms of the radius of the base and the distance from the vertex to the circumference of the base is $l = \pi kr$. Write this as an ordinary sentence.



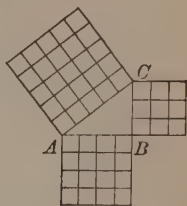
9. The formula for the surface of a sphere is $s = 4 \pi r^2$. Write this as an ordinary sentence.

10. The formula for the volume of a sphere is $v = \frac{4}{3} \pi r^3$, where r^3 ("r cube") equals rrr . Write this as an ordinary sentence.



11. The square on the hypotenuse of a right triangle has been shown to be equal to the sum of the squares on the other two sides. Letting $AB = x$, $BC = y$, and $AC = z$, we have $z^2 = x^2 + y^2$, or $z = \sqrt{x^2 + y^2}$. Suppose $x = 4$ and $y = 3$, find the value of z .

(The expression $\sqrt{x^2 + y^2}$ means that we are to square x , and to square y , then to add these results, and then to extract the square root of the sum.)



12. Given $z = \sqrt{x^2 + y^2}$, find the value of z when $x = 24$ and $y = 32$.

13. In the same formula find the value of z when $x = 36$ and $y = 27$.

14. In the same formula find the value of z when $x = 66$ and $y = 88$; when $x = 36$ and $y = 48$.

15. In the same formula find the value of z when $x = 56$ and $y = 42$; when $x = 8\frac{1}{2}$, and $y = 11\frac{1}{2}$.

404. The Formula as an Equation. A formula may be thought of as an equation, and one of the chief uses of algebra is the solution of equations of this nature.

For example, if the rate of interest is r , the interest for 1 yr. on p dollars is rp . Since this is the interest for 1 yr., for t years it is trp . Therefore

$$i = trp.$$

Here i stands for interest, t for time, r for rate, and p for principal.

If from this formula we wish to find t , we divide both sides of the equation by rp . Then

$$\frac{i}{rp} = t.$$

Likewise we may find r or p , thus: $r = \frac{i}{tp}$, and $p = \frac{i}{tr}$.

EXERCISE 261

Given $i = trp$, find:

1. i , when $t = 2$, $r = 6\%$, $p = \$500$.
2. t , when $i = \$20$, $r = 4\%$, $p = \$250$.
3. r , when $i = \$52.50$, $t = 3$, $p = \$350$.
4. p , when $i = \$135$, $t = 5$, $r = 6\%$.
5. What principal, put at interest at 6% , will produce \$135 in 3 years?
6. How long will it take \$1250 to produce \$250 interest at 5% ? at $2\frac{1}{2}\%$?
7. How long will it take \$2750 to produce \$1155 interest at 6% ? at 3% ? at $4\frac{1}{2}\%$?
8. The formula for the amount (sum of principal and interest) is $a = p(1 + tr)$. From this formula deduce the formula for p .

APPENDIX

I. LONGITUDE AND TIME

405. Longitude and Time. One class of problems that arises when we think of the change in time as we travel or telegraph east or west relates to longitude and time.

406. Meridian. The shortest line on the earth's surface passing from one pole to the other is called a *meridian*.

407. Longitude. The distance in circular measure east or west of a certain meridian agreed upon is called *longitude*.

408. Prime Meridian. A meridian agreed upon as the one from which longitude is reckoned is called a *prime meridian*.

The prime meridian now used by most of the maritime nations of the world is that which passes through the National Observatory at Greenwich, England.

A place $75^{\circ} 30' 20''$ west of this meridian is said to be in $75^{\circ} 30' 20''$ west longitude. A place $15^{\circ} 25'$ east of this meridian is said to be in $15^{\circ} 25'$ east longitude.

Since 180° is halfway around the earth from the prime meridian there can be no more than 180° E. (that is, east longitude) or 180° W. (that is, west longitude).

409. Difference in Longitude. When two places are both east or both west of the prime meridian, the difference in longitude is evidently found by subtracting one longitude from the other.

When one place is east and the other is west of the prime meridian, the difference in longitude is evidently found by adding the two longitudes.

This is clearly seen by looking at a map or globe of the world.

410. Relation of Longitude to Time. Since the earth makes one complete revolution upon its axis in 24 hr., a point on the surface passes through 360° in 24 hr. That is,

360° corresponds to 24 hr.

1° corresponds to $\frac{1}{360}$ of 24 hr., or $\frac{1}{15}$ hr., or 4 min.

$1'$ corresponds to $\frac{1}{60}$ of 4 min., or $\frac{1}{15}$ min., or 4 sec.

$1''$ corresponds to $\frac{1}{60}$ of 4 sec., or $\frac{1}{15}$ sec.

Hence we have the following correspondence :

<i>Longitude to Time</i>	<i>Time to Longitude</i>
360° . . . 24 hr.	24 hr. . . . 360°
1° . . . $\frac{1}{15}$ hr. = 4 min.	1 hr. . . . 15°
$1'$. . . $\frac{1}{15}$ min. = 4 sec.	1 min. . . . $\frac{1}{4}^\circ = 15'$
$1''$. . . $\frac{1}{15}$ sec.	1 sec. . . . $\frac{1}{4}' = 15''$

411. Finding Difference in Time. Given the difference in longitude, the difference in time may be found.

For example, two ships at sea are $25^\circ 24' 15''$ of longitude apart. What is the difference in time ?

Looking at the above table, we find the difference in time corresponding to 1° , $1'$, and $1''$, and then we obtain the following :

$$25 \times 4 \text{ min.} = 100 \text{ min.} = 1 \text{ hr. } 40 \text{ min.}$$

$$24 \times 4 \text{ sec.} = 96 \text{ sec.} = 1 \text{ min. } 36 \text{ sec.}$$

$$15 \times \frac{1}{15} \text{ sec.} = 1 \text{ sec.}$$

Therefore the difference in time is 1 hr. 41 min. 37 sec.

Therefore, *given the difference in longitude to find the difference in time, multiply 4 minutes of time by the number of degrees, 4 seconds of time by the number of minutes of longitude, and $\frac{1}{15}$ of a second of time by the number of seconds of longitude, and add the products.*

412. Later to the East, Earlier to the West. Because the earth turns from the west to the east, the sun appears to go from east to west. It is therefore 1 hr. later 15° east of here, and 1 hr. earlier 15° west.

EXERCISE 262

Find the difference in time between two places, given the following differences in longitude :

- | | | |
|------------------------|--------------------------|---------------------------------------|
| 1. $15^{\circ} 15'$. | 6. $25^{\circ} 15'$. | 11. $5^{\circ} 10' 15''$. |
| 2. $15' 15''$. | 7. $32^{\circ} 30'$. | 12. $7^{\circ} 25' 30''$. |
| 3. $30^{\circ} 15'$. | 8. $48^{\circ} 45'$. | 13. $28^{\circ} 42' 30''$. |
| 4. $45^{\circ} 30'$. | 9. $127^{\circ} 15''$. | 14. $62^{\circ} 27' 45''$. |
| 5. $60^{\circ} 30''$. | 10. $168^{\circ} 30''$. | 15. $128^{\circ} 6' 7\frac{1}{2}''$. |

16. Two ships are $32^{\circ} 40' 30''$ of longitude apart. What is the difference in time ?

17. Two observatories are $42^{\circ} 27' 45''$ of longitude apart. What is the difference in time ?

18. A ship in 15° W. sends a wireless telegram at noon. If the telegram reaches Greenwich immediately, at what time does it arrive at Greenwich ?

19. A ship in $58^{\circ} 17' 20''$ W. receives at noon a wireless telegram from one in $43^{\circ} 2' 5''$ W. When was it sent ?

20. Italy uses the time of 15° E., and Indiana that of 90° W. When it is noon in Italy, what time is it in Indiana ?

21. Constantinople uses for railway purposes the time of 30° E., and New England uses that of 75° W. When it is 1 P.M. in Constantinople, what time is it in New England ?

22. A ship at sea carries a chronometer that shows Greenwich time. A ship is in longitude $37^{\circ} 42' 30''$ W. at noon. What is the time by the Greenwich chronometer ?

23. A traveler sailing from New York has his watch set by 75th meridian time. Two days later he finds that the ship is in longitude 55° W. Is his watch faster or slower than ship time, and how much ?

413. Finding Difference in Longitude. If the difference in time between two ships is 2 hr. 17 min. 22 sec., what is the difference in longitude?

Consulting the table (§ 410), we find the difference in longitude corresponding to 1 hr., 1 min., and 1 sec., and then obtain :

$$\begin{aligned} 2 \times 15^\circ &= 30^\circ \\ 17 \times 15' &= 255' = 4^\circ 15' \\ 22 \times 15'' &= 330'' = 5' 30'' \end{aligned}$$

Therefore the difference in longitude is $34^\circ 20' 30''$.

Given the difference in time to find the difference in longitude, multiply 15° , $15'$, $15''$ respectively by the number of hours, minutes, and seconds of time, and add the products.

EXERCISE 263

Find the difference in longitude, given the following differences in time :

- | | |
|------------------|----------------------------|
| 1. 2 hr. 3 min. | 6. 4 hr. 20 min. |
| 2. 4 hr. 7 min. | 7. 5 hr. 25 min. |
| 3. 6 hr. 9 min. | 8. 7 hr. 6 min. 8 sec. |
| 4. 10 hr. 4 sec. | 9. 9 hr. 35 min. 40 sec. |
| 5. 11 hr. 8 sec. | 10. 11 hr. 58 min. 48 sec. |

11. When it is noon by the Greenwich chronometer it is 2 P.M. where a ship is. What is the ship's longitude?

12. When it is 3 P.M. by the Greenwich chronometer it is noon where a ship is. What is the ship's longitude?

13. The captain of another ship finds that when it is noon where the ship is it is 4 P.M. by the Greenwich chronometer. What is the longitude of the ship?

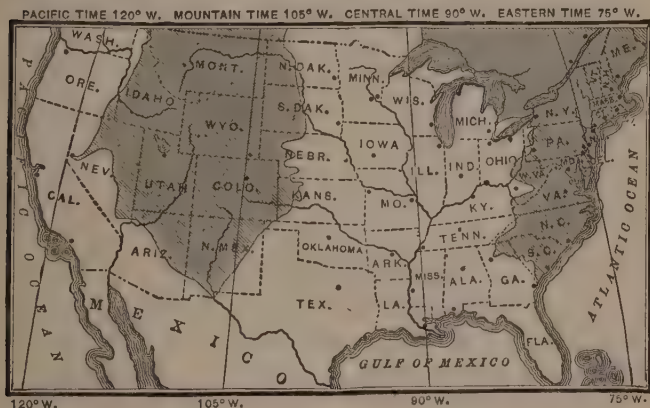
14. An ocean traveler whose watch is set by Greenwich time finds that when it is noon by ship time it is 1.45 P.M. by his watch. What is his longitude?

414. Standard Time. For practical purposes most of the civilized world is now divided into sections, and all places in the same section use the time of a certain meridian in that section. Generally this meridian is 15° , 30° , 45° , or other multiple of 15° east or west of Greenwich.

Thus England uses the time of 0° (Greenwich meridian); Germany, Switzerland, Italy, and central Europe in general use that of 15° E.; Japan uses that of 135° E.; and Australia, on account of its size, is divided into different sections.

A system in which all places in a section of territory near some chosen meridian use the time of that meridian is called a system of *standard time*.

The following is the standard-time map of the United States:



The irregular boundaries of the sections are caused by the desire to have the division lines pass through important railway centers.

When it is noon by Eastern Time (75° W.) it is 11 A.M. by Central Time (90° W.), 10 A.M. by Mountain Time (105° W.), 9 A.M. by Pacific Time (120° W.), and 5 P.M. by Greenwich (0°) time.

In Exercise 264 every example refers to standard time.

EXERCISE 264

1. When it is noon in Wisconsin, what time is it in Wyoming? in California? in New York? in Missouri?

2. When it is 11 A.M. in Minnesota, what time is it in Colorado? in eastern Nebraska? in Vermont? in England?

3. When it is 1 P.M. in Montana, what time is it in Iowa? in Indiana? in Seattle? in Boston? in London?

4. When it is 2.30 P.M. in Utah, what time is it in southern Idaho? in Arkansas? in New Hampshire?

5. When it is 9.45 A.M. in San Francisco, what time is it in eastern Oklahoma? in Louisiana? in Florida? in Alabama? in Maine?

6. When it is 11.45 P.M. in Los Angeles, what time is it in western Tennessee and Kentucky? in Mississippi? in Virginia? in Connecticut? in England?

7. At 9 P.M. a telegram is sent from London to San Francisco. Allowing an hour for transmission and delivery, at what time will it reach its destination?

8. At 2 A.M. on January 1 a telegram is sent from Constantinople (time longitude 30° E.) to Portland, Oregon. Allowing an hour for transmission, on what day and at what hour will it reach its destination?

9. If an event happens in Japan (135° E.) at 10 P.M. and the news is telegraphed without delay, at what time will it reach Italy? England? New York? San Francisco?

10. The longitude of San Francisco is $122^{\circ} 25' 40\frac{1}{2}''$ W. What is the difference between local (the real meridian) time and standard time? Which is the faster?

11. The longitude of New York is $73^{\circ} 58' 25\frac{1}{2}''$ W. What is the difference between local time and standard time? Which is the faster?

EXERCISE 265

PROBLEMS IN LONGITUDE AND TIME

1. The longitude of Cincinnati is $84^{\circ} 26'$ W. and that of Berlin is $13^{\circ} 23' 45''$ E. Find the difference in local time.

2. The longitude of Boston is $71^{\circ} 4'$ W. and that of Rochester is $77^{\circ} 51'$ W. Find the difference in local time.

3. The longitude of St. Paul is $93^{\circ} 5'$ W. and that of St. Petersburg $30^{\circ} 18'$ E. Find the difference in local time.

4. New York is $74^{\circ} 0' 3''$ west longitude and Chicago is $87^{\circ} 38'$ west; how much sooner does the sun rise at New York than at Chicago?

5. When it is noon at Greenwich it is 5 hr. 20 min. 59.4 sec. P.M., local time, at Madras, India. What is the longitude of Madras?

6. When it is noon at Portland, Me., local time, it is 4 hr. 41 min. 1.2 sec. P.M. at Greenwich. What is the longitude of Portland?

7. When it is noon at Greenwich it is 6 hr. 12 min. 48 sec. A.M., local time, at Nashville, Tenn. What is the longitude of Nashville?

8. When it is noon at the Cape Hatteras lighthouse, local time, it is 5 hr. 2 min. 5 sec. P.M. at Greenwich. What is the longitude of the Cape Hatteras lighthouse?

9. When it is noon at Greenwich it is 6 hr. 2 min. 22.2 sec. A.M., local time, at Madison, Wis. What is the difference between local and standard time at Madison?

10. On a certain day a sea captain found his Greenwich chronometer had lost 2 hr. 45 min. 30 sec., compared with the sun. Find his longitude at the time of the observation. Explain fully.

11. When it is noon in Greenwich it is 9 P.M. in Japan. What longitude does Japan use in calculating time?

12. When it is noon in Melbourne it is 2 A.M. in Greenwich. What longitude does Melbourne use in calculating time?

13. A traveler sails from New York with his watch set by the time of 75° W. If the watch indicates 2.30 P.M. when it is 5 P.M. by the ship's time, in what longitude is he?

14. A traveler sails from England with his watch set by Greenwich time. When he reaches $52^{\circ} 30' 45''$ W., is his watch faster or slower than the time of that place, and how much?

15. Suppose a telegram sent from an observatory $121^{\circ} 32' 48''$ W. at 8.40 A.M. to another observatory $73^{\circ} 8' 16''$ W., and to require 16 min. for transmission. At what time will it be received?

16. Suppose a telegram sent at 4 P.M. from a place in Germany 15° E. to a place in California 120° W. At what time will it reach its destination, allowing 30 min. for repeating at various terminal offices?

17. A ship's captain observing the sun finds that when the ship is on the meridian (that is, at noon) the time is 3 hr. 7 min. 35 sec. P.M. by a chronometer set by Greenwich time. What is the ship's longitude?

18. Two observatories A and B are connected by a telegraph line. A telegram sent from A at 2 hr. 4 min. 6 sec. P.M. reaches B at 4 hr. 8 min. P.M. The longitude of B is $79^{\circ} 12' 42.2''$ W. What is the longitude of A?

19. The officers of a steamer in the Mediterranean Sea find that when the sun is on the meridian the time is 10 hr. 50 min. 15 sec. A.M. by a chronometer set by Greenwich time. What is the ship's longitude?

II. METRIC MEASURES

415. The Metric System. The metric system is now used in most of the European countries, in parts of America, and in scientific work everywhere. Since it is on the decimal scale, it is much simpler than our system. We need to know it because we frequently see the terms used in newspapers and books.

416. Meter. The unit of length is the *meter*.

The meter is a little more than 1 yd.

417. Liter. The unit of capacity is the *liter* (pronounced lee'ter), a cube 0.1 of a meter on an edge.

A hollow cube 4 in. on an edge holds a little more than a liter.

418. Gram. The unit of weight is the *gram*. It is the weight of a cube of water 0.01 of a meter on an edge at the greatest density of water, a little above freezing.

We may think of a gram as about the weight of a cube of ice $\frac{2}{3}$ of an inch on an edge.

419. The Prefixes. The system is understood as soon as the prefixes are known.

Just as 1 mill = $\frac{1}{1000}$ of a dollar,
so 1 millimeter = 0.001 of a meter.

Just as 1 cent = $\frac{1}{100}$ of a dollar,
so 1 centimeter = 0.01 of a meter.

Just as decimal means tenths,
so 1 decimeter = 0.1 of a meter.

	THE PREFIX	MEANS	AS IN	WHICH MEANS	
From the Greek	myria-	10,000	myriameter	10,000	meters.
	kilo-	1,000	kilogram	1,000	grams.
	hekto-	100	hektoliter	100	liters.
	deka-	10	dekameter	10	meters.
From the Latin		1		1	
	deci-	0.1	decimeter	0.1	of a meter.
	centi-	0.01	centigram	0.01	of a gram.
	milli-	0.001	millimeter	0.001	of a meter.

420. Important Names. As in United States money we seldom speak of anything except dollars and cents, so in metric measures only those measures printed in black letters in the tables of this chapter are in common use.

All the units of the system are derived from the meter. Every compound name is accented on the first syllable ; thus, *mil'limeter*. The teacher should be supplied with a meter stick, a liter, and a cubic centimeter, and these can easily be made in school if necessary. The work should be made as practical as possible by actual measurements.

421. Measures of Length. The table of measures of length is as follows :

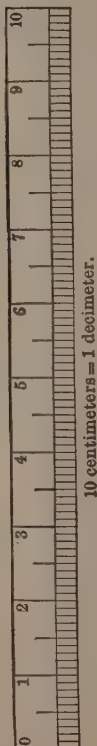
A myriameter	=	10,000 meters
A kilometer (km.)	=	1,000 meters
A hektometer	=	100 meters
A dekameter	=	10 meters
Meter (m.)		
A decimeter (dm.)	=	0.1 of a meter
A centimeter (cm.)	=	0.01 of a meter
A millimeter (mm.)	=	0.001 of a meter

The meter is about 39.37 in., or $3\frac{1}{4}$ ft., or a little over a yard ; the kilometer is about 0.62 of a mile. When the metric system was invented in France over a century ago, the meter was intended to be one ten-millionth of the distance on the surface of the earth from the equator to the pole, but it varies slightly from this standard.

The abbreviations in this book are in common use. Some, however, use Km., Dm., dm., for kilometer, dekameter, and decimeter. We do not need abbreviations for the units not commonly used, such as the myriameter.

422. Reduction. Any one of these measures may be expressed in terms of another measure by *simply moving the decimal point to the right or left*.

Thus, as $245\phi = 24.5 \text{ d.} = \$2.45,$
 so $2475 \text{ mm.} = 247.5 \text{ cm.} = 24.75 \text{ dm.} = 2.475 \text{ m.},$
 and $35.25 \text{ km.} = 35,250 \text{ m.} = 35,250,000 \text{ mm.}$



EXERCISE 266

Express as meters and decimals :

- | | | |
|------------------------|--------------|-----------------|
| 1. 25 km. | 6. 237 dm. | 11. 2965 cm. |
| 2. 3.7 km. | 7. 47.5 dm. | 12. 48,750 cm. |
| 3. 0.7 km. | 8. 2.75 dm. | 13. 29,375 mm. |
| 4. $62\frac{1}{2}$ km. | 9. 725 cm. | 14. 96,400 mm. |
| 5. $57\frac{3}{4}$ km. | 10. 68.5 cm. | 15. 250,000 mm. |

Allowing 39.37 in. to a meter, express as meters :

- | | | |
|----------------|-------------|---------------------|
| 16. 19.685 in. | 19. 25 ft. | 22. 3 ft. 3 in. |
| 17. 78.74 in. | 20. 4.8 ft. | 23. 3 ft. 3.37 in. |
| 18. 118.11 in. | 21. 0.6 ft. | 24. 29 ft. 6.33 in. |

Express as feet :

- | | | |
|-------------------------|-------------|----------------|
| 25. 75 m. | 28. 35 dm. | 31. 4500 cm. |
| 26. $48\frac{1}{2}$ m. | 29. 7.5 dm. | 32. 6875 mm. |
| 27. $2.7\frac{1}{2}$ m. | 30. 685 cm. | 33. 21,475 mm. |

Allowing 0.62 mi. to a kilometer, express as miles :

- | | | |
|------------|------------------------|--------------|
| 34. 5 km. | 36. $2\frac{1}{2}$ km. | 38. 272 km. |
| 35. 75 km. | 37. 4.75 km. | 39. 48.9 km. |

Express as kilometers :

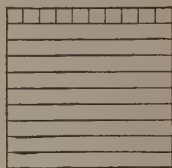
- | | | |
|-------------|--------------|--------------|
| 40. 325 mi. | 41. 42.7 mi. | 42. 0.75 mi. |
|-------------|--------------|--------------|

43. If sound travels 340.5 m. a second, how many kilometers will it travel in 17 sec. ?

44. If the distance from Paris to Brussels is 313 km., how many miles is it ?

45. If the distance from Lyons to Marseilles is 218 mi., how many kilometers is it ?

423. Square Measure. In square measure the scale is by hundreds instead of by tens. If the square here shown represents a square meter, it is evident that each of the ten equal horizontal bands is one tenth of a square meter, and each of the ten small squares therefore represents $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{100}$, of a square meter. That is, 100 square decimeters = 1 square meter.



The table is as follows:

A square myriameter	=	100,000,000 square meters
A square kilometer (sq. km.)	=	1,000,000 square meters
A square hektometer	=	10,000 square meters
A square dekameter	=	100 square meters
Square meter (sq. m.)		
A square decimeter	=	0.01 of a square meter
A square centimeter (sq. cm.)	=	0.0001 of a square meter
A square millimeter (sq. mm.)	=	0.000001 of a square meter

424. Land Measure. In the measurement of land the square dekameter is called an *are* (pronounced är); the square hektometer is called a hektare (ha.). The hektare equals 2.47 acres, or nearly $2\frac{1}{2}$ acres.

EXERCISE 267

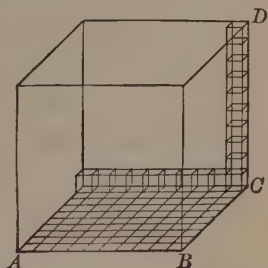
Express as square meters:

- | | | |
|-----------------|-------------------|-----------------------|
| 1. 275 sq. km. | 4. 4875 sq. cm. | 7. 123,450 sq. mm. |
| 2. 3.45 sq. km. | 5. 9235 sq. cm. | 8. 2,250,000 sq. mm. |
| 3. 0.75 sq. km. | 6. 64,275 sq. cm. | 9. 37,375,000 sq. mm. |

Express as hektares:

- | | | |
|--|--------------------|-----------------|
| 10. 10,000 sq. m. | 12. 175,000 sq. m. | 14. 24.7 acres. |
| 11. 45,000 sq. m. | 13. 325,500 sq. m. | 15. 1482 acres. |
| 16. How long is a square field that contains 0.81 ha.? | | |

425. Cubic Measure. In cubic measure the scale is by thousands. If the cube here shown represents a cubic meter, it is evident that each of the ten equal layers represents one tenth of a cubic meter; that each of the rectangular solids, DC , represents $\frac{1}{10}$ of $\frac{1}{10}$ of a cubic meter, and that each of the small cubes represents $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{1000}$, of a cubic meter. That is, 1000 cubic decimeters = 1 cubic meter.



The table is as follows:

A cubic hektometer	= 1,000,000 cubic meters
A cubic dekameter	= 1,000 cubic meters
Cubic meter (cu. m.)	
A cubic decimeter (cu. dm.)	= 0.001 of a cubic meter
A cubic centimeter (cu. cm.)	= 0.000001 of a cubic meter
A cubic millimeter (cu. mm.)	= 0.000000001 of a cubic meter

We may also have cubic kilometers and cubic myriameters. The terms above the cubic meter are seldom used.

426. Wood Measure. In measuring wood the cubic meter is called a *stere* (st.). Stere is pronounced stair.

EXERCISE 268

Express as cubic meters:

- 725.25 st.
- 2,750,000 cu. dm.
- 4,625,750 cu. dm.
- 9,125,000 cu. dm.
- 8,125,000,000 cu. mm.
- 9,275,000,000 cu. mm.
- 25,325,000,000 cu. mm.
- 37,250,000,000 cu. mm.
- A pile of wood is 2.5 m. high, 1 m. wide, and 28.7 m. long. How many steres are there?

427. Measures of Capacity. The table of capacity is as follows:

A hektoliter (hl.)	=	100 liters
A dekaliter	=	10 liters
Liter (l.)		
A deciliter (dl.)	=	0.1 of a liter
A centiliter (cl.)	=	0.01 of a liter
A milliliter (ml.)	=	0.001 of a liter

A liter is a cubic decimeter. It is practically the same as a quart.

EXERCISE 269

Express as liters:

- | | | | |
|------------|--------------|------------|-------------|
| 1. 750 hl. | 3. 92.75 hl. | 5. 750 dl. | 7. 7500 ml. |
| 2. 275 hl. | 4. 9.375 hl. | 6. 975 cl. | 8. 6250 ml. |

Express as hektoliters:

- | | | | |
|-------------|--------------|-------------------------|--------------------------|
| 9. 3750 l. | 11. 978.5 l. | 13. $97\frac{1}{2}$ qt. | 15. 65 gal. |
| 10. 4875 l. | 12. 25.75 l. | 14. $38\frac{3}{4}$ qt. | 16. $31\frac{1}{2}$ gal. |

Express as quarts:

- | | | | |
|------------|-------------|-------------|--------------|
| 17. 37 hl. | 18. 9.5 hl. | 19. 750 dl. | 20. 8250 cl. |
|------------|-------------|-------------|--------------|
21. How many liters are there in a cubic meter?
22. How many hektoliters of air are there in a room 7 m. long, 6 m. wide, and 3 m. high?
23. A cubical tank in a zoölogical garden is 2.25 m. on an edge. How many liters will it hold?
24. A bin is 11.25 m. long, 4.8 m. wide, and 1.6 m. deep. How many hektoliters will it hold? How many liters?
25. The average depth of a pond is 1.75 m., and its area is 3.5 ha. How many hektoliters of water does it contain?
26. The height of a cylindrical tank is 12 m., and the diameter of the base is 9 m. How many liters will it hold?

428. Measures of Weight. The table of weight is as follows :

A metric ton (t.)	=	1,000,000 grams
A quintal (q.)	=	100,000 grams
A myriagram	=	10,000 grams
A kilogram (kg.)	=	1,000 grams
A hektogram	=	100 grams
A dekagram	=	10 grams
Gram (g.)		
A decigram	=	0.1 of a gram
A centigram (cg.)	=	0.01 of a gram
A milligram (mg.)	=	0.001 of a gram

429. Relation of Weight to Capacity.

1 cu. cm. of water weighs 1 g.

1 l. of water weighs 1 kg.

1 cu. m. of water weighs 1 t.



The standard units vary slightly from these theoretical units.

Cubic Centimeter Gram Weight

A kilogram is about 2.2 lb.; and a metric ton is about 2204.6 lb.

EXERCISE 270

Express as grams :

- | | | | |
|------------|-------------|------------|-------------|
| 1. 1.7 kg. | 3. 0.25 kg. | 5. 700 cg. | 7. 7500 mg. |
| 2. 0.9 kg. | 4. 3.75 kg. | 6. 950 cg. | 8. 8125 mg. |

Express as milligrams :

- | | | | |
|------------|-------------|-------------|-------------|
| 9. 7 g. | 11. 0.27 g. | 13. 15 kg. | 15. 75 cg. |
| 10. 8.7 g. | 12. 4.25 g. | 14. 2.7 kg. | 16. 9.8 cg. |

Express as kilograms :

- | | | | |
|-----------|------------|------------|--------------|
| 17. 27 t. | 18. 0.5 t. | 19. 250 g. | 20. 4400 lb. |
|-----------|------------|------------|--------------|

Express as pounds :

- | | | | |
|-----------|------------|------------|--------------|
| 21. 3 kg. | 22. 75 kg. | 23. 300 g. | 24. 9000 mg. |
|-----------|------------|------------|--------------|

430. Specific Gravity. The ratio of the weight of a given substance to the weight of an equal volume of water is called the *specific gravity* of the substance.

Thus, if a cubic decimeter of copper weighs 8.9 kg., the specific gravity of copper is 8.9, because 1 cu. dm. = 1 l., and 1 l. of water weighs 1 kg. Therefore copper is 8.9 times as heavy as water.

It is usually easy to find the volume of any substance by immersing it in a vessel full of water and measuring the amount that runs over.

By means of specific gravity the purity of metals and other substances may be determined.

In the case of gases the weight of the gas is usually compared with that of air or hydrogen instead of water.

EXERCISE 271

1. If 2 l. of alcohol weigh 1.58 kg., what is the specific gravity of alcohol?

2. If 5 l. of petroleum weigh $3\frac{1}{2}$ kg., what is the specific gravity of petroleum?

3. If 7 cu. cm. of cork weigh 1.68 g., what is the specific gravity of cork?

4. If 6.5 l. of milk weigh $6.69\frac{1}{2}$ kg., what is the specific gravity of milk?

5. The specific gravity of silver is 10.5. Some spoons weighing 371.7 g., thought to be silver, are immersed in a jar full of water and 35.4 cu. cm. of water overflow. Do the spoons stand the specific-gravity test?

6. A body immersed in a liquid always weighs less than it does in air by the weight of the liquid that it displaces. If a piece of gold weighs 9.65 g. in air and 9.15 g. in water, what is its specific gravity?

7. If the specific gravity of steel is 7.8, what is the weight of a cylindrical steel rod 1 m. long and 5 cm. in diameter?

EXERCISE 272

REVIEW PROBLEMS

1. Find the area of a square 1.37 m. on a side.
2. Find the area of a rectangle 3.95 m. by 1.36 m.
3. Find the area of a circle whose radius is 75 cm.
4. Find the area of a circle whose diameter is 324 mm.
5. Find the area of a parallelogram whose base is 2.75 m. and whose altitude is 1.65 m.
6. Find the area of a triangle whose base is 325 cm. and whose altitude is 76 cm.
7. Find the altitude of a parallelogram whose area is 3.75 sq. m. and whose base is 1.875 m.
8. Find the base of a triangle whose area is 9.106 sq. m., and whose altitude is 2.9 m.
9. Find the weight of the water required to fill a tank 2.3 m. long, 0.98 m. wide, and 0.56 m. deep.
10. A liter of air weighs 1.292 g. Find the weight of air in a schoolroom 6.8 m. long, 5.2 m. wide, and 3 m. high.
11. How many liters of water will a tank hold that is 2.7 m. long, 1.4 m. wide, and 0.75 m. deep?
12. If a barrel full of water weighs 123 kg., and when empty weighs 10 kg., how many liters will it hold?
13. What part of a hektare is there in a piece of land 125 m. long and 65 m. wide?
14. If a 40-liter keg of olive oil weighs 40.5 kg., and the empty keg weighs 3.9 kg., what does a liter of oil weigh?
15. In a shop where machines are made for foreign trade an order is received to make some wheels 1.34 m. in diameter. How would the workmen express the diameter in inches?

431. Metric System Equivalents.

Length

Meter	= 39.370 in.	Inch	= 0.0254 m.
	= 3.281 ft.	Foot	= 0.3048 m.
	= 1.093 yd.	Yard	= 0.9144 m.
	= about 1.1 yd.		= about 0.9 m.
Millimeter	= 0.039 in.	Inch	= 25.400 mm.
Centimeter	= 0.3937 in.		= 2.540 cm.
Kilometer	= 0.62137 mi.	Mile	= 1.60935 km.
	= about $\frac{3}{5}$ mi.		= about 1.6 km.

Surface

Sq. meter	= 10.764 sq. ft.	Sq. foot	= 0.0929 sq. m.
	= 1.1960 sq. yd.	Sq. yard	= 0.8361 sq. m.
Hektare	= 2.471 A.	Acre	= 0.4047 ha.
	= about $2\frac{1}{2}$ A.		= about $\frac{2}{5}$ ha.
Sq. kilometer	= 0.3861 sq. mi.	Sq. mile	= 2.59 sq. km.

Volume

Cu. centimeter	= 0.061 cu. in.	Cu. inch	= 16.3872 cu. cm.
Cu. meter	= 35.314 cu. ft.	Cu. foot	= 0.0283 cu. m.
	= 1.3079 cu. yd.	Cu. yard	= 0.7646 cu. m.
Stere	= 0.2759 cd.	Cord	= 3.6245 st.

Capacity

Liter	= 1.05668 liq. qt.	Liq. quart	= 0.94636 l.
	= 0.9081 dry qt.	Dry quart	= 1.1012 l.
	= 0.26417 gal.	Gallon	= 3.78543 l.
	= 0.11351 pk.	Peck	= 8.80982 l.
Hektoliter	= 2.83774 bu.	Bushel	= 0.35239 hl.

Weight

Gram	= 15.4324 gr.	Grain	= 0.0648 g.
	= about $15\frac{1}{2}$ gr.		= about $\frac{3}{50}$ g.
	= 0.03527 oz. av.	Ounce av.	= 28.3495 g.
Kilogram	= 2.20462 lb. av.	Pound av.	= 0.45359 kg.
	= about $2\frac{1}{5}$ lb.		= 0.00045 t.
Met. ton.	= 2204.62 lb. av.	Short ton	= 0.90718 t.

III. TABLES OF DENOMINATE NUMBERS

Table of Length

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
 $5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods or 5280 feet = 1 mile (mi.)

		ft.	in.
	yd.	1 =	12
	rd.	1 = 3 =	36
mi.	1 =	$5\frac{1}{2}$ = $16\frac{1}{2}$ =	198
	1 = 320 = 1760 = 5280 =		63,360

A *hand* (4 in.) is used in measuring the height of horses ; a *fathom* (6 ft.) and a *cable length* (120 fathoms) in measuring depths of water ; a *knot* (nautical mile, 1.152 common or statute miles, or 6080.27 ft.) in measuring distances at sea. Carpenters and mechanics usually write 2' 6" for 2 ft. 6 in.

Table of Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
 $30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

		sq. yd.	sq. ft.	sq. in.
		1 =	1 =	144
	sq. rd.	1 =	9 =	1296
A.	1 =	$30\frac{1}{4}$ =	$272\frac{1}{4}$ =	39,204
sq. mi.	1 =	160 =	4840 =	43,560 = 6,272,640
	1 = 640 = 102,400 = 3,097,600 =		27,878,400 =	4,014,489,600

Carpenters and mechanics often write *sq.'* for square feet, and *sq."* for square inches.

The unit of measure in plastering, painting, and calcimining is usually the square yard. Flooring is measured by the *square*, 100 sq. ft. Carpenters allow about 1000 shingles (4 bunches) to 100 sq. ft.

*Table of Cubic Measure***1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)****27 cubic feet = 1 cubic yard (cu. yd.)****128 cubic feet = 1 cord (cd.)**

	cu. ft.	cu. in.
cu. yd.	1 =	1728
	1 =	27 = 46,656

A perch of stone or masonry is usually 1 rd. long, 1 ft. high, and $1\frac{1}{2}$ ft. thick, and contains $24\frac{3}{4}$ cu. ft., but this varies in different parts of the country. A cubic yard of earth is considered a load.

*Table of Avoirdupois Weight***16 ounces (oz.) = 1 pound (lb.)****2000 pounds = 1 ton (T.)**

	lb.	oz.
T.	1 =	16
	1 =	2000 = 32,000

100 lb. is sometimes called a *hundredweight* (cwt.). The ton of 2000 lb. is sometimes called the *short ton*, there being an old ton of 2240 lb., known as the *long ton*, which is used in the customhouse and in some wholesale transactions in mining products. There is also the *long hundredweight* of 112 lb., but it is rarely used except in English trade.

*Table of Troy Weight***24 grains (gr.) = 1 pennyweight (pwt. or dwt.)****20 pennyweights = 1 ounce (oz.)****12 ounces = 1 pound (lb.)**

The avoirdupois pound contains 7000 gr., the troy pound 5760 gr. A *carat* weight is commonly taken in the United States as 3.2 gr.

*Table of Apothecaries' Weight***20 grains (gr.) = 1 scruple (sc. or $\mathfrak{S}$)****3 scruples = 1 dram (dr. or $\mathfrak{D}$)****8 drams = 1 ounce (oz. or $\mathfrak{Z}$)****12 ounces = 1 pound (lb.)**

*Table of Liquid Measure***4 gills (gi.) = 1 pint (pt.)****2 pints = 1 quart (qt.)****4 quarts = 1 gallon (gal.)** **$31\frac{1}{2}$ gallons = 1 barrel (bbl.)****63 gallons = 1 hogshead (hhd.)**

		pt.	gi.
	qt.	1 =	4
gal.	1 =	2 =	8
bbl.	1 =	4 =	8 = 32
	1 =	$31\frac{1}{2}$ =	126 = 252 = 1008

A gallon contains 231 cubic inches.

Casks holding from 28 gal. to 43 gal. are called barrels, and casks holding from 54 gal. to 63 gal. are called hogsheads. Whenever barrels or hogsheads are used as *measures*, a barrel means $31\frac{1}{2}$ gallons, and a hogshead means 63 gallons. A barrel contains about $4\frac{1}{3}$ cu. ft.

Apothecaries and physicians use the following table of liquid measure, but it need not be learned:

60 minims (℥) = 1 dram (℥ lx. or f. 3)

8 fluid drams = 1 ounce (fl. dr. viij. or f. 3)

16 ounces = 1 pint (fl. oz. xvj. or O)

8 pints = 1 gallon (cong.)

*Table of Dry Measure***2 pints (pt.) = 1 quart (qt.)****8 quarts = 1 peck (pk.)****4 pecks = 1 bushel (bu.)**

	qt.	pt.
pk.	1 =	2
bu.	1 =	8 = 16
	1 =	4 = 32 = 64

A bushel contains 2150.42 cu. in., or about $1\frac{1}{4}$ cu. ft. A dry quart contains 67.2 cu. in., while the liquid quart contains only 57.75 cu. in. In measuring grain, seeds, and small fruits the measure must be *even* full. In measuring apples, potatoes, and other large articles the measure must be *heaping* full.

Table of Surveyors' Linear Measure

7.92 inches (in.) = 1 link (li.)
25 links = 1 rod (rd.)
100 links = 1 chain (ch.)
80 chains = 1 mile (mi.)

Table of Surveyors' Square Measure

16 square rods (sq. rd.) = 1 square chain (sq. ch.)
10 square chains = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
1 square mile = 1 section (sec.)
36 sections = 1 township (T.)

Table of Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days = 1 common year (yr.)
366 days = 1 leap year

			min.	sec.
		hr.	1 =	60
	da.	1 =	60 =	3600
wk.	1 =	24 =	1440 =	86,400
yr. mo.	1 =	7 =	168 =	10,080 =
1 = 12 =	{ 365 = 8760 = 525,600 = 31,536,000 { 366 = 8784 = 527,040 = 31,622,400			

Angle Measure

60 seconds (") = 1 minute (')
60 minutes = 1 degree (°)
360 degrees = 4 right angles

Arc Measure

60 seconds (") = 1 minute (')
60 minutes = 1 degree (°)
360 degrees = 1 circumference

<i>Numbers</i>	<i>Paper</i>
12 units = 1 dozen (doz.)	24 sheets = 1 quire
12 dozen = 1 gross (gr.)	20 quires = 1 ream
12 gross = 1 great gross	2 reams = 1 bundle
20 units = 1 score	5 bundles = 1 bale

The great gross and score are becoming obsolete. For convenience in counting, 500 sheets are now commonly called a ream.

United States Money

10 mills = 1 cent (ct. or ¢)
10 cents = 1 dime (d.)
10 dimes = 1 dollar (\$)
10 dollars = 1 eagle (E.)

The term *eagle* is not used. The mill is not coined, although the name is frequently used in speaking of fractional parts of a cent.

English Money

4 farthings = 1 penny (d.)
12 pence = 1 shilling (s. or /)
20 shillings = 1 pound (£) or sovereign

Notes on the Common Units of Measure. The smaller units of length were originally derived from the measure of the human body. The *inch* was the length from the end of the thumb to the first joint. Four inches were taken as the *palm* or *hand-breadth*, and three hand-breadths as the *foot*. The entire length of the arm was the *yard*. The distance to which a man's two hands can extend, across the shoulders, was the *fathom*. The ancient *pace* was the distance covered in two steps, and was taken to be six feet, and a thousand paces (*mille passus*, in Latin) was the *mile*. The *pound* is a very ancient weight. The Roman *gallon* of water weighed ten pounds. The Roman pound was divided into twelve *ounces* (*uncia*). In 1266 the English law decided that the penny should weigh as much as 32 grains of wheat "in the midst of the ear," and that 20 pennies should regulate the weight of the ounce, 12 ounces should make a pound, 8 pounds a gallon, and 8 gallons a bushel. Our word *quart* means quarter of a gallon. *Acre* is an old name for a cultivated field. The word *yard* is from an old word meaning stick. *Pint* means a mark, there having been a mark made halfway up the old quart measures.

SPECIMEN OF EXAMINATION SET BY THE NEW YORK
STATE EDUCATION DEPARTMENT

Answer question 1 and nine other questions. No credit will be allowed unless all operations necessary to find results are given. Reduce each result to its simplest form and mark each answer "Ans."

1. See separate sheet. (The addition of 20 numbers, the largest being \$8743.45 and the smallest being \$9.43. The multiplication of 22,568 by 607.) [No credit is allowed unless the answer is correct.]

2. Multiply 78,092 by 105.68 [no credit will be allowed unless the answer is fully correct].

3. Define circumference, denominator, denominate number, commission, payee.

4. A man has three fields containing respectively $52\frac{3}{4}$, $75\frac{4}{5}$, and $67\frac{2}{3}$ acres; if he sells $84\frac{3}{5}$ acres, how much of the land will he have left?

5. I need 273 planks 4 ft. 8 in. long, 1 ft. wide, and $1\frac{1}{2}$ in. thick to build a sidewalk; how much will they cost at \$24.75 per thousand feet?

6. A man owning .375 of a ship sold $\frac{1}{3}$ of his share for \$25,896.45; what would be the total value of the ship at the same rate?

7. Find the cost of carpeting a room 26 ft. long by 18 ft. wide, with carpet $\frac{3}{4}$ of a yard wide, if the strips run lengthwise of the room and the carpet costs \$1.87 per yard. Illustrate [2 credits will be allowed for illustration].

8. A speculator buys 56 shares of railroad stock at 98 and sells them at $123\frac{3}{4}$; find his gain, the brokerage in each case being $\frac{1}{8}\%$.

9. A boy purchased $3\frac{1}{2}$ pounds of butter at 32 cents a pound, 3 dozen of eggs at 26 cents a dozen, 5 pounds of prunes at 19 cents a pound, and 8 bars of soap at $6\frac{1}{2}$ cents each; how much change should he receive if he gives a \$5 bill in payment?

10. The diagonal of a square is 30 ft.; find to two decimal places the length of one side.

11. The dimensions of a tank are 8 m. long, 6 m. 5 dm. wide, and 4 m. 2 dm. deep; how many liters of water will it hold?

12. A note for \$600 dated December 2, 1909, is discounted at a bank December 27, 1909, at 6%; find the bank discount and the proceeds.

SPECIMEN OF EXAMINATION SET BY THE NEW YORK
STATE EDUCATION DEPARTMENT

(See the directions at the top of page 510.)

1. See separate sheet. (The addition of 18 numbers of from 3 to 6 figures each, and the division of 322,560 by 672.)

2. Define *five* of the following : factor, prime factor, denominate number, proper fraction, trapezoid, bank discount.

3. (a) Express in figures one million one thousand fifty-six and four hundred twenty-five hundred-thousandths.

(b) Express 1911 in Roman notation.

4. (a) From $96\frac{3}{4}$ subtract $78\frac{3}{4}$. (b) Multiply $75\frac{3}{4}$ by $25\frac{3}{4}$. (c) Divide 225 by $18\frac{3}{4}$. (d) Divide $25\frac{3}{4}$ by $16\frac{3}{4}$.

5. The cost of food purchased for 12 hens for the six months from April 1 to October 1 was \$1.48 per month ; during that entire period the owner collected on an average 7 eggs a day ; these eggs were sold at $24\frac{1}{2}$ cents per dozen. Allowing 30 days to the month, how much money was gained by keeping the hens ?

6. (a) Write the table of (1) time measure ; (2) liquid measure.

(b) If a cubic yard of earth weighs $2787\frac{1}{2}$ lb., how many tons will 10 cubic yards 18 cubic feet of earth weigh ?

7. At 21 cents a square yard, what will be the cost of painting the walls of a kitchen 15 ft. long by 12 ft. wide by 9 ft. 4 in. high ?

8. Find the simple interest at 5% on a note for \$5375 dated September 15, 1905, and payable June 12, 1911.

9. Same as page 247, Ex. 113.

10. (a) The triangular gable of a house has a base 13 ft. and an altitude $10\frac{1}{2}$ ft. ; find the area of the triangle.

(b) Compute the cost of fencing a field 58 rd. $12\frac{1}{2}$ ft. long and 39 rd. wide, if the fence costs \$3.25 per rod.

11. (a) In a certain quantity of milk the ratio of the cream to the rest of the milk is as 2 to 9 ; how many pounds of cream are there 253 pounds of the milk ?

(b) The inside dimensions of a car are 8 ft. by 36 ft. by 4 ft. ; find the cost of a carload of oats at 31 cents per bushel.

12. An example like those on page 172.

SPECIMEN OF EXAMINATION SET BY THE NEW YORK
STATE EDUCATION DEPARTMENT

(See the directions at the top of page 510.)

1. See separate sheet. (The addition of 20 numbers, the largest being \$936.25, and the smallest \$1.78. The multiplication of 36.49 by 8.79.)

2. Write in words *each* of the following: (a) 90,909, (b) 125.00036, (c) $149\frac{2}{3}$, (d) 500,000,000, (e) XCIV.

3. A farmer sold 200 bushels of potatoes at 75 cents a bushel, 10 dozen eggs at 24 cents a dozen, and 40 pounds of butter at 30 cents a pound; he invested $\frac{2}{5}$ of the proceeds in coal and deposited the balance in the bank. How much money did he deposit?

4. (a) Reduce $\frac{720}{884}$ to its lowest terms.

(b) Express .625 and .0625 as common fractions in their lowest terms.

5. Perform *each* of the following operations: $7 \times \frac{5}{8}$; $19 \div \frac{3}{4}$; $\frac{5}{8} \times \frac{2}{3}$; $\frac{5}{7} \div \frac{5}{8}$; $1\frac{8}{7} \div 4$.

6. Find the cost, at \$25 per thousand, of 100 boards, each 18 ft. long, 8 in. wide, and $\frac{1}{2}$ in. thick.

7. A dealer bought 300 tons of ice for \$750; he sold the ice at the rate of 25 cents per hundred pounds. Find (a) how much he gained, (b) what per cent he gained.

8. Charles Jones bought of the George N. Johnson Company an automobile for \$4000; he paid \$2500 in cash and for the balance gave a promissory note for 3 months, with interest at 5%. Write the note, dating it January 15, 1912.

9. Find, at date of maturity, the interest at 6% on a note for \$375, dated June 15, 1907, and payable September 27, 1912.

10. Richard, James, and Henry formed a partnership to sell fruit; Richard invested \$48, James \$60, and Henry \$72. If they gained \$60 the first week, what was each partner's share of the profit?

11. Extract the square root of 44,521.

12. A man purchased a house for \$5000; the first year the expense for repairs was \$45, for taxes \$75, for insurance \$10. If the house was rented for \$40 a month, what per cent did he gain on his investment that year?

DEFINITIONS OF COMMON TERMS

Abstract Number. A number used without reference to any particular unit.

Addends. Numbers added.

Addition. The operation of finding a number equal to two or more numbers taken together.

Agent. A person who transacts business for another.

Aliquot Part. An exact divisor of another number.

Amount. The sum of principal and interest.

Analysis. Any process of reasoning by which we prove a mathematical truth.

Antecedent. The first term of a ratio.

Area. The number of square units in a surface.

Arithmetic. The science that treats of numbers and the methods of using them.

Assets. All the property of an estate, individual, or corporation.

Austrian Division. Long division with the quotient written above the dividend.

Austrian Subtraction. Subtraction by the addition or making-change method.

Bank. A corporation authorized by law to receive and loan money and to perform certain other financial operations.

Bank Discount. Interest paid in advance on a note.

Base. The number of which some per cent is to be taken.

Bond. A written or printed promise to pay a specified sum at a given time, signed by the maker and often bearing his seal.

Broker. An agent who makes sales or purchases for his principal without having actual possession of the goods.

Brokerage. A sum paid a broker for transacting business.

Cancellation. The process of shortening work in division by removing or canceling equal factors from both dividend and divisor.

Capital Stock. The amount contributed by the members of a corporation to carry on the business.

Check. Any method tending to verify an operation. A written order on a bank to pay money.

Commercial Discount. A reduction from the list price of an article, from the amount of a bill of goods, or from the amount of a debt.

Commercial Draft. A draft made by one business house on another to secure the payment of a debt.

Commission. A sum paid an agent for transacting business.

Commission Merchant. An agent who has actual possession of goods to be sold for his principal.

Common Denominator. A denominator common to two or more fractions.

Common Divisor. The same as Common Measure.

Common Fraction. A fraction expressed by two numbers, one under the other with a line between them.

Common Measure. A number that exactly divides each of two or more numbers.

Common Multiple. A multiple of two or more numbers.

Composite Number. A number that is not a prime number.

Compound Interest. The total interest when interest is added to the principal and becomes a part of it at specified intervals.

Compound Number. A denominate number involving two or more different units.

Concrete Number. If the unit of a number is named, the number together with the unit is called a *concrete number*.

Consequent. The second term of a ratio.

Consignee. The person to whom goods are sent.

Consignor. The person who sends goods to another.

Corporation. A company composed of a number of persons authorized by law to do business of a certain nature.

Creditor. A person to whom money is due.

Cube. The third power of a number. A six-sided solid, all the sides of which are squares.

Customs. Taxes imposed by law on merchandise imported into a country.

Debtor. A person who owes money to another.

Decimal Fraction. A fraction whose denominator is not written, but is some power of 10.

Decimal Point. The period written at the left of tenths.

Decimal Scale. The common mode of representing numbers, each order having a value ten times that of the order to the right.

Denominate Number. A concrete number expressing measure. (See Compound Number.)

Denominator. The number which is written below the line in a common fraction. It shows the number of parts into which the unit has been divided.

Difference. The result found in subtraction.

Discount. Allowance made for the payment of money before it becomes due. Also the difference between the market value and the face value when the market value is *below* the face value. A reduction from the list price of goods (trade discount).

Discount Series. A series of trade discounts.

Dividend. In division, the given number which is equal to the product of a given factor (called divisor) and a required factor (called quotient). In business, the sum paid on each share of stock from the profits of the business.

Divisible Number. A number is *divisible* by another when it is exactly divisible, that is, divisible without a remainder.

Division. The process by which, when a product and one of two factors are given, the other factor is found.

Divisor. The number by which a given dividend is divided.

Draft. A written order directing one person to pay a specified sum of money to another.

Drawee of a Draft. The person to whose order the sum of money named in the draft is to be paid.

Drawer of a Draft. The person who signs the draft.

Duties. The same as customs.

Even Number. A number that is exactly divisible by 2.

Exact Divisor. A number that divides another without a remainder.

Exact Interest. Interest computed for exact days, and at 365 days to the year.

Exchange. A system of paying debts, due to persons living at a distance, without actually sending the money.

Exponent. A small figure placed at the right of a number to show how many times the number is taken as a factor.

Extremes. The first and last numbers of a proportion.

Factors. Integral numbers whose product is a given number.

Figures. Symbols used to represent numbers in the common system of notation. The word is also used to mean the diagrams used to represent geometric forms.

Fraction. A number that shows what part or what number of parts of a unit is taken.

Greatest Common Divisor. The same as Greatest Common Measure.

Greatest Common Measure. The greatest number that exactly divides each of two or more numbers.

Improper Fraction. A fraction whose numerator is not less than the denominator.

Indorse. To write across the back of a note, draft, or check.

Insurance. An agreement by which, for a lawful consideration, a company agrees to pay an indemnity for loss by specified cause.

Integers. Numbers applied to whole units.

Interest. Money paid for the use of money.

Inverse Ratio. The result of interchanging the terms of a ratio.

Least Common Multiple. The smallest common multiple of two or more numbers.

License. A permit to engage in some specified act or to enjoy some special privilege.

Like Numbers. Numbers applied to the same unit.

Long Division. The method of dividing in which the processes are written in full.

Longitude. The distance in circular measure east or west of a certain meridian agreed upon.

Lowest Terms. If the terms of a fraction are prime to each other, the fraction is expressed in *lowest terms*.

Maker of a Note. The person who signs a note.

Market Value. The price at which goods or stocks sell.

Maturity of a Note. The date at which a note legally becomes due.

Means. The two middle numbers of a proportion.

Meridian. The shortest line on the earth's surface passing from one pole to the other.

Minuend. The number from which the subtrahend is taken.

Mixed Number. A number composed of an integer and a fraction taken together.

Multiple. The product of two or more integers is called a *multiple* of each.

Multiplicand. A number to be multiplied.

Multiplication. The process of taking a number as many times as there are units in another number.

Multiplier. A number by which another is multiplied.

Net Price. The amount of a bill after the discount is made.

Net Proceeds. The money that remains of the money received for property after all expenses and discounts are paid.

Notation. A system of expressing numbers by symbols.

Note. A written promise to pay a specified sum of money on demand or at a specified time. •

Number. That which shows how many times a unit is taken, or what part of a unit is taken.

Numerals. The characters used in any system of notation.

Numeration. The reading of numbers.

Numerator. The number above the line in a common fraction.

Odd Number. A number that is not an even number.

Order of Units. Each successive place in a number that may be occupied by a figure.

Par Value. The face value of stocks.

Partial Payments. Payments of a part of a note.

Payee. The person to whom a note, check, or draft is payable.

Per Cent. Another name for hundredths.

Percentage. The part of arithmetic treating of per cent. The result found by taking a certain per cent of the base.

Perimeter. The distance around a polygon.

Period. A group of three figures.

Policy. A written contract of insurance.

Poll Tax. A tax levied upon a person.

Power. The result of taking a number any number of times as a factor.

Premium. Money paid for insurance. Also the excess of market value above par value.

Prime Factor. A factor that is a prime number.

Prime Number. A number that has no factors.

Prime to Each Other. Two integers that have no common measure, always excepting 1.

Principal. The person for whom another transacts business. Money loaned on which interest is paid.

Proceeds. The balance after deducting the discount from the face of a note.

Product. The result found in multiplication.

Promissory Note. A written promise to pay a specified sum of money on demand or at a specified time.

Proper Fraction. A fraction whose numerator is less than the denominator.

Property Tax. A tax levied upon property.

Proportion. An expression of equality between two ratios.

Quotient. The number found by division.

Rate of Interest. The rate per cent of the principal paid in interest for one year.

Rate Per Cent. Rate in hundredths.

Ratio. The relative magnitude of two numbers, as expressed by the fraction which has the first number for the numerator and the second for the denominator.

Reciprocal of a Fraction. The fraction inverted.

Reduction. The process of changing the unit of a number without changing the value.

Remainder. The part of the dividend that is left in division.

Root. One of the equal factors of a number.

Security. Property used to guarantee the payment of any debt.

Short Division. The process of dividing in which the operations of multiplying and subtracting are performed mentally.

Similar Decimals. Decimals that have the same number of decimal places.

Similar Fractions. Fractions with the same denominator.

Simple Fraction. A fraction whose terms are both integers.

Specific Gravity. The ratio of the weight of a given substance to the weight of an equal volume of water.

Square. The result of taking a number twice as a factor.

Square Root. One of the two equal factors of a number.

Stock. Capital invested in business.

Subtraction. The process of taking one number from another, or of finding what number must be added to one of two numbers to make the other.

Subtrahend. The number taken away in subtraction.

Sum. The result obtained in addition.

Taxes. Money levied by a government for its support and for public purposes.

Terms of a Fraction. The numerator and denominator.

Terms of a Proportion. The extremes and means.

Unitary Analysis. An analysis that proceeds first to the unit and then to the required number of units.

Unit Fraction. A fraction whose numerator is 1.

Units. The standards by which we count or measure.

Volume. The number of cubic units a body contains.

INDEX

	PAGE		PAGE
Abstract number	1	Check on operations, 10, 17,	36, 158
Acceptance	315	Circle	117, 370, 483
Accounts	174, 426, 428	Circumference	117, 482
Accuracy	158	Cisterns	122, 387
Ad valorem duty	451	Collector	448
Addends	9	Commercial bank	307
Addition	9, 65, 145, 197, 401	discount	255
Agent	269	draft	315
Algebraic notation	249, 479	Commission	269
Aliquot parts	87, 408	Common denominator	156
Amount	277	fraction	191
Analysis	165	measure	182
Angle measure	140	multiple	185
Antecedent	327	Composite number	179
Arabic notation	2	Compound interest	319
Arc	140	interest table	320
Area	103	numbers	99
Assessor	447	Concrete number	1
Bank	307	Cone	388
discount	312	Consequent	327
note	311	Corporation	429
Base	236	Creditor	171
Bill	171	Cube	29, 112
Bins	115	Cubic measure	112, 499
Board measure	360	Customs	451
Bond	438	Cylinder	120, 384, 484
Brick work	123	Debtor	171
Broker	269, 431	Decimal fraction	57, 58
Business forms	423	Definitions	513
Cancellation	95, 163, 188, 222	Denominate number	99
Capacity	115, 500	number tables	505
of bins	115	Denominator	191
of cisterns	122, 387	Deposit slip	307
Capital	429	Difference	17
Carpeting	111	between dates	281, 297
Cash accounts	176	Digits	2
discount	255	Direct tax	447
Check, bank	309	Directly proportional	332

	PAGE		PAGE
Discount	255, 262, 312	Land measure	498, 508
series	262	Lateral area	120
Discounting bills	259	Lathing	363
notes	313	Least common multiple	185
Dividend	35, 430	Length	99, 496
Divisibility tests	180	Lever	335
Division	35, 76, 153, 209, 416	Life insurance	445
check for	36	Like numbers	9
exact	179	Liquid measure	131
principles of	51	List price	255
Divisor	35	Long division	42
exact	179	Longitude and time	487
Draft	315	Lowest terms	192
Drawing to scale	376		
Dry measure	129	Maker	282, 311
Duties	451	Means	330
		Merchants' rule	301
English money	135	Metric measures	495, 504
Equation	249	Minuend	17
Equivalent measures	134	Miscellaneous measures	142
Even number	180	Mixed number	194
Exact interest	300	Multiple	23, 185
Examination papers	510	Multiplicand	23
Extremes	330	Multiplication 23, 70, 151, 201, 408	
		table	24
Face	318, 443	Multiplier	23
Factors	23, 179		
Figures	2	Negotiable note	282
Fire insurance	443	paper	315
Flooring	362	Net price	255
Formula	484	Notation	2
Formulas in interest	287	Note	282, 311
Fraction	191	Numbers	1
French money	136	Numerals	2
		Numeration	2
German money	136	Numerator	191
Greatest common measure	182		
		Odd number	180
Hypotenuse	351	Orders of units	2
Improper fraction	191	Painting	109
Indirect tax	447, 451	Papering	358
Indorsement	282, 309, 311	Par value	430
Insurance	443	Parallelogram	106, 369, 480
Integers	2	Partial payments	301
Interest	277, 319	Payee	282, 315
tables	299, 320	Per cent	231
Inverse ratio	332	Per cents, important	234
Invoice	423	Percentage	231, 236

	PAGE		PAGE
Periods	3	Short division	36
Perpendicular	140	methods	401, 418
Place value	2	Similar decimals	60
Plastering	109	figures	334, 353
Policy	443	fractions	196
Power	29, 343	Six per cent method	293
Powers and roots	343	Solving problems	157
Premium	443	Specific duty	451
Prime factor	179	gravity	502
meridian	487	Sphere	391
number	179	Square	29, 103, 343
Principal	269, 277	measure	103, 498
Principles of division	51	on the hypotenuse	351
of fractions	218	root	343, 351
Prism	374, 379	Standard time	491
Proceeds	312	Statements	425
Product	23	Stock	429, 431
tables	44	certificate	429
Profit and loss	243	exchange	429
Promissory note	282	Subtraction 17, 68, 148, 199, 407	
Proper fraction	191	Subtrahend	17
Property tax	447	Sum	9
Proportion	330	Surveyors' measure	498, 508
Proportional, directly	332		
indirectly	332	Tariff	451
Pyramid	382	Tax rate	447
		table	449
Quotient	35	Taxation	447
		Taxes	447
Rapid calculation	401	Terms of a fraction	191
Rate	236, 437	of a proportion	330
Rate of interest	277	Time measure	137
Ratio and proportion	327	Trade discount	255
Reciprocal	212	Trapezoid	372, 481
Rectangle	105, 357	Triangle	106, 353, 369, 480
Rectangular solid	112		
Reduction of compound		Unit fraction	191
numbers	99	Unitary analysis	165
of decimals	60	United States money 7, 16,	
of fractions	60, 192	rule	27, 49, 63
of ratios	327	Units	1
Remainder	17, 35		
Right angle	140	Vocational problems	461
triangle	351	Volume 112, 379, 382, 384, 388, 393	
Roman notation	2, 8		
		Weight	125, 501
Savings bank	323	Whole numbers	2
Separatrix	3	Wood measure	143, 499
Shingling	364		

ANSWERS

Exercise 5. Page 10

1. 26.	12. 26.	23. 30.	34. 29.	45. 44.	56. 36.
2. 22.	13. 22.	24. 33.	35. 31.	46. 43.	57. 37.
3. 26.	14. 17.	25. 30.	36. 40.	47. 35.	58. 52.
4. 30.	15. 32.	26. 31.	37. 49.	48. 39.	59. 57.
5. 30.	16. 25.	27. 36.	38. 38.	49. 36.	60. 60.
6. 35.	17. 24.	28. 44.	39. 45.	50. 44.	61. 62.
7. 19.	18. 35.	29. 34.	40. 55.	51. 44.	62. 85.
8. 28.	19. 28.	30. 35.	41. 52.	52. 36.	63. 79.
9. 23.	20. 35.	31. 40.	42. 64.	53. 45.	64. 82.
10. 17.	21. 37.	32. 41.	43. 50.	54. 37.	65. 79.
11. 19.	22. 33.	33. 42.	44. 36.	55. 44.	

Exercise 6. Page 12

1. 99.	10. 193.	19. 358.	28. 500.	37. 238.	46. 410.
2. 99.	11. 283	20. 470.	29. 489.	38. 218.	47. 435.
3. 124.	12. \$130.	21. 438.	30. 544.	39. 268.	48. 512.
4. 111.	13. 147.	22. 394.	31. 390.	40. 253.	49. 609.
5. 228.	14. 159.	23. 265.	32. 400.	41. 359.	50. 726.
6. \$102.	15. 172.	24. 293.	33. 410.	42. 499.	51. 810.
7. 269.	16. 271.	25. 276.	34. 220.	43. 507.	52. 780.
8. 184.	17. 438.	26. 439.	35. 195.	44. 400.	53. 788.
9. 137.	18. 420.	27. 461.	36. 226.	45. 487.	

Exercise 7. Page 14

1. 1388.	9. 23,058.	17. 265,241.	25. 9,436,416.
2. 2210.	10. 16,886.	18. 249,853.	26. \$102,354,579.
3. 1652.	11. 99,936.	19. 92,097.	27. \$752,856,806.
4. 1662.	12. 116,404.	20. 134,487.	28. \$2,055,150,998.
5. 2210.	13. 156,827.	21. 22,027.	29. 76,303,387.
6. 20,000.	14. 142,877.	22. 1157.	
7. 6024.	15. 198,887.	23. \$34,593,348.	
8. 14,030.	16. 210,264.	24. 3,026,789.	

Exercise 8. Page 16

1. \$13.67.	6. \$21.10.	11. \$29.50.
2. \$6.46.	7. \$24.41.	12. \$40.39.
3. \$23.30.	8. \$11.49.	13. \$30.41.
4. \$14.	9. \$16.27.	14. \$40.04.
5. \$22.	10. \$91.	15. \$247.91.

Exercise 9. Page 18

1. 412.	14. \$5.50.	27. \$121.63.	40. 52,371.
2. 322.	15. \$6.51.	28. \$134.55.	41. 320.
3. 292.	16. \$13.55.	29. \$361.90.	42. 325.
4. 313.	17. \$21.10.	30. \$415.23.	43. \$345.
5. 330.	18. \$22.35.	31. 51,113.	44. 1411.
6. 206.	19. \$31.15.	32. 11,341.	45. 240.
7. 531.	20. \$62.36.	33. 34,225.	46. 1112.
8. 333.	21. \$11.54.	34. 13,463.	47. 115.
9. 111.	22. \$80.61.	35. 11,221.	48. 832 miles.
10. 246.	23. \$72.50.	36. 12,363.	49. 35,100 square miles.
11. \$6.23.	24. \$60.	37. 7505.	50. 106,110 square miles
12. \$3.21.	25. \$24.42.	38. 24,152.	51. 140 square miles.
13. \$8.23.	26. \$154.10.	39. 12,920.	52. 31,602,024 bushels.

Exercise 10. Page 21

1. 212.	17. 685.	33. 135.	49. 172.	65. 4087.
2. 309.	18. 436.	34. 259.	50. 161.	66. 6082.
3. 408.	19. 287.	35. 205.	51. 268.	67. 6185.
4. 538.	20. 377.	36. 404.	52. 213.	68. 2087.
5. 608.	21. 478.	37. 126.	53. 2662.	69. 1287.
6. 717.	22. 670.	38. 104.	54. 3209.	70. 4895.
7. 829.	23. 557.	39. 115.	55. 5108.	71. 1869.
8. 892.	24. 244.	40. 209.	56. 8267.	72. 14,859.
9. 292.	25. 310.	41. 208.	57. 2093.	73. 9339.
10. 183.	26. 401.	42. 89.	58. 4492.	74. 21,833.
11. 262.	27. 512.	43. 179.	59. 6172.	75. 13,389.
12. 342.	28. 300.	44. 287.	60. 4091.	76. 17,887.
13. 672.	29. 318.	45. 178.	61. 7093.	77. 16,789.
14. 782.	30. 117.	46. 189.	62. 2487.	78. 10,083.
15. 289.	31. 217.	47. 176.	63. 5078.	
16. 578.	32. 309.	48. 166.	64. 3057.	

Exercise 11. Page 22

1. 1943.	4. 28,571.	7. \$2413.75.	10. \$43.50.	13. \$165.
2. 14,529.	5. 19,148.	8. \$61.75.	11. \$555.61.	14. \$240.90.
3. 2768.	6. 24,110.	9. \$370.95.	12. \$782.50.	

Exercise 12. Page 26

1. 414.	14. 5691.	27. 54,464.	40. 89,991.	53. 433,662.
2. 1227.	15. 4896.	28. 38,763.	41. 82,406.	54. 444,003.
3. 1224.	16. 6570.	29. 5526.	42. 78,621.	55. 386,368.
4. 3520.	17. 1498.	30. 13,488.	43. 142,820.	56. 327,924.
5. 3048.	18. 2052.	31. 33,108.	44. 213,045.	57. 473,274.
6. 5614.	19. 6216.	32. 39,395.	45. 185,556.	58. 629,902.
7. 4840.	20. 4536.	33. 55,416.	46. 282,023.	59. 454,312.
8. 8181.	21. 12,174.	34. 62,601.	47. 563,544.	60. 899,991.
9. 874.	22. 18,222.	35. 75,584.	48. 725,625.	61. \$336.
10. 1578.	23. 32,384.	36. 66,303.	49. 154,468.	62. \$2400.
11. 3292.	24. 35,240.	37. 59,892.	50. 247,887.	63. 3240.
12. 3575.	25. 20,454.	38. 51,429.	51. 190,904.	64. 3934.
13. 2472.	26. 32,942.	39. 71,104.	52. 419,605.	

Exercise 13. Page 27

1. \$3.75.	21. \$3468.	41. \$26,901.	61. \$540,519.
2. \$8.10.	22. \$45.75.	42. \$46,736.	62. \$544,581.
3. \$10.60.	23. \$65.10.	43. \$364.50.	63. \$508,488.
4. \$13.	24. \$50.48.	44. \$813.60.	64. \$111.
5. \$13.15.	25. \$128.60.	45. \$501.	65. \$22.75.
6. \$31.55.	26. \$86.	46. \$1226.80.	66. \$13.50.
7. \$14.88.	27. \$115.55.	47. \$1607.55.	67. \$25.90.
8. \$22.32.	28. \$113.40.	48. \$1243.75.	68. \$31.50.
9. \$29.96.	29. \$225.	49. \$1973.40.	69. \$142.
10. \$40.25.	30. \$288.12.	50. \$3256.50.	70. \$171.
11. \$28.96.	31. \$264.25.	51. \$901.25.	71. \$6915.
12. \$51.28.	32. \$173.04.	52. \$2136.40.	72. \$71.
13. \$29.25.	33. \$662.08.	53. \$1694.	73. \$6.84.
14. \$65.52.	34. \$319.50.	54. \$3402.	74. \$77.46.
15. \$3164.	35. \$428.67.	55. \$3354.48.	75. \$2300.
16. \$1956.	36. \$12,852.	56. \$8889.12.	76. \$339.75.
17. \$1405.	37. \$15,710.	57. \$84,682.	77. \$112.50.
18. \$3432.	38. \$12,378.	58. \$64,263.	78. \$29.75.
19. \$1596.	39. \$4214.	59. \$105,604.	79. \$65.95.
20. \$7191.	40. \$30,380.	60. \$158,100.	

Exercise 14. Page 29

- | | | | |
|------------|---------------|---------------|-----------------|
| 1. 1770. | 6. 15,300. | 11. \$186.20. | 16. 320,000. |
| 2. 19,080. | 7. 32,100. | 12. \$345.60. | 17. 2,940,000. |
| 3. 14,280. | 8. \$30,100. | 13. \$546. | 18. 3,150,000. |
| 4. 23,500. | 9. \$284. | 14. \$504. | 19. \$3000. |
| 5. 55,800. | 10. \$439.20. | 15. 128,100. | 20. \$2400. |
| | | | 21. 89,100,000. |

Exercise 15. Page 30

- | | | | |
|--------------|----------------|------------------|-------------------|
| 1. 55,608. | 23. 410,720. | 45. 4,480,852. | 67. \$5,250,000. |
| 2. 151,032 | 24. 397,467. | 46. 2,811,468. | 68. \$1,920,000. |
| 3. 110,628. | 25. 627,480. | 47. 5,915,182. | 69. \$4,800,000. |
| 4. 192,696. | 26. 383,808. | 48. 7,411,656. | 70. \$13,654.75. |
| 5. 95,788. | 27. 639,878. | 49. 4,005,980. | 71. \$10,174.50. |
| 6. 195,642. | 28. 859,068. | 50. 7,728,993. | 72. \$13,639.92. |
| 7. 184,522. | 29. 989,901. | 51. 5,485,172. | 73. \$25,502.40. |
| 8. 436,408. | 30. 1,553,171. | 52. 3,403,944. | 74. \$32,181.32. |
| 9. 200,129. | 31. 1,515,192. | 53. 3,471,794. | 75. \$43,421.84. |
| 10. 273,208. | 32. 1,469,264. | 54. 6,195,441. | 76. \$13,048.82. |
| 11. 644,982. | 33. 1,389,789. | 55. 39,930,966. | 77. \$32,484.06. |
| 12. 433,519. | 34. 3,071,880. | 56. 23,887,808. | 78. \$52,706.70. |
| 13. 338,706. | 35. 4,015,920. | 57. 26,611,944. | 79. \$734,447.04. |
| 14. 287,616. | 36. 2,980,670. | 58. 50,024,883. | 80. \$350,513.52. |
| 15. 656,197. | 37. 3,376,360. | 59. \$22,605. | 81. \$337,085.60. |
| 16. 366,816. | 38. 2,398,484. | 60. \$32,900. | 82. \$411.25. |
| 17. 452,898. | 39. 3,044,672. | 61. \$30,660. | 83. \$1940.40. |
| 18. 720,802. | 40. 1,644,032. | 62. \$26,010. | 84. \$529.20. |
| 19. 268,128. | 41. 5,019,258. | 63. \$19,411. | 85. \$1482. |
| 20. 653,315. | 42. 1,542,940. | 64. \$41,088. | 86. \$2386.30. |
| 21. 508,776. | 43. 6,778,982. | 65. \$75,078. | |
| 22. 651,817. | 44. 3,517,701. | 66. \$3,360,000. | |

Exercise 16. Page 32

- | | | | |
|-------------------|--------------|---------------|--------------|
| 1. \$1050. | 6. \$23.80. | 11. \$3000. | 16. 2100. |
| 2. 42,240. | 7. \$195. | 12. 1960. | 17. 126,720. |
| 3. 88,000 pounds. | 8. \$1200. | 13. \$4875. | 18. 13,392. |
| 4. 264 feet. | 9. \$1020. | 14. \$13,875. | 19. 19,456. |
| 5. 22,500 feet. | 10. \$25.50. | 15. 15,504. | 20. 4900. |

21. 2448.	30. \$1575.	39. 5568 pounds.
22. \$4200.	31. \$4668.75.	40. 6,065,400.
23. 86,400.	32. \$5378.75.	41. 62.
24. 620,400 pounds.	33. \$2047.	42. \$1008.
25. 20,664 pounds.	34. \$257.50.	43. \$369.60.
26. 62,060 pounds.	35. \$1424.44.	44. \$31.36.
27. 8640 ounces.	36. 135,000 feet.	45. The horses ;. \$136.
28. 52,095.	37. 16,092 pounds.	46. 17,280,000 miles.
29. 28,224 pounds.	38. 16,524 pounds.	

Exercise 17. Page 37

1. 116.	12. 603.	23. 1140 $\frac{3}{8}$.	34. 5801.	45. 5298 $\frac{1}{8}$.
2. 115.	13. 672.	24. 897 $\frac{4}{9}$.	35. 5211.	46. 8467 $\frac{6}{9}$.
3. 104.	14. 439.	25. \$23.62.	36. 5720.	47. \$17.60 $\frac{1}{2}$.
4. 65.	15. 921.	26. \$11.05.	37. 8791.	48. \$24.14 $\frac{1}{5}$.
5. 123.	16. 891.	27. \$7.18.	38. 3557.	49. \$15.30 $\frac{3}{4}$.
6. 132.	17. 2777 $\frac{1}{2}$.	28. \$6.55.	39. 15,600 $\frac{1}{2}$.	50. \$14.47 $\frac{1}{5}$.
7. 74.	18. 2004 $\frac{2}{3}$.	29. \$3.55.	40. 11,359 $\frac{1}{3}$.	51. \$13.66 $\frac{5}{6}$.
8. 93.	19. 1053 $\frac{1}{4}$.	30. \$4.21.	41. 6557 $\frac{3}{4}$.	52. \$6.00 $\frac{3}{7}$.
9. 1621.	20. 578 $\frac{3}{5}$.	31. \$8.21.	42. 14,004 $\frac{3}{5}$.	53. \$8.12 $\frac{4}{8}$.
10. 1374.	21. 1045 $\frac{1}{6}$.	32. \$9.29.	43. 13,389 $\frac{1}{6}$.	54. \$10.40 $\frac{5}{9}$.
11. 1501.	22. 1050 $\frac{6}{7}$.	33. 20,251.	44. 4188 $\frac{5}{7}$.	

Exercise 18. Page 38

1. \$39.	4. 144.	7. 124.	10. 576.	13. \$1.35.
2. 27 feet.	5. 59.	8. 288.	11. 720.	14. \$2.48.
3. 52.	6. 44 feet.	9. \$136.	12. \$1.65.	

Exercise 19. Page 39

1. 34.	10. 8.	19. 3 $\frac{8}{90}$.	28. 52 $\frac{748}{8000}$.	37. 24.
2. 36.	11. 27 $\frac{3}{10}$.	20. 8 $\frac{57}{90}$.	29. 109 $\frac{107}{900}$.	38. 17.
3. 18.	12. 21 $\frac{3}{20}$.	21. 203.	30. 42.	39. 27.
4. 16.	13. 10 $\frac{25}{30}$.	22. 137.	31. 82.	40. 37.
5. 15.	14. 63 $\frac{5}{40}$.	23. 214.	32. 93.	41. 165.
6. 14.	15. 43 $\frac{6}{50}$.	24. 181.	33. 94 $\frac{7000}{60000}$.	42. 17.
7. 12.	16. 44 $\frac{7}{70}$.	25. 63 $\frac{1}{500}$.	34. 61 $\frac{2820}{70000}$.	43. 175 ; \$250.
8. 12.	17. 75 $\frac{2}{70}$.	26. 45 $\frac{403}{6000}$.	35. 92 $\frac{6623}{80000}$.	44. 17 ; \$1100.
9. 4.	18. 47 $\frac{1}{80}$.	27. 85 $\frac{21}{700}$.	36. 95 $\frac{2542}{90000}$.	45. 291 ; 1260.

Exercise 20. Page 41

1. 13 quarts. 3. 14. 5. 385 pounds. 7. 63 pounds.
 2. 15 quarts. 4. 65. 6. 8120 pounds. 8. 12 pounds; 3 pounds.
 9. Red, 14 pounds; crimson, 15 pounds. 10. 10.

Exercise 21. Page 43

- | | | | | | |
|--------|---------|----------|----------|----------|----------|
| 1. 32. | 10. 14. | 19. 17. | 28. 219. | 37. 803. | 46. 987. |
| 2. 41. | 11. 21. | 20. 32. | 29. 407. | 38. 809. | 47. 998. |
| 3. 62. | 12. 32. | 21. 22. | 30. 107. | 39. 456. | 48. 777. |
| 4. 88. | 13. 37. | 22. 33. | 31. 511. | 40. 862. | 49. 888. |
| 5. 78. | 14. 39. | 23. 37. | 32. 462. | 41. 877. | 50. 999. |
| 6. 87. | 15. 47. | 24. 111. | 33. 531. | 42. 888. | 51. 567. |
| 7. 67. | 16. 28. | 25. 123. | 34. 617. | 43. 880. | 52. 909. |
| 8. 83. | 17. 19. | 26. 112. | 35. 609. | 44. 990. | 53. 808. |
| 9. 12. | 18. 15. | 27. 101. | 36. 701. | 45. 999. | 54. 809. |

Exercise 22. Page 44

- | | | | | |
|---------|----------|-----------|----------------|-----------------|
| 1. 73. | 8. 77. | 15. 990. | 22. 352 miles. | 29. 696. |
| 2. 92. | 9. 129. | 16. 709. | 23. 720. | 30. 504. |
| 3. 105. | 10. 129. | 17. 809. | 24. 3075. | 31. 625. |
| 4. 63. | 11. 333. | 18. 707. | 25. 2009. | 32. 4025. |
| 5. 204. | 12. 426. | 19. 2111. | 26. 640. | 33. 329 miles. |
| 6. 65. | 13. 916. | 20. 4706. | 27. 225. | 34. 3168; 8800. |
| 7. 48. | 14. 862. | 21. 9009. | 28. 346. | |

Exercise 23. Page 47

- | | | | | | |
|--|--|---|---------------|-----------|-----------------------------------|
| 1. 84. | 6. 8760. | 11. 365. | 16. 207. | 21. 1332. | 26. 79. |
| 2. 82. | 7. 503. | 12. 467. | 17. 783. | 22. 1875. | 27. $111\frac{2}{3}\frac{5}{9}$. |
| 3. 76. | 8. 1060. | 13. 675. | 18. 858. | 23. 2397. | 28. $112\frac{1}{7}\frac{9}{6}$. |
| 4. 726. | 9. 626. | 14. 4268. | 19. 367. | 24. 2086. | 29. $372\frac{6}{23}$. |
| 5. 950. | 10. 1050. | 15. 468. | 20. 780. | 25. 98. | 30. $365\frac{1}{5}\frac{2}{6}$. |
| 31. $283\frac{1}{6}\frac{4}{4}$. | 40. $612\frac{6}{4}\frac{1}{1}\frac{2}{2}$. | 49. 98. | 58. 1350. | | |
| 32. $259\frac{3}{5}\frac{4}{7}$. | 41. $185\frac{2}{6}\frac{5}{5}\frac{3}{3}$. | 50. $564\frac{4}{2}\frac{8}{8}\frac{3}{3}\frac{6}{6}$. | 59. 6480. | | |
| 33. $416\frac{4}{4}\frac{6}{6}$. | 42. $617\frac{6}{4}\frac{2}{2}\frac{2}{2}$. | 51. $384\frac{4}{6}\frac{0}{2}\frac{5}{5}\frac{7}{7}$. | 60. \$25. | | |
| 34. $564\frac{1}{4}\frac{5}{7}$. | 43. $831\frac{1}{2}\frac{0}{3}\frac{0}{2}$. | 52. $873\frac{4}{5}\frac{5}{9}\frac{4}{4}\frac{7}{7}$. | 61. 5746. | | |
| 35. $467\frac{6}{5}\frac{9}{9}$. | 44. 36. | 53. \$144. | 62. \$4938. | | |
| 36. $682\frac{2}{6}\frac{0}{6}\frac{9}{9}$. | 45. 54. | 54. \$21. | 63. \$18,246. | | |
| 37. $617\frac{8}{8}\frac{2}{2}$. | 46. 37. | 55. \$41. | 64. 476. | | |
| 38. $436\frac{8}{2}\frac{3}{3}\frac{2}{2}$. | 47. 75. | 56. \$81. | 65. \$780. | | |
| 39. $890\frac{1}{1}\frac{0}{2}\frac{4}{4}$. | 48. 77. | 57. 1411. | 66. \$90. | | |

Exercise 24. Page 49

1. \$8.76.	11. \$481.20.	21. \$77.	31. \$2.07.
2. \$1.88.	12. \$681.39.	22. \$43.	32. \$3.10.
3. \$29.35.	13. \$502.75.	23. \$70.40.	33. \$3.15.
4. \$625.72.	14. \$491.76.	24. \$873.	34. \$2.84.
5. \$572.30.	15. \$123.13.	25. \$574.	35. \$2.67.
6. \$4213.01.	16. \$33.87.	26. \$390.	36. \$4.93.
7. \$682.02.	17. \$879.17.	27. \$1198.34.	37. \$2.75.
8. \$617.20.	18. \$932.60.	28. \$2.17.	38. \$3.20.
9. \$527.30.	19. \$49.50.	29. \$1.57.	39. \$3.10.
10. \$309.87.	20. \$79.	30. \$9.45.	40. \$10.08.
			41. \$2.50.

Exercise 25. Page 52

1. \$9.45.	8. \$0.21.	15. \$12.70.	22. 60 miles.
2. \$8.75; \$1.25.	9. 12; 36.	16. 8538 feet.	23. 348 miles.
3. \$2.10.	10. 31.	17. 36,605 miles.	24. 576 miles.
4. \$3.50.	11. 7425.	18. \$30.15.	25. 884; 272.
5. \$1.65.	12. \$71.55.	19. 33,645 feet.	26. \$0.93.
6. \$58.50; \$120.25.	13. \$168.75.	20. \$5.49.	27. 2 hours.
7. \$79.60.	14. \$201.60.	21. 475 tons.	

Exercise 29. Page 60

1. 0.20; 0.17.	9. 0.120; 0.103.	17. 0.001; 0.110.
2. 0.80; 0.08.	10. 0.720; 0.008.	18. 0.2070; 0.2376.
3. 0.700; 0.002.	11. 0.69; 0.07.	19. 0.412; 0.932.
4. 0.90; 0.13.	12. 0.3; 0.7.	20. 0.6810; 0.6881.
5. 0.600; 0.123.	13. 0.990; 0.999.	21. 0.9; 0.7.
6. 0.5; 0.7.	14. 0.75; 0.63.	22. 0.62; 0.57.
7. 0.40; 0.05.	15. 0.5; 0.4.	23. 0.090; 0.655.
8. 0.100; 0.357.	16. 0.850; 0.392.	24. 0.5840; 0.5834.

Exercise 30. Page 61

1. $\frac{1}{5}$.	10. $\frac{2}{5}$.	19. $1\frac{1}{5}$.	28. $1\frac{1}{3}$.	37. $187\frac{1}{2}$.	46. $2\frac{1}{4}$.
2. $\frac{2}{5}$.	11. $\frac{1}{5}$.	20. $1\frac{2}{5}$.	29. $2\frac{2}{5}$.	38. $3\frac{3}{8}$.	47. $3\frac{1}{5}$.
3. $\frac{3}{5}$.	12. $\frac{5}{8}$.	21. $1\frac{1}{8}$.	30. $2\frac{1}{6}$.	39. $3\frac{3}{8}$.	48. $5\frac{3}{4}$.
4. $\frac{4}{5}$.	13. $\frac{1}{4}$.	22. $1\frac{1}{40}$.	31. $2\frac{1}{4}$.	40. $3\frac{3}{8}$.	49. $2\frac{2}{5}$.
5. $\frac{1}{10}$.	14. $\frac{1}{2}$.	23. $1\frac{3}{8}$.	32. $2\frac{1}{2}$.	41. $\frac{1}{16}$.	50. $6\frac{4}{5}$.
6. $\frac{3}{20}$.	15. $\frac{3}{8}$.	24. $1\frac{1}{20}$.	33. $3\frac{3}{4}$.	42. $\frac{1}{16}$.	51. $4\frac{1}{20}$.
7. $\frac{1}{4}$.	16. $1\frac{4}{200}$.	25. $1\frac{5}{8}$.	34. $6\frac{1}{4}$.	43. $\frac{4}{16}$.	52. $5\frac{7}{20}$.
8. $\frac{1}{8}$.	17. $\frac{7}{8}$.	26. $1\frac{3}{4}$.	35. $1\frac{7}{8}$.	44. $\frac{8}{16}$.	
9. $\frac{1}{6}$.	18. $\frac{1}{3}$.	27. $1\frac{9}{50}$.	36. $18\frac{3}{4}$.	45. $1\frac{1}{2}$.	

Exercise 31. Page 62

1. 0.5.	7. 0.875.	13. 0.1875.	19. $0.333\frac{1}{3}$.	25. 1.36.
2. 0.25.	8. 0.2.	14. 0.3125.	20. $0.666\frac{2}{3}$.	26. $3.053\frac{1}{3}$.
3. 0.125.	9. 0.4.	15. 0.4375.	21. $0.166\frac{2}{3}$.	27. 0.1375.
4. 0.75.	10. 0.6.	16. 0.9375.	22. $0.833\frac{1}{3}$.	28. 6.2875.
5. 0.375.	11. 0.8.	17. 0.03125.	23. 0.12.	29. 1.38.
6. 0.625.	12. 0.0625.	18. 0.09375.	24. 0.28.	30. 6.125.

Exercise 32. Page 64

1. 700¢.	10. 965¢.	19. \$5.75.	28. \$96.54.
2. 2900¢.	11. 805¢.	20. \$6.06.	29. \$0.02.
3. 3500¢.	12. 2963¢.	21. \$7.32.	30. \$0.05.
4. 17,500¢.	13. 14,175¢.	22. \$5.22 $\frac{1}{2}$.	31. \$0.55.
5. 240¢.	14. 24,385¢.	23. \$6.33 $\frac{1}{3}$.	32. \$0.75.
6. 360¢.	15. 69,205¢.	24. \$1.66 $\frac{2}{3}$.	33. 27,630 m.
7. 970¢.	16. 90,009¢.	25. \$26.42.	34. \$5.24.
8. 1280¢.	17. \$3.	26. \$49.	35. \$276 ; 27,600¢.
9. 345¢.	18. \$4.50.	27. \$72.75.	

Exercise 33. Page 65

1. 59.5.	12. 40.	23. 34.86.	34. $368.33\frac{1}{3}$.
2. 80.73.	13. 40.	24. 30.	35. 365.08.
3. 85.46.	14. 40.08.	25. 35.45.	36. 206.
4. $30.82\frac{1}{4}$.	15. 35.55.	26. 20.606.	37. 201.
5. 28.12.	16. 51.166.	27. 2.041.	38. 88.1 miles.
6. 214.47.	17. 40.61.	28. 83.087.	39. 148.1 miles.
7. 271.28.	18. 60.1.	29. 334.87.	40. 236.2 miles.
8. 637.59.	19. 50.2.	30. 918.4546.	41. 498.64 miles.
9. $167.88\frac{3}{4}$.	20. 46.96.	31. 194.04.	42. 141.3 miles ; 84.6
10. $151.75\frac{1}{5}$.	21. 50.	32. 99.986.	miles ; 88.1 miles.
11. 40.	22. 59.	33. 52.9.	43. $207.57\frac{2}{3}$.
			44. Yes.

Exercise 34. Page 68

1. 22.79.	8. 44.62.	15. 24.757.	22. 2.7291.
2. 35.92.	9. 14.59.	16. 67.115.	23. 4.9727.
3. 53.06.	10. 34.05.	17. 11.933.	24. $4.86\frac{2}{3}$.
4. 58.69.	11. 18.67.	18. 39.2235.	25. $6.73\frac{1}{3}$.
5. 67.13.	12. 24.518.	19. 7.175.	26. $5.37\frac{1}{2}$.
6. 20.48.	13. 18.975.	20. 0.53074.	27. 162.75 acres.
7. 28.47.	14. 65.727.	21. 6.7302.	28. \$0.1335.

- | | | |
|------------------------------------|---------------------------------|------------------|
| 29. 318.912 $\frac{2}{3}$ barrels. | 32. 267.96 $\frac{2}{3}$ acres. | 35. 510.8 miles. |
| 30. 3.0997 grains. | 33. 9.16 feet. | 36. 67.5 miles. |
| 31. 46.25 feet. | 34. 11.1 miles. | 37. 177.8 miles. |

Exercise 35. Page 70

- | | | | | |
|-----------------------------|---------------------------------|-------------|-------------|------------------|
| 1. 13.4. | 7. 20. | 13. 31.425. | 19. 42.624. | 25. 60. |
| 2. 39.2. | 8. 60. | 14. 10. | 20. 42.924. | 26. 9.98. |
| 3. 6.3. | 9. 17. | 15. 10. | 21. 18.496. | 27. 3.125. |
| 4. 4.8. | 10. 25.5. | 16. 40. | 22. 0.3. | 28. 4. |
| 5. 16.25. | 11. 35.175. | 17. 33. | 23. 1.94. | 29. 210. |
| 6. 19.25. | 12. 56.072. | 18. 32.643. | 24. 0.222. | 30. 100.198. |
| 31. 1002.001. | 39. 111.11. | | | 47. \$131.3955. |
| 32. 127.75. | 40. 57.75 inches; 49 inches. | | | 48. 331.6 miles. |
| 33. 702.675. | 41. 24.5 inches; 21.245 inches. | | | 49. 158 miles. |
| 34. 49.9125. | 42. \$5; \$9. | | | 50. 27.6. |
| 35. 83.754. | 43. 472.44; 984.25; 1456.69. | | | 51. 468.8 feet. |
| 36. 130.725 $\frac{1}{3}$. | 44. 165.3; 279.908. | | | 52. 12.24 acres. |
| 37. 115.635. | 45. \$24.125. | | | 53. \$8171.625. |
| 38. 318.3231. | 46. 3.2515. | | | |

Exercise 36. Page 72

- | | | | |
|----------|--------------|----------------|---------------------|
| 1. 684. | 6. 2843.4. | 11. 50. | 16. 157,480. |
| 2. 1884. | 7. 253.6. | 12. 100. | 17. \$11,325. |
| 3. 4680. | 8. 3128.4. | 13. 10. | 18. 156,250 pounds. |
| 4. 496. | 9. 31,284. | 14. 5584.25. | 19. 70 inches. |
| 5. 4960. | 10. 312,840. | 15. 5,394,400. | 20. 125.664 feet. |

Exercise 37. Page 73

- | | | | |
|---------------------|------------------------|----------------|------------------|
| 1. 0.49. | 8. 18.24. | 15. 13.3774. | 22. 0.4. |
| 2. 0.81. | 9. 83.52. | 16. 42.9359. | 23. 2.4615. |
| 3. 1.36. | 10. 0.186. | 17. 67.47. | 24. 21.7761. |
| 4. 2.34. | 11. 0.576. | 18. 37.7798. | 25. 81.68325. |
| 5. 3.75. | 12. 0.264. | 19. 0.0889. | 26. 0.53801. |
| 6. 5.36. | 13. 0.774. | 20. 0.1. | 27. 0.007416. |
| 7. 9.43. | 14. 1.284. | 21. 0.1. | 28. 57.5 pounds. |
| 29. 15 pounds. | 32. 131.9472 inches. | 35. \$9.8904. | |
| 30. 173.75 pounds. | 33. 25,132.8 miles. | 36. \$12.0897. | |
| 31. 559.375 pounds. | 34. 2,722,196.4 miles. | 37. \$10.3385. | |

Exercise 38. Page 75

1. 1.46.	17. 5.0532.	33. 3.	49. 1124.12.	64. 154,000.
2. 5.	18. 8.2362.	34. 2841.3.	50. 5.9049.	65. 15.4.
3. 36.	19. 3366.	35. 28.413.	51. 10.3518.	66. 5.577153.
4. 23.94.	20. 3366.	36. 16.4836.	52. 553.311.	67. 12.561256.
5. 30.	21. 67.552.	37. 1648.36.	53. 16.7265.	68. 6.060505.
6. 105.	22. 67.552.	38. 13.104.	54. 10.4247.	69. 121.60608.
7. 148.	23. 6.7552.	39. 13.104.	55. 6.39.	70. 10.271611.
8. 220.	24. 37.2375.	40. 10.6436.	56. 0.639.	71. 49.972.
9. 356.25.	25. 1.1025.	41. 54.912.	57. 0.000035.	72. 17.2788.
10. 478.5.	26. 0.00665.	42. 11.718.	58. 0.000369.	73. 2.59809.
11. 6.02.	27. 26.162.	43. 0.7098.	59. 4.720472.	74. 13.4442.
12. 1100.	28. 0.63.	44. 0.9604.	60. 11.125497.	75. 35.552.
13. 3200.	29. 6.3.	45. 334.645.	61. 3201.64.	76. 8.888.
14. 14.625.	30. 63.	46. 3.66.	62. 32.0164.	77. 39.715.
15. 4.3967.	31. 300.	47. 1.9588.	63. 1,540,000.	78. 42.42.
16. 315.99.	32. 30.	48. 9.604.		

Exercise 39. Page 76

1. 2.26.	5. 0.045.	9. 5.873.	13. 0.07564.	17. 45.15 miles.
2. 0.62.	6. 0.2937.	10. 6.324.	14. 0.09852.	18. $27\frac{3}{9}$ miles.
3. 0.45.	7. 1.089.	11. 0.7059.	15. 0.00479.	19. 62.5.
4. 4.5.	8. 3.684.	12. 465.2.	16. \$0.43.	

Exercise 40. Page 78

1. 2.8.	13. 91.	25. 0.29.	37. 80,000.
2. 12.5.	14. 143.	26. 0.207.	38. 174.
3. 7.2.	15. 460.	27. 0.333.	39. 2200 bushels.
4. 7.3.	16. 520.	28. 0.66.	40. 32.
5. 7.8.	17. 0.17.	29. 0.44.	41. 164.
6. 12.1.	18. 0.24.	30. 0.29.	42. \$7.20.
7. 0.0422.	19. 0.03.	31. 20.	43. 720 +.
8. 0.241.	20. 280.	32. 0.03125.	44. 2.78.
9. 0.234.	21. 11.	33. 2.03.	45. 1.92.
10. 0.0017.	22. 1.85.	34. 146.61 feet.	46. 1.20 ounces.
11. 136.	23. 76.	35. 20.	47. 275.
12. 178.	24. 340.	36. 24,500.	

Exercise 41. Page 80

1. 2.5.	14. 1000.	27. 0.34.	40. 65.	53. 2.95.	66. 2.03.
2. 25.	15. 10,000.	28. 1.7.	41. 91.	54. 3.1.	67. 2.03.
3. 2.5.	16. 10,000.	29. 17.	42. 0.7.	55. 493.	68. 24,500.
4. 1.7.	17. 2090.	30. 17.	43. 21.	56. 493.	69. 800.
5. 0.17.	18. 20.9.	31. 8.2.	44. 1.2.	57. 0.493.	70. 174.
6. 17.	19. 2090.	32. 11.792.	45. 25.	58. 0.502.	71. 2205.
7. 1.7.	20. 50.	33. 9100.	46. 2.5.	59. 5.02.	72. 2.16.
8. 2.3.	21. 0.5.	34. 51.	47. 9.1.	60. 1.1.	73. 216.
9. 23.	22. 7.6.	35. 6.	48. 0.39.	61. 11.	74. 7.059.
10. 2.3.	23. 76.	36. 980.	49. 5.	62. 1.85.	75. 4652.
11. 2.1.	24. 0.76.	37. 78.	50. 182.	63. 1076.	76. 14.118.
12. 21.	25. 340.	38. 7.1.	51. 290.	64. 10.76.	77. 35.295.
13. 2.1.	26. 3.4.	39. 81.	52. 28.5.	65. 2000.	78. 111.

Exercise 42. Page 81

1. \$0.60.	13. \$728.	24. \$48.75.	36. 3.
2. \$0.70.	14. 0.26.	25. \$3.60.	37. 27.
3. \$14.	15. \$552; \$624;	26. \$3.60.	38. 51.
4. 10¢.	\$240; \$1416.	27. \$35.52.	39. 1.095.
5. 22¢.	16. \$528.	28. \$46.61.	40. 93.33½.
6. \$1.14.	17. \$83.09.	29. \$680; \$746.67.	41. 0.913.
7. 500; 450.	18. 586.4.	30. \$26; \$29.60.	42. 13.11.
8. 90¢.	19. \$15,000.	31. \$121.60.	43. 637.5625.
9. 0.23.	20. 4.43.	32. 1446.3 feet.	44. 72.45.
10. \$360.	21. 3.28.	33. 31.3 feet.	45. 0.05.
11. 0.25.	22. 360.9375.	34. 12.4 feet.	46. \$50.74.
12. 0.28.	23. \$45.	35. \$1497.	47. 212.25.

Exercise 44. Page 89

1. \$2.50.	10. \$37,610.	19. \$61,000.	28. \$10,605.
2. \$82,510.	11. \$43,077.50.	20. \$80,000.	29. 238,324.
3. \$62.50.	12. \$3600.	21. \$800,000.	30. \$1,053,250.
4. \$55.	13. \$16,000.	22. \$350,000.	31. \$10,560,000.
5. \$37.	14. \$8125.	23. 1200.	32. \$105,600.
6. \$287.50.	15. \$16,875.	24. 1600.	33. 80,000,000.
7. \$250.	16. \$637.50.	25. 4800.	34. \$30.
8. \$6375.	17. \$1881.25.	26. \$10,410.	35. \$2900.
9. \$3,211,200.	18. \$80,000.	27. 210,500.	36. \$2700.

- | | | | |
|--------------|---------------|--------------------|----------------|
| 37. \$62.50. | 40. \$12. | 43. \$900; \$1200. | 45. \$7687.50; |
| 38. \$310. | 41. \$1200. | 44. 300 pounds; | \$10,250. |
| 39. \$4.50. | 42. \$71,320. | 600 pounds. | |

Exercise 45. Page 90

- | | | | | |
|------------|------------|------------|--------------|---------------------------|
| 1. 513. | 6. \$324. | 11. \$244. | 16. 19.24. | 21. \$5400. |
| 2. 612. | 7. 5163. | 12. \$29. | 17. 366,000. | 22. \$11,000. |
| 3. \$1.11. | 8. \$276. | 13. 651. | 18. 0.02. | 23. \$72. |
| 4. \$2.40. | 9. \$460. | 14. 2244. | 19. \$3600. | 24. \$171 $\frac{7}{8}$. |
| 5. \$186. | 10. \$721. | 15. 192.4. | 20. \$4125. | 25. \$363. |
| | | | | 26. \$273. |

Exercise 46. Page 92

- | | | | | |
|--------------|--------------|-----------|---------------|---------------|
| 1. \$746.80. | 10. 502. | 19. 496. | 28. 14; 11. | 37. 1760. |
| 2. \$28.96. | 11. \$45.75. | 20. 640. | 29. 675. | 38. \$2250. |
| 3. \$4.96. | 12. \$98.25. | 21. 720. | 30. \$54. | 39. 2700. |
| 4. \$443.10. | 13. 228. | 22. 405. | 31. \$1375. | 40. 4000. |
| 5. \$624.47. | 14. 261. | 23. 495. | 32. \$1400. | 41. 63. |
| 6. 34,446.4. | 15. 144. | 24. 39. | 33. \$46,875. | 42. 87. |
| 7. 365. | 16. 102. | 25. 3450. | 34. \$742.50. | 43. \$800. |
| 8. 493. | 17. 128. | 26. 5030. | 35. \$108. | 44. 1448. |
| 9. 250.6. | 18. 92. | 27. 144. | 36. \$156.25. | 45. 1111. |
| | | | | 46. \$41,500. |

Exercise 47. Page 94

- | | | | | |
|------------|-------------|--------------|-------------|------------|
| 1. 122. | 15. \$43. | 29. \$248. | 43. 12.4. | 57. 4512. |
| 2. 1220. | 16. \$97. | 30. \$328. | 44. 1.24. | 58. 8.25. |
| 3. 12,200. | 17. 159. | 31. \$93. | 45. 180. | 59. 97.8. |
| 4. 1.22. | 18. 1590. | 32. \$29. | 46. 18. | 60. 870. |
| 5. 483. | 19. 15,900. | 33. 637. | 47. 1.8. | 61. 642. |
| 6. 4830. | 20. 159. | 34. 6.37. | 48. 0.02. | 62. 78.6. |
| 7. 48,300. | 21. 365. | 35. 6370. | 49. 1940. | 63. 63.6. |
| 8. 4.83. | 22. 3650. | 36. 63.7. | 50. 4208. | 64. 8320. |
| 9. 1827. | 23. 36.5. | 37. 637,000. | 51. 3443.2. | 65. 780. |
| 10. 178.7. | 24. 0.365. | 38. 364. | 52. 17.44. | 66. 90. |
| 11. 134. | 25. \$518. | 39. 3640. | 53. 211.2. | 67. 384. |
| 12. 1.34. | 26. \$46. | 40. 3640. | 54. 1.856. | 68. 0.384. |
| 13. 0.134. | 27. \$112. | 41. 147.6. | 55. 18.56. | 69. 888. |
| 14. \$176. | 28. \$99. | 42. 12.4. | 56. 262.4. | |

Exercise 48. Page 95

1. $\frac{21}{100}$.	4. 3.	7. 56.	10. 7.	13. $\frac{1}{10}$.	16. 1.
2. $1\frac{1}{3}$.	5. $3\frac{1}{2}$.	8. 1.	11. 5.	14. $\frac{3}{10}$.	17. 5.
3. $\frac{6}{35}$.	6. $3\frac{1}{3}$.	9. 10.	12. 9.	15. $\frac{1}{10}$.	

Exercise 49. Page 96

10. 19.809.	34. \$12,144.50.	57. 15.76204.	81. 0.0001052.
11. 1.3263.	35. 0.3.	58. 282.9.	82. 0.01624.
12. 15.151.	36. \$1829.23.	59. 53.9352.	83. 0.01031.
13. 1.0087.	37. \$100.	60. 53.63205.	84. \$4.06.
14. 0.159.	38. \$496.40.	61. 4392.63.	85. \$1.05.
15. 0.122.	39. 15.836 gallons.	62. 0348.861.	86. \$1.19.
16. 79.572.	40. 17.444 inches.	63. 1244.35.	87. \$7.11.
17. 27.006.	41. 42.432 feet.	64. 408.04.	88. \$6.85.
18. 57.529.	42. 3494.54 square feet;	65. 5431.69.	89. 74.
19. 30.09.	\$5241.81.	66. 227.151.	90. 7.198.
20. 10.06.	43. 132.795432 feet.	67. 830.025.	91. 196.54.
21. 0.35.	44. \$29.75.	68. 2546.2116.	92. 317.2.
22. \$1566.96.	45. 138.8 feet.	69. 1744.6.	93. 235.68.
23. \$3524.04.	46. \$1925.25.	70. 52.5525.	94. 11.853.
24. \$1114.72.	47. \$24.15.	71. 2782.35.	95. 1568.889.
25. \$2287.22.	48. \$9037.50.	72. 154.8504.	96. 7810.72.
26. \$341.71.	49. 9.296.	73. 3238.17.	97. 45.351.
27. \$775.14.	50. 70.992.	74. 225.024.	98. 270.565.
28. \$2447.13.	51. 4259.52.	75. 329.4308.	99. 11.326.
29. \$572.88.	52. 2424.558.	76. 453.22.	100. 6.213.
30. \$701.25.	53. 1563.25.	77. 14.63775.	101. 3.115.
31. \$397.38.	54. 2165.44.	78. 0.170625.	102. 66.104.
32. 1876.2345.	55. 5517.54.	79. 0.0345.	
33. \$136.69.	56. 32.7096.	80. 1.168.	

Exercise 50. Page 100

1. 655 in.	6. 1760 yd.;	10. 880 yd.	15. 2640 ft.
2. 873 in.	5280 ft.	11. 1600 rd.	16. 3168 ft.
3. 84 ft.; 1008 in.	7. 41 mi. 4224 ft.	12. 12,320 yd.	17. 3432 ft.
4. $16\frac{1}{2}$ ft.; 198 in.	8. 25 mi. 3960 ft.	13. 7200 rd.	18. 17.6 yd.
5. 143 yd.; 429 ft.	9. 126 mi. 3696 ft.	14. 17,160 ft.	19. 552.96 mi.

Exercise 51. Page 101

- | | | |
|----------------------------|------------------|-------------------|
| 1. 2 yd. 2 ft. | 11. 3 mi. | 21. 20 yd. |
| 2. 2 yd. 2 ft. 4 in. | 12. 4 mi. | 22. 21 yd. 2 ft. |
| 3. 3 yd. 7 in. | 13. 12 ft. 6 in. | 23. 166 yd. 2 ft. |
| 4. 1 yd. 2 ft. | 14. 13 ft. 9 in. | 24. 250 yd. |
| 5. 3 yd. | 15. 30 ft. | 25. 2 yd. |
| 6. 1 rd. 2 yd. 2 ft. 6 in. | 16. 33 ft. 4 in. | 26. 2 yd. 1 ft. |
| 7. 2 rd. | 17. 50 ft. | 27. 1 yd. 14 in. |
| 8. 2 rd. 4 yd. | 18. 60 ft. | 28. 2 yd. 8 in. |
| 9. 6 rd. 2 yd. | 19. 80 ft. | |
| 10. 2 mi. 60 rd. | 20. 81 ft. | |

Exercise 52. Page 102

- | | | | |
|-------------|--------------------------|---------------------------|--------------------------------|
| 1. 114 in. | 15. 101 ft. | 29. $3833\frac{1}{2}$ yd. | 43. 426 yd. 2 ft. |
| 2. 436 in. | 16. 124 ft. | 30. 5720 yd. | 44. 2550 yd. |
| 3. 513 in. | 17. 100 ft. | 31. 160 rd. | 45. 1760 yd. |
| 4. 347 in. | 18. 200 ft. | 32. 384 rd. | 46. 598 yd. 1 ft. |
| 5. 30 in. | 19. 200 ft. | 33. 1200 rd. | 47. 11 yd. 29 in. |
| 6. 66 in. | 20. 227 ft. | 34. 4 ft. | 48. 24 yd. 27 in. |
| 7. 44 in. | 21. $82\frac{1}{2}$ ft. | 35. 40 ft. | 49. 4 rd. |
| 8. 56 in. | 22. $848\frac{1}{2}$ ft. | 36. 30 ft. | 50. 20 rd. |
| 9. 110 in. | 23. 16,560 ft. | 37. 22 ft. 11 in. | 51. 6 rd. |
| 10. 159 in. | 24. 21,252 ft. | 38. 27 ft. 1 in. | 52. 4 rd. 9 ft. |
| 11. 238 in. | 25. 121 yd. | 39. 37 ft. 6 in. | 53. 1 rd. |
| 12. 318 in. | 26. 154 yd. | 40. 52 ft. 11 in. | 54. 63 rd. $10\frac{1}{2}$ ft. |
| 13. 14 ft. | 27. 5280 yd. | 41. 49 ft. 2 in. | |
| 14. 67 ft. | 28. 1815 yd. | 42. 104 ft. 2 in. | |

Exercise 53. Page 103

- | | | |
|--|-----------------|--------------------|
| 1. 1872 sq. in. | 6. 3751 sq. yd. | 11. 16,000 A. |
| 2. 4032 sq. in. | 7. 960 sq. rd. | 12. 96,000 A. |
| 3. 8640 sq. in. | 8. 2400 sq. rd. | 13. 10,836 sq. in. |
| 4. 387 sq. ft. | 9. 5600 sq. rd. | 14. 2650 sq. in. |
| 5. 558 sq. ft. | 10. 3200 A. | 15. 476 sq. ft. |
| 16. 1 sq. rd. 21 sq. yd. 8 sq. ft. 20 sq. in. | | |
| 17. 1 A. 16 sq. rd. | | |
| 18. 4 A. 102 sq. rd. 25 sq. yd. 4 sq. ft. 72 sq. in. | | |

Exercise 54. Page 104

- | | | | |
|------------------|---------------------------------|-------------------------------|----------------------------|
| 1. 172.8 sq. in. | 16. 382.5 sq. ft. | 31. 2420 sq. yd. | 46. 3 A. |
| 2. 244.8 sq. in. | 17. 3 sq. ft. | 32. 3630 sq. yd. | 47. 5 A. |
| 3. 252 sq. in. | 18. $1361\frac{1}{4}$ sq. ft. | 33. $1613\frac{1}{3}$ sq. yd. | 48. 7 A. |
| 4. 648 sq. in. | 19. 1769.625 sq. ft. | 34. 1 sq. rd. | 49. 1 A. |
| 5. 3240 sq. in. | 20. 1973.8125 sq. ft. | 35. 4 sq. rd. | 50. 5 A. |
| 6. 907.2 sq. in. | 21. $35,392\frac{1}{2}$ sq. ft. | 36. 20 sq. rd. | 51. $5\frac{1}{2}$ A. |
| 7. 507 sq. in. | 22. 1 sq. yd. | 37. 40 sq. rd. | 52. 4480 A. |
| 8. 368 sq. in. | 23. 5 sq. yd. | 38. 4 sq. rd. | 53. 1600 A. |
| 9. 648 sq. in. | 24. 12 sq. yd. | 39. 8 sq. rd. | 54. 2400 A. |
| 10. 2 sq. ft. | 25. 89 sq. yd. | 40. 20 sq. rd. | 55. 2 sq. mi. |
| 11. 5 sq. ft. | 26. 1830.125 sq. yd. | 41. 10 sq. rd. | 56. 3 sq. mi. |
| 12. 6 sq. ft. | 27. $3660\frac{1}{4}$ sq. yd. | 42. 1120 sq. rd. | 57. $\frac{1}{2}$ sq. mi. |
| 13. 7 sq. ft. | 28. $18,301\frac{1}{4}$ sq. yd. | 43. 240 sq. rd. | 58. $3\frac{1}{2}$ sq. mi. |
| 14. 531 sq. ft. | 29. $7320\frac{1}{2}$ sq. yd. | 44. 280 sq. rd. | 59. 7 sq. mi. |
| 15. 78.3 sq. ft. | 30. 4840 sq. yd. | 45. 260 sq. rd. | 60. 9 sq. mi. |

Exercise 55. Page 105

- | | | | |
|-------------------|----------------|-----------------------------|----------------------------------|
| 1. 5842 sq. ft. | 4. 220 sq. ft. | 7. $4\frac{2}{3}$ sq. ft. | 10. $29\frac{3}{4}\frac{7}{6}$. |
| 2. 5320 sq. in. | 5. 19 sq. ft. | 8. $101\frac{1}{2}$ sq. ft. | 11. $3334\frac{5}{8}$. |
| 3. 18,450 sq. yd. | 6. 430 sq. in. | 9. 512. | 12. 18 rd. |

Exercise 56. Page 106

- | | | | |
|-----------------------------|----------------------------|-----------|-----------|
| 1. 15,072 sq. ft. | 3. $92\frac{1}{2}$ A. | 5. 27 ft. | 7. 32 ft. |
| 2. $503\frac{3}{4}$ sq. ft. | 4. $91\frac{2}{3}$ sq. ft. | 6. 26 ft. | 8. 64 rd. |

Exercise 57. Page 107

- | | | |
|------------------------------|------------------------------|--|
| 1. 658 sq. ft. | 11. 8 ft. | 21. $154\frac{3}{8}$ sq. ft. |
| 2. $937\frac{1}{2}$ sq. ft. | 12. 20 ft. | 22. 52.25 sq. ft. |
| 3. 1443.75 sq. ft. | 13. 22 ft. | 23. 636 ; 2544. |
| 4. 2000 sq. rd. | 14. 54 in. | 24. 60 rd. |
| 5. $1093\frac{1}{2}$ sq. in. | 15. \$10,260.25. | 25. $127\frac{1}{2}$ sq. ft. |
| 6. $10\frac{1}{2}$ sq. ft. | 16. 15.57 sq. in. | 26. 41,925 sq. in. |
| 7. 1591 sq. in. | 17. $157\frac{1}{2}$ sq. ft. | 27. $6254\frac{3}{4}$ sq. ft. |
| 8. $19\frac{5}{8}$ sq. ft. | 18. $10\frac{1}{3}$ in. | 28. $18\frac{1}{16}$ sq. in. ; |
| 9. $47\frac{1}{16}$ sq. ft. | 19. $4\frac{1}{2}$ in. | $9\frac{1}{3}\frac{1}{2}$ sq. in. ; $18\frac{1}{16}$ sq. in. ; |
| 10. $16\frac{5}{8}$ sq. ft. | 20. $3\frac{1}{3}$ ft. | $18\frac{1}{16}$ sq. in. |

Exercise 58. Page 110

1. 28.	9. 109.	17. \$6.60.	25. \$0.34.	33. \$1.66.
2. 32.	10. 103.	18. \$6.16.	26. \$0.38.	34. \$2.34.
3. 22.	11. 98.	19. \$6.16.	27. \$0.46.	35. \$3.12.
4. 25.	12. 103.	20. \$5.06.	28. \$0.42.	36. \$4.44.
5. 30.	13. \$5.94.	21. \$5.50.	29. \$0.68.	37. \$37.67.
6. 42.	14. \$7.26.	22. \$5.72.	30. \$0.80.	38. \$24.30.
7. 56.	15. \$7.04.	23. \$10.78.	31. \$0.76.	39. \$26.50.
8. 89.	16. \$8.80.	24. \$13.42.	32. \$1.16.	

Exercise 59. Page 111

1. 56.	3. $58\frac{2}{3}$.	5. 34.	7. $66\frac{2}{3}$.	9. $106\frac{2}{3}$.	11. 8; 24 in.
2. $46\frac{2}{3}$.	4. 30.	6. $44\frac{1}{3}$.	8. $69\frac{1}{3}$.	10. \$34.42.	12. \$26.63.

Exercise 60. Page 113

1. 3456 cu. in.	17. 1 cu. ft. 272 cu. in.	32. $46\frac{1}{4}$ cu. ft.
2. 43,200 cu. in.	18. 2 cu. ft. 804 cu. in.	33. $183\frac{1}{2}\frac{1}{4}$ cu. ft.
3. 129,600 cu. in.	19. 43,265 cu. in.	34. 1292.
4. 216,000 cu. in.	20. $415\frac{1}{2}$ cu. ft.	35. \$83.20.
5. 57,600 cu. in.	21. 10 cd. 10 cu. ft. ;	36. \$68.95.
6. 115,200 cu. in.	47 cu. yd. 21 cu. ft.	37. 1830.
7. 675 cu. ft.	22. 10,336 cu. in.	38. 4096 cu. in.
8. 972 cu. ft.	23. 9504 cu. in.	39. 7840.
9. 3375 cu. ft.	24. 8160 cu. in.	40. 262 sq. in.
10. 3200 cu. ft.	25. 9367 cu. in.	41. $36\frac{3}{4}$ sq. ft.
11. 16,000 cu. ft.	26. 5220 cu. ft.	42. $218\frac{2}{7}\frac{9}{2}$.
12. 64,000 cu. ft.	27. 1143 cu. ft.	43. $62\frac{1}{2}$ lb.
13. 2 cu. ft.	28. 1302 cu. ft.	44. 2025 lb.
14. 5 cu. ft.	29. $1462\frac{1}{2}$ cu. ft.	45. 6 ft.
15. 91 cu. yd.	30. $2016\frac{2}{3}$ cu. ft.	46. 12 ft.
16. 181 cu. yd. 13 cu. ft.	31. 35 cu. ft.	47. 8 sq. in.

Exercise 61. Page 115

1. 288.	7. 240.	13. 1875.	19. 20,000.
2. $403\frac{1}{5}$.	8. 527.	14. $2031\frac{1}{4}$.	20. 27,900.
3. 540.	9. $681\frac{2}{4}\frac{9}{10}$.	15. 2325.	21. 35,000.
4. 768.	10. 750.	16. 4000.	22. 46,875.
5. 1152.	11. $1062\frac{1}{2}$.	17. 7000.	23. 83,700.
6. $1209\frac{3}{5}$.	12. $1218\frac{3}{4}$.	18. 9375.	24. 74,000.

- | | | | |
|----------------------------|------------------------|--------------------------|--------------------------|
| 25. 41,218 $\frac{3}{4}$. | 31. 8 $\frac{1}{7}$. | 37. 6 $\frac{1}{8}$. | 43. 156 $\frac{4}{7}$. |
| 26. \$273.60. | 32. 10. | 38. 10 $\frac{1}{2}$. | 44. 5555 $\frac{5}{9}$. |
| 27. \$4896. | 33. 10 $\frac{4}{5}$. | 39. 6 $\frac{3}{4}$. | 45. \$3280.50. |
| 28. 12. | 34. 15. | 40. 71 $\frac{3}{7}$ lb. | 46. 59.84. |
| 29. 24. | 35. 13 $\frac{5}{7}$. | 41. 59 $\frac{1}{2}$ lb. | |
| 30. 13. | 36. 11 $\frac{2}{3}$. | 42. 1666 $\frac{2}{3}$. | |

Exercise 62. Page 117

- | | | | |
|---------------|------------------|------------------|------------------|
| 1. 330 ft. | 10. 484 in. | 19. 100.5312 yd. | 28. 30.15936 ft. |
| 2. 59.4 ft. | 11. 616 in. | 20. 14.76552 yd. | 29. 11.93808 yd. |
| 3. 81.4 ft. | 12. 1320 in. | 21. 1.947792 ft. | 30. 16.96464 yd. |
| 4. 9.24 ft. | 13. 387.2 in. | 22. 2.3562 ft. | 31. 3.015936 ft. |
| 5. 1.144 in. | 14. 431.2 in. | 23. 7.0686 ft. | 32. 3.895584 ft. |
| 6. 13,420 in. | 15. 536.8 ft. | 24. 3.989832 in. | 33. 20 ft. |
| 7. 15,620 in. | 16. 55.44 ft. | 25. 169.6464 in. | 34. 110 in. |
| 8. 160.6 in. | 17. 84.8232 ft. | 26. 238.7616 in. | 35. 160 in. |
| 9. 396 in. | 18. 135.0888 ft. | 27. 383.2752 in. | |

Exercise 63. Page 118

- | | | |
|---------------------|----------------------------------|--------------------------------|
| 1. 3.1416 sq. in. | 14. 2827.44 sq. in. | 27. 19.635 sq. in. |
| 2. 28.2744 sq. in. | 15. 3.1416 sq. in. | 28. 33.18315 sq. in. |
| 3. 19.635 sq. in. | 16. 50.2656 sq. in. | 29. 16.619064 sq. in. |
| 4. 50.2656 sq. in. | 17. 28.2744 sq. in. | 30. 12.5664 in. |
| 5. 201.0624 sq. in. | 18. 78.54 sq. in. | 31. 7.0686 sq. in. |
| 6. 132.7326 sq. in. | 19. 314.16 sq. in. | 32. 10.9956 in. |
| 7. 78.54 sq. in. | 20. 1256.64 sq. in. | 33. 47.124 in. |
| 8. 63.6174 sq. in. | 21. 0.7854 sq. in. | 34. 10 in. |
| 9. 50.2656 sq. in. | 22. 0.3490 $\frac{2}{3}$ sq. in. | 35. 31.416 ft. |
| 10. 38.4846 sq. in. | 23. 0.031416 sq. in. | 36. 21.46 sq. ft. |
| 11. 95.0334 sq. in. | 24. 0.19635 sq. in. | 37. 220 ft.; 38,013.36 sq. ft. |
| 12. 314.16 sq. in. | 25. 1.76715 sq. in. | 38. 8.2957875 sq. in. |
| 13. 1256.64 sq. in. | 26. 1.3962 $\frac{2}{3}$ sq. in. | 39. 11.0446875 sq. in. |

Exercise 64. Page 121

- | | | |
|------------------------------|---------------------------|-----------------------|
| 1. 351 cu. in. | 7. 2 $\frac{3}{8}$ ft. | 13. 1357.1712 cu. in. |
| 2. 612 cu. in. | 8. 6 ft. 3 in. | 14. 2827.44 cu. in. |
| 3. 4750 cu. in. | 9. 18 sq. ft. | 15. 6433.9968 cu. in. |
| 4. 365 $\frac{1}{2}$ cu. in. | 10. 19 sq. ft. | 16. 9160.9056 cu. in. |
| 5. 405 cu. in. | 11. 13 sq. ft. | 17. 376.992 cu. in. |
| 6. 35 cu. ft. 1320 cu. in. | 12. 15 sq. ft. 36 sq. in. | 18. 10.472 cu. ft. |

Exercise 65. Page 122

- | | | |
|--------------------------|--------------------------------------|---|
| 1. 60. | 8. 400. | 15. 637,215 $\frac{1}{2}$ $\frac{5}{8}$. |
| 2. 75. | 9. 265 $\frac{8}{3}$ $\frac{5}{6}$. | 16. 424 $\frac{2}{7}$. |
| 3. 50 $\frac{5}{8}$. | 10. 2121 $\frac{3}{7}$. | 17. 318 $\frac{3}{4}$. |
| 4. 101 $\frac{1}{4}$. | 11. 15,910 $\frac{5}{7}$. | 18. 754 $\frac{2}{7}$. |
| 5. 7031 $\frac{1}{4}$. | 12. 21,120. | 19. 433 $\frac{1}{8}$. |
| 6. 9445 $\frac{5}{16}$. | 13. 288,750. | 20. 435 $\frac{1}{2}$ $\frac{5}{8}$. |
| 7. 282 $\frac{6}{7}$. | 14. 776,242 $\frac{1}{2}$. | 21. 725 $\frac{2}{2}$ $\frac{5}{8}$. |

Exercise 66. Page 123

- | | | | |
|------------------------|-----------------------|-----------------------|-----------------------|
| 1. 248. | 5. 1066. | 9. 18 $\frac{1}{3}$. | 13. 1152; 46.08; |
| 2. 585. | 6. 1248. | 10. 50. | \$414.72. |
| 3. 408 $\frac{1}{3}$. | 7. 18 $\frac{1}{3}$. | 11. \$105.60. | 14. 61,424; \$509.82. |
| 4. 900. | 8. 26 $\frac{2}{3}$. | 12. 36,740; \$303.11. | 15. 210,364. |

Exercise 67. Page 125

- | | | |
|--------------------------------------|--------------------------------------|---|
| 1. 9 $\frac{1}{2}$ T. | 19. $\frac{1}{4}$; $\frac{1}{7}$. | 39. 437 oz. |
| 2. 510,000 lb. | 20. $\frac{1}{5}$; $\frac{7}{15}$. | 40. 54,000 lb. |
| 3. 7000 lb. | 21. \$362.50. | 41. 5400 lb. |
| 4. 12,000 lb. | 22. \$984.38. | 42. 9 lb. |
| 5. 45,000 lb. | 23. \$770.25. | 43. 6700 lb. |
| 6. 40,000 lb. | 24. \$14.70. | 44. 98 $\frac{3}{4}$ lb. |
| 7. 100,500 lb. | 25. 375 lb.; | 45. 2023 $\frac{7}{8}$ lb. |
| 8. 4800 lb.; 13,200 lb. | 4375 lb. | 46. 31.25 lb.; 6.25 lb.; |
| 9. 4800 lb.; 1200 lb.; | 26. 1 $\frac{3}{4}$ T. | 50 lb. |
| 120,000 lb. | 27. 0.925. | 47. $\frac{1}{5}$; $\frac{2}{5}$; $\frac{4}{5}$; $\frac{1}{2}$. |
| 10. 5600 lb.; 1400 lb.; | 28. 256 oz. | 48. 625 lb.; 1875 lb. |
| 140,000 lb. | 29. 40 oz. | 49. 31 $\frac{1}{4}$ lb.; 15 $\frac{5}{8}$ lb.; |
| 11. 800 lb.; 8000 lb.; | 30. 32,000 lb. | 7 $\frac{3}{8}$ lb. |
| 80,000 lb. | 31. 57,000 lb. | 50. 125 lb. |
| 12. 30; 16. | 32. 11 lb. | 51. 15 lb. |
| 13. 33 $\frac{1}{3}$. | 33. 12 lb. 8 oz. | 52. 1206 $\frac{1}{4}$ lb. |
| 14. 62 $\frac{1}{2}$. | 34. 1 T. | 53. \$5250. |
| 15. $\frac{1}{5}$; $\frac{1}{15}$. | 35. 5 T. | 54. 70 lb. |
| 16. $\frac{1}{2}$; $\frac{1}{3}$. | 36. 476 oz. | 55. 15. |
| 17. $\frac{1}{6}$; $\frac{1}{8}$. | 37. 906 oz. | 56. 4975 lb. |
| 18. $\frac{1}{2}$; $\frac{1}{3}$. | 38. 867 oz. | 57. 768 $\frac{9}{32}$. |

Exercise 68. Page 129

- | | | | | |
|-----------------------|--------------|-----------------------|--------------|-----------------------|
| 1. 74 pt. | 4. 512 pt. | 7. 175 pt. | 10. 472 qt. | 13. 188 pk. |
| 2. 98 pt. | 5. 33 pt. | 8. 497 pt. | 11. 538 qt. | 14. 226 pk. |
| 3. 198 pt. | 6. 75 pt. | 9. 384 qt. | 12. 2150 qt. | 15. 371 pk. |
| 16. 1709 pk. | | 21. 1 bu. 3 pk. 4 qt. | | 26. 2 bu. 1 pk. 6 qt. |
| 17. 1 bu. 4 qt. | | 22. 3 bu. 2 qt. | | 27. 4 bu. 3 pk. 4 qt. |
| 18. 1 bu. 6 qt. | | 23. 24 bu. | | 28. 39 bu. |
| 19. 1 bu. 2 pk. 1 qt. | | 24. 22 bu. 1 pk. | | 29. 144 pt. |
| 20. 2 bu. | | 25. 18 bu. 3 pk. | | 30. 171 pt. |
| 31. 137 pt. | 40. 144 pt. | 49. 288 pk. | | 58. 61 bu. |
| 32. 784 pt. | 41. 304 qt. | 50. 358 pk. | | 59. 31 bu. |
| 33. 856 pt. | 42. 388 qt. | 51. 18 pk. | | 60. 99 bu. |
| 34. 1212 pt. | 43. 478 qt. | 52. 36 pk. | | 61. 300. |
| 35. 128 pt. | 44. 625 qt. | 53. 84 bu. | | 62. 720. |
| 36. 2240 pt. | 45. 2400 qt. | 54. 129 bu. | | 63. 43,200 lb. |
| 37. 1824 pt. | 46. 2800 qt. | 55. 193 bu. | | 64. 5. |
| 38. 38 pt. | 47. 88 qt. | 56. 239 bu. | | 65. 22,400 lb. |
| 39. 94 pt. | 48. 296 qt. | 57. 11 bu. | | |

Exercise 69. Page 131

- | | | | | |
|--|------------------------------|--|------------|------------------------------|
| 1. 108 gi. | 3. 138 pt. | 5. 356 qt. | 7. 257 qt. | 9. 3 gal. |
| 2. 184 gi. | 4. 174 pt. | 6. 272 qt. | 8. 125 pt. | 10. 12 gal. |
| 11. 1 gal. 3 gi. | | 13. 75 gal. | | 15. 23 gal. 1 pt. |
| 12. 10 gal. 2 qt. 1 pt. | | 14. 59 gal. 1 qt. 1 pt. | | 16. 9 gal. 1 qt. 1 pt. 2 gi. |
| 17. 99 pt. | 28. 1080 sq. in. | 39. 6 T. 500 lb. | | 50. 131,600 lb. |
| 18. 223 pt. | 29. 192,000 lb. | 40. 37 T. 1550 lb. | | 51. 7200 lb. |
| 19. 253 pt. | 30. $196\frac{2}{3}$ sq. yd. | 41. 12 oz. | | 52. $6666\frac{2}{3}$ lb. |
| 20. 636 pt. | 31. 97,500 lb. | 42. $3\frac{1}{2}$ oz. | | 53. 27 lb. |
| 21. 99 qt. | 32. 400 sq. rd. | 43. 1158 oz. | | 54. 302 lb. |
| 22. 149 qt. | 33. 33 lb. | 44. $466\frac{1}{2}$ oz. | | 55. 4800 lb. |
| 23. 3707 qt. | 34. 39 lb. | 45. 1550 oz. | | 56. 10,650 lb. |
| 24. 291 qt. | 35. 23 lb. 7 oz. | 46. 476 oz. | | 57. 59,360. |
| 25. 1024 oz. | 36. 28 lb. 2 oz. | 47. 35,200 oz. | | 58. 175,000. |
| 26. 9504 cu. in. | 37. 1 T. 187 lb. 8 oz. | 48. 157 oz. | | 59. 1032. |
| 27. 156 oz. | 38. 1 T. 11 lb. | 49. 85,500 lb. | | 60. 625,000 lb. |
| 61. One of 50 lb., and one of 25 lb. | | 64. 27.9 lb.; 15.45 lb.; 21.675 lb. | | |
| 62. $207\frac{1}{2}$ lb.; 398.4 lb. | | 65. 1037.5 lb.; 2946.5 lb.; 380.14 lb.; 12,450 lb. | | |
| 63. 4.15 lb.; 1.0375 lb.; 6.225 lb.; 20.75 lb. | | 66. 261.45 lb. | | |

- | | |
|---|---|
| 67. 8.549 lb.; 2.13725 lb.;
1.068625 lb. | 72. 192 cu. ft.; 331,776 cu. in.;
1436.26 gal. |
| 68. 6.557 lb.; 206.5455 lb. | 73. 132 cu. ft.; 228,096 cu. in.;
987.428 gal. |
| 69. 5.81 lb.; 183.015 lb. | 74. 864. |
| 70. 559.375 lb.; 74.285 lb. | 75. 10.292 lb. |
| 71. 8.217 lb.; 258.8355 lb. | |

Exercise 70. Page 134

9. 4; 8. 10. 3; 4; 6; 5. 11. 12; 24; 20; 28. 12. 8; 48; 16; 32; 2.

Exercise 71. Page 135

- | | | | | |
|----------|--------------------------|-----------|---------------|----------------|
| 1. 240d. | 8. 833d. | 15. 139s. | 22. £1. | 29. \$365.25. |
| 2. 480d. | 9. 1108 $\frac{1}{2}$ d. | 16. 4s. | 23. £2. | 30. \$2313.25. |
| 3. 60d. | 10. 40s. | 17. 8s. | 24. £5. | 31. £5. |
| 4. 264d. | 11. 100s. | 18. 20s. | 25. £35. | 32. £7. |
| 5. 348d. | 12. 140s. | 19. £3. | 26. £3. | 33. £110. |
| 6. 672d. | 13. 30s. | 20. £7. | 27. £75. | |
| 7. 270d. | 14. 75s. | 21. £24. | 28. \$233.76. | |

Exercise 72. Page 136

- | | | | |
|--------------|-------------|---------------------|---------------|
| 1. 8.50 fr. | 5. 4 M. | 9. $\frac{3}{4}$. | 13. 0.952. |
| 2. 3.75 fr. | 6. 1.75 M. | 10. $\frac{3}{4}$. | 14. 0.952. |
| 3. 4.80 fr. | 7. 16 M. | 11. 0.772. | 15. 18.85 fr. |
| 4. 12.50 fr. | 8. 92.75 M. | 12. 0.965. | 16. 1.53 M. |

Exercise 73. Page 138

- | | | | |
|--------------------------------|-----------------------|----------------------------|---------------------------------|
| 1. 21,600 sec. | 10. 96 mo. | 19. 0.25 of a | 26. Saturday. |
| 2. 1080 min. | 11. 192 mo. | century. | 27. 55,937 sec. |
| 3. 2160 min. | 12. 384 mo. | 20. 52 wk. 1 da. | 28. 3329 min. |
| 4. 384 hr. | 13. 6 min. | 21. 92. | 29. 87,963 min. |
| 5. 864 hr. | 14. 12 hr. | 22. 153. | 30. 29; 696. |
| 6. 1164 hr. | 15. 3 wk. | 23. 18,262 $\frac{1}{2}$. | 31. 4 wk. 1 da. 12 hr. |
| 7. 364 da. | 16. 1 wk. | 24. 500. | 44 min. 3 sec. |
| 8. 182 da. | 17. 11 yr. | 25. 1928; 1940; 1960; | 32. \$71.50. |
| 9. 325 da. | 18. $\frac{1}{2}$ yr. | 1972; 2000. | 33. 30. |
| 34. 1 hr. 5 $\frac{1}{4}$ min. | 38. $\frac{5}{11}$. | | 42. 1 hr. 26 $\frac{1}{4}$ min. |
| 35. 1 hr. 21 min. | 39. 1.55 P.M. | | 43. 1.50 P.M. |
| 36. 2 hr. 55 min. | 40. 4.30 P.M. | | 44. 100; 135; 210. |
| 37. 13 hr. | 41. 5.30 P.M. | | |

Exercise 74. Page 141

- | | | | |
|--------------|-----------------------|----------------------|------------------------|
| 1. 1020". | 13. 324,000". | 25. 549'. | 37. 14°. |
| 2. 1920". | 14. 162,000". | 26. 610'. | 38. 30°. |
| 3. 2880". | 15. 109,830". | 27. 2130'. | 39. 60°. |
| 4. 3600". | 16. 8'. | 28. 1676'. | 40. 180°; 90°. |
| 5. 10,800". | 17. $9\frac{1}{3}$ '. | 29. 2930'. | 41. 21,600'. |
| 6. 97,200". | 18. 7'. | 30. 3560'. | 42. 5400'; |
| 7. 11,100". | 19. 15'. | 31. 1° 20'. | 324,000". |
| 8. 14,880". | 20. 6'. | 32. 1°. | 43. $11\frac{1}{4}$ °. |
| 9. 29,108". | 21. 4'. | 33. $\frac{1}{2}$ °. | 44. 45°; 6°. |
| 10. 32,762". | 22. 1620'. | 34. 2° 40'. | 45. 1410'. |
| 11. 54,630". | 23. 2100'. | 35. 45°. | 46. 43°. |
| 12. 66,035". | 24. 2700'. | 36. 90°. | |

Exercise 75. Page 142

- | | | | |
|------------------------|----------------|-----------------|---------------------|
| 1. 576. | 4. \$63. | 7. \$2.10; 336. | 11. 1680. |
| 2. 216; 1296;
5040. | 5. 80; 60; 90. | 8. \$2.40. | 12. \$1.19; \$0.81; |
| | 6. 48; 840; | 9. \$27. | \$1.50. |
| 3. 6; 32; 69. | 1800. | 10. 60¢. | 13. 32¢. |

Exercise 76. Page 143

- | | | | | |
|--------------------------------|--------------------------------|-----------------------|----------------------------------|----------------------|
| 1. $1\frac{1}{4}$. | 6. $3\frac{3}{4}$. | 11. \$37.69. | 16. 24 ft. | 21. \$42.19. |
| 2. $2\frac{1}{4}$. | 7. 3. | 12. $17\frac{3}{4}$. | 17. 4 ft. | 22. 65.63. |
| 3. $2\frac{1}{3}\frac{1}{2}$. | 8. $4\frac{1}{3}\frac{5}{2}$. | 13. \$12.03. | 18. 480. | 23. $5\frac{1}{4}$. |
| 4. $2\frac{1}{2}$. | 9. $5\frac{2}{3}$. | 14. \$25.31. | 19. $12\frac{3}{8}$. | 24. $7\frac{1}{8}$. |
| 5. $2\frac{1}{4}$. | 10. \$18.75. | 15. \$4. | 20. $16\frac{1}{3}\frac{3}{2}$. | 25. \$16.41. |

Exercise 77. Page 145

- | | | |
|------------------|-------------------|-----------------------------|
| 1. 2 yd. | 11. 80 mi. 10 rd. | 21. 9 wk. 1 da. |
| 2. 37 ft. 10 in. | 12. 83 yd. | 22. 78 bu. 3 pk. |
| 3. 51 lb. 12 oz. | 13. 71 ft. 1 in. | 23. 94 bu. 1 pk. |
| 4. 35 rd. 15 ft. | 14. 51 lb. 2 oz. | 24. 24 ft. 3 in. |
| 5. 36 rd. | 15. 58 lb. | 25. 250 sq. ft. 71 sq. in. |
| 6. 143 lb. 8 oz. | 16. 71 rd. | 26. 258 cu. ft. 626 cu. in. |
| 7. 4 lb. 7 oz. | 17. 151 mi. 5 rd. | 27. 1431 lb. 11 oz. |
| 8. 50 lb. | 18. 51 yd. 2 in. | 28. 100 ft. 5 in. |
| 9. 100 rd. | 19. 64 yd. 18 in. | 29. 211 sq. ft. 65 sq. in. |
| 10. 50 mi. | 20. 24 da. 12 hr. | 30. 251 cu. ft. 920 cu. in. |

- | | | |
|---------------------|-------------------------|------------------------------|
| 31. 1857 lb. 11 oz. | 37. 237 bu. 3 pk. | 43. 104 hr. 12 min. |
| 32. 139 gal. 3 qt. | 38. 284 ft. 6 in. | 57 sec. |
| 33. 241 gal. 5 pt. | 39. $25^{\circ} 16''$. | 44. $130^{\circ} 58' 47''$. |
| 34. 139 ft. | 40. 166 qt. | 45. 10 A. |
| 35. 21 hr. 38 sec. | 41. 94 pk. 1 qt. | 46. 19 sq. rd. |
| 36. 209 gal. 3 qt. | 42. 86 wk. 2 da. | 47. 18 cu. yd. 1 cu. ft. |
| | | 48. 30 lb. 13 oz. |

Exercise 78. Page 148

- | | | |
|---------------------------------|--|-------------------------------|
| 1. 9 in. | 21. 5 yd. $31\frac{3}{4}$ in. | 41. 5 rd. $15\frac{1}{2}$ ft. |
| 2. 1 gal. 3 qt. | 22. 26 qt. 1 pt. | 42. 7 yd. 24 in. |
| 3. 3 in. | 23. 12 ft. 9 in. | 43. 15 mi. 229 rd. |
| 4. 49 ft. 4 in. | 24. 1 yd. 1 ft. | 44. 17 yd. 32 in. |
| 5. 7 lb. 14 oz. | 25. 103 lb. 6 oz. | 45. 16 ft. 9 in. |
| 6. 8 rd. $12\frac{1}{2}$ ft. | 26. 12 gal. 3 qt. | 46. 116 mi. 275 rd. |
| 7. 10 mi. 160 rd. | 27. 17 ft. $10\frac{1}{4}$ in. | 47. 48 lb. 14 oz. |
| 8. 6 yd. 27 in. | 28. 9 bu. 7 qt. | 48. 28 rd. |
| 9. 17 rd. $12\frac{1}{2}$ ft. | 29. 74 lb. 8 oz. | 49. 19 mi. 240 rd. |
| 10. 2 mi. 280 rd. | 30. 34 qt. 1 pt. | 50. 27 rd. 7 ft. |
| 11. 6 rd. $2\frac{1}{2}$ ft. | 31. 48 sq. ft. 130 sq. in. | 51. 1 yr. 10 mo. 29 da. |
| 12. 5 rd. 10 ft. | 32. 7 hr. 15 min. | 52. 4 yr. 4 mo. 18 da. |
| 13. 7 rd. $15\frac{1}{4}$ ft. | 33. 74 cu. ft. 797 cu. in. | 53. 2 yr. 5 mo. 14 da. |
| 14. 15 bu. 3 pk. | 34. 17 mi. 268 rd. | 54. $2^{\circ} 20' 47''$. |
| 15. 7 mi. 40 rd. | 35. 20 sq. ft. $48\frac{3}{4}$ sq. in. | 55. $24' 4''$. |
| 16. 9 mi. 300 rd. | 36. 15 A. 122 sq. rd. | 56. $1^{\circ} 53' 50''$. |
| 17. 46 mi. $319\frac{3}{4}$ rd. | 37. 47 cu. ft. 1453 cu. in. | 57. $48^{\circ} 36' 38''$. |
| 18. 21 gal. 2 qt. | 38. 56 min. 38 sec. | 58. $13' 37.5''$. |
| 19. 9 lb. $3\frac{1}{3}$ oz. | 39. 31 ft. 11 in. | |
| 20. 8 lb. $14\frac{2}{3}$ oz. | 40. 20 lb. 15 oz. | |

Exercise 79. Page 151

- | | | |
|--------------------|---------------------|-------------------------|
| 1. 6 ft. 8 in. | 10. 1127 lb. | 19. 60,137 mi. 1940 ft. |
| 2. 30 ft. | 11. 2725 yd. 24 in. | 20. 393 min. 36 sec. |
| 3. 73 ft. 6 in. | 12. 262 ft. 6 in. | 21. 724 min. |
| 4. 27 yd. 18 in. | 13. 748 ft. 9 in. | 22. 450 min. |
| 5. 65 yd. 4 in. | 14. 1069 ft. 9 in. | 23. 110 lb. 4 oz. |
| 6. 43 yd. 6 in. | 15. 603 qt. 1 pt. | 24. 60 rd. 6 ft. |
| 7. 1187 ft. 6 in. | 16. 1568 gal. 3 qt. | 25. 45 lb. 12 oz. |
| 8. 4709 yd. 12 in. | 17. 3007 bu. | 26. 29 gal. 1 qt. |
| 9. 481 ft. 9 in. | 18. 92 hr. 42 min. | 27. 25 gal. 2 qt. |

- | | | |
|------------------------------|-------------------------|---------------------|
| 28. 51 mi. 720 ft. | 39. 1110 min. | 50. 472 gal. 2 qt. |
| 29. 2283 lb. 12 oz. | 40. 150 bu. | 51. 21 lb. |
| 30. 3916 mi. 267 ft. | 41. 39 T. 600 lb. | 52. 4 hr. 30 min. |
| 31. 9560 ft. 10 in. | 42. 29,492 mi. 5110 ft. | 53. 2191 da. 10 hr. |
| 32. 20,723 lb. 15 oz. | 43. 10,500 mi. 500 yd. | 52 min. 36 sec. |
| 33. 30,712 yd. 11 in. | 44. 10,501 mi. 280 rd. | 54. 3000 lb. |
| 34. 191 sq. ft. 132 sq. in. | 45. 2003 lb. 12 oz. | 55. 24,375 lb. |
| 35. 57 cu. ft. 1116 cu. in. | 46. 920 ft. 10 in. | 56. 24,375 lb. |
| 36. 216 cu. ft. 1600 cu. in. | 47. 1145 ft. 10 in. | 57. 14,625 lb. |
| 37. 988 lb. 12 oz. | 48. 35 ft. | 58. 109 ft. 8 in. |
| 38. 774 min. 40 sec. | 49. 31 ft. | |

Exercise 80. Page 153

- | | | | |
|--------------------|---------|----------------------|-----------------------|
| 1. 2 ft. 5 in. | 8. 9. | 15. 1 min. 13 sec. | 22. 7. |
| 2. 21 ft. 11 in. | 9. 4. | 16. 8 ; 242. | 23. $7\frac{1}{2}$. |
| 3. 3 lb. 2 oz. | 10. 11. | 17. $1\frac{4}{5}$. | 24. 144. |
| 4. 4 yd. 33 in. | 11. 9. | 18. 72. | 25. 112. |
| 5. 3 mi. 4864 ft. | 12. 20. | 19. 8. | 26. $17\frac{1}{2}$. |
| 6. 24 min. 21 sec. | 13. 8. | 20. 35. | |
| 7. 7. | 14. 32. | 21. 6. | |

Exercise 83. Page 159

- | | | |
|---|--------------------------------------|---------------------------|
| 1. 392 in. | 14. 16,008 $\frac{1}{3}$. | 30. $195\frac{5}{8}$ lb.; |
| 2. 23 ft. 6 in.; $23\frac{1}{2}$ ft. | 15. $129\frac{1}{3}$. | 53,202 $\frac{3}{8}$ lb.; |
| 3. \$2.75. | 16. 84 ft.; $27\frac{1}{2}$ fathoms. | 4256 T. 350 lb. |
| 4. 1200. | 17. 5 hr. 52 min. | 31. \$62.08. |
| 5. 7. | 18. 1 mi. 4764 ft. | 32. \$31.62. |
| 6. 3. | 19. 25,875 T. | 33. 12 da.; |
| 7. 447. | 20. 1.85 min. | \$73.50. |
| 8. $34\frac{1}{2}\frac{3}{8}$. | 21. $1.42\frac{1}{2}$ min. | 34. \$232.75. |
| 9. 3963 mi. 1562 ft. | 22. 6 min. 47 sec. | 35. 12,375 lb. |
| 10. $21\frac{9}{16}$. | 23. 6 min. 48 sec. | 36. 1056 lb. |
| 11. 30 in.; $2\frac{1}{2}$ ft.; 2.5 ft.; | 24. 3200 ; 20. | 37. \$1518. |
| $\frac{5}{8}$ yd.; $\frac{5}{3}$ rd. | 25. $40\overline{)160}$. | 38. 62,475 lb. |
| 12. 56 in.; 4 ft. 8 in.; | 26. 1200. | 39. 216.2 lb. |
| $4\frac{2}{3}$ ft.; $4.66\frac{2}{3}$ ft. | 27. 12,243.75 lb. | 40. 24.2 T.; |
| 13. 62.5 lb.; 62 lb. 8 oz.; | 28. 12.5. | 30.25 T.; |
| $62\frac{1}{2}$ lb.; 62.5 lb. | 29. $11\frac{1}{5}$ T.; 18 T. | 36.3 T. |

Exercise 84. Page 163

- | | | | | |
|-------------|-------------|--------------|--------------------------|---------------|
| 1. \$12.50. | 4. \$13.50. | 7. \$29.70. | 10. \$4.38. | 13. \$163.80. |
| 2. \$3.90. | 5. \$27. | 8. \$115.20. | 11. \$1.50. | |
| 3. \$2.44. | 6. \$6.65. | 9. \$120. | 12. \$363; \$242; \$242. | |

Exercise 85. Page 165

- | | | | |
|------------|---------------------------------|-------------------------------|-----------|
| 1. 150. | 5. 2 min. 11 $\frac{1}{4}$ sec. | 9. 3480. | 13. \$15. |
| 2. 171. | 6. 1023 lb. | 10. 8 ft. 5 $\frac{1}{2}$ in. | 14. 63. |
| 3. 175 da. | 7. 3 hr. 46 $\frac{4}{5}$ min. | 11. 2 hr. 12 min. | 15. 5 bu. |
| 4. 27 da. | 8. 22.5. | 12. 2 $\frac{5}{8}$ mi. | |

Exercise 86. Page 167

- | | | | | |
|---------|----------|---------------|------------|------------------------|
| 1. 100. | 3. 1280. | 5. 1600. | 7. \$3200. | 9. 220. |
| 2. 53. | 4. 960. | 6. \$4809.38. | 8. \$152. | 10. 3 $\frac{1}{2}$ ¢. |

Exercise 87. Page 168

- | | | |
|-----------------------|--------------------|-----------------------|
| 1. 100,412 T. 974 lb. | 4. 19,711,840,608. | 7. \$241,600. |
| 2. 8735 T. 1000 lb. | 5. \$3000. | 8. 70. |
| 3. 5,119,737,264. | 6. 500 lb. | 9. \$138.24; \$57.60. |

Exercise 88. Page 169

- | | | | | |
|---------------------|------------|------------|------------|--------------|
| 1. \$0.90. | 3. \$0.13. | 5. \$1.62. | 7. \$2.64. | 9. 10¢; 30¢. |
| 2. $\frac{2}{5}$ ¢. | 4. \$0.06. | 6. \$0.38. | 8. 24¢. | |

Exercise 89. Page 170

- | | | | | | |
|---------|-------------|-----------|------------|-------------------------|----------------------------|
| 1. 5. | 3. 560. | 5. 16 in. | 7. \$4.32. | 9. 3055 $\frac{7}{8}$. | 11. 20 rd. |
| 2. 720. | 4. 3454.70. | 6. 24 in. | 8. 5 A. | 10. 20 rd. | 12. 660; 4 $\frac{1}{8}$. |

Exercise 90. Page 172

- | | | | | |
|-------------|--------------|----------------|---------------|----------------|
| 1. \$27.60. | 5. \$10.84. | 9. \$456.60. | 13. \$670.36. | 17. \$1086.10. |
| 2. \$6.99. | 6. \$3.76. | 10. \$1328.50. | 14. \$523.74. | 18. \$656.38. |
| 3. \$7.09. | 7. \$11.54. | 11. \$541.92. | 15. \$197.98. | 19. \$86.09. |
| 4. \$10.82. | 8. \$378.45. | 12. \$355.85. | 16. \$641.61. | |

Exercise 91. Page 174

- | | | |
|--------------|--------------|--------------|
| 1. \$8.12. | 3. \$90.26. | 5. \$288.93. |
| 2. \$182.75. | 4. \$232.60. | 6. \$252.72. |

Exercise 92. Page 176

- | | | | |
|--------------|-------------|----------------|----------------|
| 1. \$77.74. | 5. \$12.40. | 9. \$68.12. | 13. \$52.15. |
| 2. \$50.28. | 6. \$16.07. | 10. \$14.60. | 14. \$1115.60. |
| 3. \$184.95. | 7. \$18.67. | 11. \$1183.50. | 15. \$1450.59. |
| 4. \$157.80. | 8. \$13.75. | 12. \$303.60. | 16. \$1500.09. |

Exercise 94. Page 181

- | | | |
|---|--------------------|--------------------------------|
| 1. 432; 640; 1024; 3036; 8292; 4878. | | |
| 2. 636; 444; 1002; 3027; 1104; 6777. | | |
| 3. 830; 4225; 3010; 7200; 6305; 9270; 4005. | | |
| 4. 53; 61; 97. | 11. 3, 3, 5, 5. | 18. 3, 3, 3, 3, 3, 3, 5. |
| 5. 2, 2, 2, 3, 5. | 12. 3, 101. | 19. 2, 2, 2, 3, 11, 13. |
| 6. 2, 3, 5, 7. | 13. 2, 3, 5, 5, 5. | 20. 2, 2, 2, 2, 2, 2, 3, 3, 3. |
| 7. 2, 3, 5, 5. | 14. 2, 3, 3, 37. | 21. 2, 2, 2, 2, 2, 3, 5, 11. |
| 8. 2, 2, 5, 11. | 15. 7, 11, 13. | 22. 2, 3, 7, 11, 13. |
| 9. 2, 5, 17. | 16. 3, 5, 83. | 23. 37, 101. |
| 10. 5, 5, 7. | 17. 11, 11, 11. | 24. 2, 2, 2, 3, 11, 37. |

Exercise 95. Page 183

- | | | | | | |
|--------|---------|--------|---------|---------|---------|
| 1. 2. | 9. 9. | 17. 3. | 25. 5. | 33. 10. | 41. 25. |
| 2. 4. | 10. 19. | 18. 4. | 26. 12. | 34. 16. | 42. 64. |
| 3. 3. | 11. 5. | 19. 3. | 27. 6. | 35. 3. | 43. 32. |
| 4. 6. | 12. 21. | 20. 5. | 28. 16. | 36. 12. | 44. 9. |
| 5. 7. | 13. 11. | 21. 4. | 29. 6. | 37. 12. | 45. 25. |
| 6. 15. | 14. 23. | 22. 7. | 30. 13. | 38. 38. | |
| 7. 8. | 15. 6. | 23. 8. | 31. 21. | 39. 8. | |
| 8. 17. | 16. 2. | 24. 9. | 32. 11. | 40. 11. | |

Exercise 96. Page 184

- | | | | | | |
|--------|---------|---------|----------|---------|----------|
| 1. 65. | 4. 41. | 7. 61. | 10. 14. | 13. 53. | 16. 146. |
| 2. 14. | 5. 144. | 8. 101. | 11. 229. | 14. 57. | 17. 29. |
| 3. 37. | 6. 625. | 9. 29. | 12. 37. | 15. 71. | 18. 29. |

Exercise 97. Page 186

- | | | | | |
|---------|-------------|-----------|-------------|-------------|
| 1. 66. | 10. 240. | 19. 396. | 28. 27,720. | 37. 144. |
| 2. 51. | 11. 882. | 20. 756. | 29. 96. | 38. 312. |
| 3. 48. | 12. 144. | 21. 120. | 30. 1260. | 39. 300. |
| 4. 105. | 13. 180. | 22. 720. | 31. 6930. | 40. 30 ft. |
| 5. 351. | 14. 210. | 23. 66. | 32. 1260. | 41. 24 qt. |
| 6. 165. | 15. 6552. | 24. 4004. | 33. 576. | 42. 50¢. |
| 7. 105. | 16. 25,389. | 25. 560. | 34. 1890. | 43. 96 gal. |
| 8. 228. | 17. 150. | 26. 600. | 35. 660. | 44. 509. |
| 9. 390. | 18. 192. | 27. 480. | 36. 495. | 45. 127. |

Exercise 98. Page 188

1. 36.	7. 3.	13. 28.	19. 384.	25. 18.	30. 8.
2. 8.	8. 2.	14. 144.	20. 70.	26. 9.	31. 7.
3. 9.	9. 8.	15. $1\frac{1}{2}$.	21. \$2.	27. \$16.	32. 7.
4. 6.	10. 16.	16. 84.	22. 3.	28. \$75;	33. 4.
5. 10.	11. 9.	17. 40.	23. 4.	\$108.	34. 5.
6. 8.	12. 252.	18. 140.	24. 9.	29. 4.	

Exercise 99. Page 192

1. $\frac{1}{4}$.	5. $\frac{6}{11}$.	9. $\frac{2}{3}$.	13. $\frac{1}{4}$.	17. $\frac{1}{4}$.	21. $\frac{1}{2}$.	25. $\frac{2}{3}$.	29. $\frac{37}{40}$.
2. $\frac{3}{7}$.	6. $\frac{1}{3}$.	10. $\frac{8}{25}$.	14. $\frac{3}{4}$.	18. $\frac{1}{3}$.	22. $\frac{1}{3}$.	26. $\frac{9}{32}$.	30. $\frac{11}{12}$.
3. $\frac{9}{11}$.	7. $\frac{1}{3}$.	11. $\frac{1}{3}$.	15. $\frac{5}{9}$.	19. $\frac{16}{13}$.	23. $\frac{2}{5}$.	27. $\frac{1}{7}$.	31. $\frac{11}{12}$.
4. $\frac{4}{7}$.	8. $\frac{1}{2}$.	12. $\frac{1}{3}$.	16. $\frac{3}{4}$.	20. $\frac{6}{13}$.	24. $\frac{11}{48}$.	28. $\frac{2}{7}$.	32. $\frac{12}{13}$.

Exercise 100. Page 193

1. $\frac{30}{60}$.	6. $\frac{25}{80}$.	11. $\frac{30}{144}$.	16. $\frac{14}{156}$.	21. $\frac{144}{288}$.	26. $\frac{60}{320}$.	31. $\frac{16}{100}$.
2. $\frac{32}{60}$.	7. $\frac{28}{80}$.	12. $\frac{14}{144}$.	17. $\frac{128}{256}$.	22. $\frac{24}{288}$.	27. $\frac{50}{320}$.	32. $\frac{18}{100}$.
3. $\frac{27}{60}$.	8. $\frac{18}{80}$.	13. $\frac{78}{156}$.	18. $\frac{48}{256}$.	23. $\frac{60}{288}$.	28. $\frac{45}{320}$.	
4. $\frac{22}{60}$.	9. $\frac{72}{144}$.	14. $\frac{8}{156}$.	19. $\frac{40}{256}$.	24. $\frac{42}{288}$.	29. $\frac{50}{100}$.	
5. $\frac{40}{80}$.	10. $\frac{12}{144}$.	15. $\frac{10}{156}$.	20. $\frac{28}{256}$.	25. $\frac{160}{320}$.	30. $\frac{30}{100}$.	

Exercise 101. Page 194

1. 3.	8. $4\frac{1}{2}$.	15. 3.	22. $1\frac{1}{2}$.	29. $1\frac{3}{8}$.	36. 2.	43. $4\frac{1}{4}$.
2. $3\frac{1}{6}$.	9. 5.	16. 2.	23. $1\frac{4}{5}$.	30. 7.	37. $2\frac{1}{16}$.	44. $5\frac{1}{2}$.
3. $3\frac{1}{3}$.	10. 6.	17. $1\frac{1}{2}$.	24. $1\frac{9}{10}$.	31. $1\frac{1}{9}$.	38. 3.	45. $7\frac{1}{2}$.
4. 5.	11. $1\frac{1}{2}$.	18. $1\frac{1}{6}$.	25. 7.	32. $1\frac{1}{6}$.	39. 5.	46. 1.
5. 6.	12. 3.	19. $1\frac{3}{4}$.	26. 1.	33. $1\frac{1}{3}$.	40. 6.	47. $2\frac{1}{2}$; 3.
6. $2\frac{1}{4}$.	13. 4.	20. 5.	27. $1\frac{1}{8}$.	34. 2.	41. $2\frac{1}{2}$.	
7. $2\frac{1}{2}$.	14. 4.	21. $1\frac{1}{5}$.	28. $1\frac{1}{4}$.	35. 4.	42. $3\frac{3}{4}$.	

Exercise 102. Page 195

1. $\frac{31}{8}$.	10. $\frac{107}{16}$.	19. $\frac{201}{32}$.	28. $\frac{107}{48}$.	37. $\frac{439}{72}$.
2. $\frac{39}{8}$.	11. $\frac{53}{24}$.	20. $\frac{75}{32}$.	29. $\frac{305}{48}$.	38. $\frac{383}{72}$.
3. $\frac{61}{8}$.	12. $\frac{127}{24}$.	21. $\frac{293}{36}$.	30. $\frac{71}{48}$.	39. $\frac{2555}{72}$.
4. $\frac{55}{8}$.	13. $\frac{203}{24}$.	22. $\frac{223}{36}$.	31. $\frac{71}{64}$.	40. $\frac{5225}{72}$.
5. $\frac{77}{8}$.	14. $\frac{61}{24}$.	23. $\frac{155}{36}$.	32. $\frac{331}{64}$.	41. $\frac{7}{8}$; $\frac{24}{8}$; $\frac{31}{8}$.
6. $\frac{115}{16}$.	15. $\frac{161}{24}$.	24. $\frac{265}{36}$.	33. $\frac{465}{64}$.	42. $\frac{5}{16}$; $\frac{80}{16}$; $\frac{85}{16}$.
7. $\frac{85}{16}$.	16. $\frac{227}{32}$.	25. $\frac{89}{36}$.	34. $\frac{221}{64}$.	43. $\frac{7}{32}$; $\frac{96}{32}$; $\frac{103}{32}$.
8. $\frac{137}{16}$.	17. $\frac{293}{32}$.	26. $\frac{245}{48}$.	35. $\frac{421}{64}$.	44. 7; 16; 23.
9. $\frac{151}{16}$.	18. $\frac{103}{32}$.	27. $\frac{391}{48}$.	36. $\frac{77}{72}$.	

Exercise 103. Page 196

- | | | |
|------------------------------------|---|---|
| 1. $\frac{9}{24}; \frac{10}{24}$ | 13. $\frac{4}{8}; \frac{6}{8}; \frac{7}{8}$ | 25. $\frac{6}{12}; \frac{4}{12}; \frac{3}{12}; \frac{2}{12}$ |
| 2. $\frac{20}{24}; \frac{21}{24}$ | 14. $\frac{8}{24}; \frac{6}{24}; \frac{15}{24}$ | 26. $\frac{6}{12}; \frac{8}{12}; \frac{3}{12}; \frac{10}{12}$ |
| 3. $\frac{32}{32}; \frac{3}{32}$ | 15. $\frac{16}{24}; \frac{6}{24}; \frac{9}{24}$ | 27. $\frac{12}{24}; \frac{24}{24}; \frac{18}{24}; \frac{3}{24}$ |
| 4. $\frac{35}{40}; \frac{21}{40}$ | 16. $\frac{16}{24}; \frac{18}{24}; \frac{15}{24}$ | 28. $\frac{12}{24}; \frac{16}{24}; \frac{18}{24}; \frac{9}{24}$ |
| 5. $\frac{20}{48}; \frac{21}{48}$ | 17. $\frac{24}{120}; \frac{15}{120}; \frac{10}{120}$ | 29. $\frac{12}{24}; \frac{16}{24}; \frac{18}{24}; \frac{21}{24}$ |
| 6. $\frac{28}{48}; \frac{27}{48}$ | 18. $\frac{48}{120}; \frac{45}{120}; \frac{50}{120}$ | 30. $\frac{8}{16}; \frac{12}{16}; \frac{10}{16}; \frac{3}{16}$ |
| 7. $\frac{44}{48}; \frac{45}{48}$ | 19. $\frac{72}{120}; \frac{105}{120}; \frac{70}{120}$ | 31. $\frac{24}{32}; \frac{28}{32}; \frac{10}{32}; \frac{3}{32}$ |
| 8. $\frac{45}{48}; \frac{38}{48}$ | 20. $\frac{15}{120}; \frac{12}{120}; \frac{10}{120}$ | 32. $\frac{12}{48}; \frac{28}{48}; \frac{3}{48}; \frac{2}{48}$ |
| 9. $\frac{26}{32}; \frac{29}{32}$ | 21. $\frac{45}{120}; \frac{84}{120}; \frac{70}{120}$ | 33. $\frac{36}{48}; \frac{18}{48}; \frac{9}{48}; \frac{10}{48}$ |
| 10. $\frac{52}{96}; \frac{15}{96}$ | 22. $\frac{45}{120}; \frac{36}{120}; \frac{50}{120}$ | 34. $\frac{40}{64}; \frac{4}{64}; \frac{6}{64}; \frac{1}{64}$ |
| 11. $\frac{9}{96}; \frac{22}{96}$ | 23. $\frac{30}{48}; \frac{15}{48}; \frac{14}{48}$ | 35. $\frac{56}{64}; \frac{20}{64}; \frac{10}{64}; \frac{7}{64}$ |
| 12. $\frac{45}{96}; \frac{58}{96}$ | 24. $\frac{42}{48}; \frac{21}{48}; \frac{14}{48}$ | 36. $\frac{168}{192}; \frac{8}{192}; \frac{6}{192}; \frac{15}{192}$ |

Exercise 104. Page 197

- | | | | | | |
|-------------------|---------------------|---------------------|---------------------|----------------------|-----------------------|
| 1. $\frac{3}{4}$ | 8. $\frac{7}{8}$ | 15. $\frac{11}{16}$ | 22. $1\frac{5}{24}$ | 29. $\frac{25}{48}$ | 36. $1\frac{1}{12}$ |
| 2. $1\frac{1}{4}$ | 9. $1\frac{1}{8}$ | 16. $1\frac{9}{16}$ | 23. $\frac{2}{3}$ | 30. $\frac{37}{48}$ | 37. $\frac{13}{64}$ |
| 3. $\frac{5}{8}$ | 10. $1\frac{1}{8}$ | 17. $\frac{19}{32}$ | 24. $1\frac{1}{12}$ | 31. $1\frac{7}{8}$ | 38. $\frac{83}{192}$ |
| 4. $1\frac{1}{8}$ | 11. $1\frac{3}{8}$ | 18. $\frac{27}{32}$ | 25. $1\frac{1}{48}$ | 32. $\frac{31}{48}$ | 39. $\frac{139}{192}$ |
| 5. $1\frac{3}{8}$ | 12. $1\frac{5}{8}$ | 19. $\frac{27}{32}$ | 26. $\frac{25}{48}$ | 33. $\frac{47}{48}$ | 40. $1\frac{59}{64}$ |
| 6. $\frac{3}{8}$ | 13. $\frac{6}{16}$ | 20. $1\frac{9}{32}$ | 27. $\frac{43}{48}$ | 34. $1\frac{17}{48}$ | |
| 7. $\frac{5}{8}$ | 14. $1\frac{3}{16}$ | 21. $\frac{11}{24}$ | 28. $1\frac{5}{48}$ | 35. $\frac{5}{12}$ | |

Exercise 105. Page 198

- | | | | | |
|--------------------|---------------------|-----------------------|----------------------|---------------------------|
| 1. $6\frac{1}{8}$ | 5. $11\frac{1}{6}$ | 9. $62\frac{4}{15}$ | 13. $91\frac{7}{24}$ | 17. $15\frac{1}{3}$ |
| 2. $10\frac{5}{8}$ | 6. $12\frac{7}{12}$ | 10. $81\frac{5}{16}$ | 14. $10\frac{3}{8}$ | 18. $13\frac{15}{16}$ |
| 3. $10\frac{7}{8}$ | 7. $70\frac{1}{8}$ | 11. $89\frac{19}{24}$ | 15. $12\frac{5}{8}$ | 19. $5\frac{1}{48}$ in. |
| 4. $8\frac{5}{6}$ | 8. $64\frac{2}{15}$ | 12. $109\frac{3}{16}$ | 16. $15\frac{1}{16}$ | 20. $478\frac{1}{4}$ |
| | | | | 21. $11\frac{85}{96}$ qt. |

Exercise 106. Page 199

- | | | | | |
|--------------------|---------------------|---------------------|---------------------|------------------------|
| 1. $\frac{1}{8}$ | 10. $\frac{5}{16}$ | 19. $\frac{13}{64}$ | 28. $\frac{13}{64}$ | 37. $\frac{1}{18}$ |
| 2. $\frac{3}{8}$ | 11. $\frac{5}{16}$ | 20. $\frac{17}{64}$ | 29. $\frac{1}{6}$ | 38. $\frac{3}{8}$ |
| 3. $\frac{1}{8}$ | 12. $\frac{13}{32}$ | 21. $\frac{13}{64}$ | 30. $\frac{5}{24}$ | 39. $\frac{7}{144}$ |
| 4. $\frac{3}{8}$ | 13. $\frac{3}{32}$ | 22. $\frac{9}{64}$ | 31. $\frac{31}{72}$ | 40. $\frac{17}{144}$ |
| 5. $\frac{3}{8}$ | 14. $\frac{5}{32}$ | 23. $\frac{5}{64}$ | 32. $\frac{25}{72}$ | 41. $\frac{31}{144}$ |
| 6. $1\frac{3}{16}$ | 15. $\frac{1}{32}$ | 24. $\frac{19}{64}$ | 33. $\frac{11}{72}$ | 42. $\frac{83}{144}$ |
| 7. $\frac{5}{16}$ | 16. $\frac{31}{64}$ | 25. $\frac{19}{64}$ | 34. $\frac{19}{72}$ | 43. $\frac{95}{144}$ |
| 8. $\frac{1}{16}$ | 17. $\frac{17}{64}$ | 26. $\frac{5}{64}$ | 35. $\frac{1}{9}$ | 44. $\frac{33}{100}$ |
| 9. $\frac{7}{16}$ | 18. $\frac{9}{64}$ | 27. $\frac{9}{64}$ | 36. $\frac{1}{18}$ | 45. $\frac{787}{1000}$ |

Exercise 107. Page 200

- | | | | | |
|---------------------|---------------------|----------------------|-------------------------|--------------------------|
| 1. $5\frac{5}{8}$. | 5. $2\frac{1}{8}$. | 9. $5\frac{7}{8}$. | 13. $13\frac{1}{5}$. | 17. $185\frac{3}{10}$ A. |
| 2. $6\frac{1}{8}$. | 6. $8\frac{7}{8}$. | 10. $4\frac{7}{8}$. | 14. $15\frac{3}{5}$. | 18. 1 hr. |
| 3. $4\frac{1}{8}$. | 7. $6\frac{5}{8}$. | 11. $4\frac{1}{2}$. | 15. $24\frac{5}{2}$. | 45 min. A.M. |
| 4. $6\frac{3}{8}$. | 8. $7\frac{7}{8}$. | 12. $7\frac{1}{2}$. | 16. $41\frac{1}{4}$ yd. | |

Exercise 108. Page 201

- | | | | | |
|----------------------|-----------------------|------------------------|------------------------|-------------------------|
| 1. $3\frac{3}{8}$. | 8. $6\frac{9}{16}$. | 15. 42. | 22. $81\frac{2}{3}$. | 29. 63. |
| 2. 6. | 9. $13\frac{1}{8}$. | 16. $58\frac{1}{2}$. | 23. $137\frac{1}{2}$. | 30. $114\frac{7}{12}$. |
| 3. 10. | 10. $24\frac{1}{2}$. | 17. 77. | 24. 45. | 31. $\$12\frac{3}{4}$. |
| 4. $13\frac{1}{2}$. | 11. $67\frac{1}{2}$. | 18. $112\frac{1}{2}$. | 25. $66\frac{1}{2}$. | 32. 110 mi. |
| 5. $8\frac{1}{4}$. | 12. 88. | 19. 216. | 26. $115\frac{1}{2}$. | |
| 6. $16\frac{1}{4}$. | 13. 15. | 20. $263\frac{1}{2}$. | 27. 143. | |
| 7. 28. | 14. $27\frac{1}{2}$. | 21. $52\frac{1}{2}$. | 28. $292\frac{1}{2}$. | |

Exercise 109. Page 202

- | | | | | | |
|----------------------|----------------------|-----------------------|-----------------------|-----------------------|--------------------------------------|
| 1. 12. | 5. 60. | 9. $46\frac{2}{3}$. | 13. 42. | 17. $38\frac{1}{2}$. | 21. $43\frac{3}{4}$. |
| 2. 18. | 6. $77\frac{1}{2}$. | 10. 88. | 14. $56\frac{1}{4}$. | 18. 52. | 22. 45ϕ ; 54ϕ . |
| 3. 14. | 7. 9. | 11. $13\frac{1}{2}$. | 15. 15. | 19. $70\frac{5}{8}$. | 23. 84ϕ ; 60ϕ . |
| 4. $28\frac{4}{5}$. | 8. 30. | 12. 25. | 16. $23\frac{1}{3}$. | 20. $15\frac{5}{8}$. | 24. 66ϕ ; 33ϕ ; 55ϕ . |

Exercise 110. Page 203

- | | | | | |
|----------------------|------------------------|-----------------------|--|---|
| 1. $\frac{2}{3}$. | 13. $\frac{1}{40}$. | 25. $\frac{4}{5}$. | 37. $\frac{1}{3}$. | 49. $1\frac{9}{40}$ yd. |
| 2. $\frac{3}{5}$. | 14. $\frac{1}{50}$. | 26. $\frac{1}{4}$. | 38. $\frac{9}{160}$. | 50. $\frac{7}{12}$ yd. |
| 3. $\frac{5}{8}$. | 15. $\frac{1}{9}$. | 27. $\frac{5}{16}$. | 39. $\frac{9}{40}$. | 51. $\frac{3}{8}$ yd. |
| 4. $\frac{11}{36}$. | 16. $\frac{1}{16}$. | 28. $\frac{2}{9}$. | 40. $\frac{105}{512}$. | 52. $\frac{1}{16}$ yd. |
| 5. $\frac{25}{48}$. | 17. $\frac{5}{12}$. | 29. $\frac{13}{24}$. | 41. $\frac{7}{32}$ in. | 53. $\frac{3}{8}$; $\frac{9}{16}$. |
| 6. $\frac{5}{32}$. | 18. $\frac{9}{20}$. | 30. $\frac{3}{8}$. | 42. $\frac{7}{24}$ in. | 54. $\frac{5}{18}$ yd.; $\frac{5}{8}$ yd. |
| 7. $\frac{1}{10}$. | 19. $\frac{3}{8}$ yd. | 31. $\frac{7}{27}$. | 43. $\frac{15}{128}$. | 55. $\frac{9}{20}$ hr.; 27. |
| 8. $\frac{21}{64}$. | 20. $\frac{5}{12}$ A. | 32. $\frac{1}{4}$. | 44. $\frac{9}{20}$. | 56. $\frac{2}{9}$. |
| 9. $\frac{1}{2}$. | 21. $\frac{7}{32}$ mi. | 33. $\frac{11}{15}$. | 45. $\$1\frac{1}{2}$; $\$3\frac{3}{8}$. | 57. $\frac{5}{8}$ in. |
| 10. $\frac{3}{16}$. | 22. $\frac{1}{28}$ in. | 34. $\frac{1}{4}$. | 46. $\frac{3}{8}$ yd. | 58. $\frac{8}{15}$. |
| 11. $\frac{1}{6}$. | 23. $\frac{3}{5}$. | 35. $\frac{2}{5}$. | 47. $\frac{59}{80}$ min. | 59. $\frac{11}{20}$. |
| 12. $\frac{1}{6}$. | 24. $\frac{2}{7}$. | 36. $\frac{1}{4}$. | 48. $\$1\frac{1}{2}$; $\$4\frac{1}{4}$; $\$3\frac{3}{8}$. | 60. $\frac{5}{8}$ yd. |

Exercise 111. Page 206

- | | | | | |
|-----------------------|-----------------------------------|-----------------------------------|--------------------------|-------------------------|
| 1. $35\frac{1}{3}$. | 8. $133\frac{2}{3}$. | 15. $471\frac{1}{2}$. | 22. $\$148\frac{3}{4}$. | 29. 2680. |
| 2. $78\frac{3}{4}$. | 9. $76\frac{3}{4}$. | 16. $90\frac{3}{16}$. | 23. $\$213$. | 30. $4245\frac{1}{2}$. |
| 3. $56\frac{1}{4}$. | 10. $244\frac{7}{12}$. | 17. $296\frac{5}{8}$. | 24. $\$222$. | 31. $\$20\frac{1}{4}$. |
| 4. 143. | 11. $32\frac{3}{8}$. | 18. $775\frac{3}{8}\frac{1}{2}$. | 25. 1410. | 32. $104\frac{3}{4}$. |
| 5. $86\frac{3}{4}$. | 12. $51\frac{1}{16}\frac{5}{8}$. | 19. $554\frac{1}{4}$. | 26. 3333. | 33. 60¢. |
| 6. $76\frac{7}{8}$. | 13. $101\frac{3}{4}$. | 20. $584\frac{1}{16}$. | 27. $3661\frac{1}{2}$. | 34. 1386. |
| 7. $341\frac{1}{4}$. | 14. $317\frac{1}{2}$. | 21. $\$131$. | 28. 3390. | 35. $9\frac{5}{8}$. |
| | | | | 36. $43\frac{3}{4}$ mi. |

Exercise 112. Page 207

- | | | | | |
|-----------------------|------------------------|------------------------|-------------------------|--------------|
| 1. 285. | 7. 354. | 13. 66. | 19. $830\frac{1}{2}$. | 25. 3289. |
| 2. 322. | 8. $468\frac{3}{4}$. | 14. 204. | 20. $468\frac{3}{8}$. | 26. 3573. |
| 3. 378. | 9. 360. | 15. 425. | 21. 495. | 27. 3960. |
| 4. 651. | 10. $533\frac{3}{4}$. | 16. 906. | 22. 2660. | 28. 4825. |
| 5. $193\frac{3}{4}$. | 11. 372. | 17. $197\frac{1}{8}$. | 23. $1270\frac{1}{2}$. | 29. 158,477. |
| 6. $341\frac{1}{4}$. | 12. $534\frac{5}{8}$. | 18. $258\frac{3}{4}$. | 24. $5432\frac{1}{2}$. | 30. 192,054. |

Exercise 113. Page 208

- | | | | | | |
|----------------------|----------------------|----------------------|-------------------------|-------------------------|-----------------------------------|
| 1. $8\frac{7}{16}$. | 4. $43\frac{2}{5}$. | 7. $88\frac{2}{3}$. | 10. $152\frac{3}{4}$. | 13. 2210. | 16. $\$400$. |
| 2. $15\frac{5}{8}$. | 5. $27\frac{5}{8}$. | 8. 250. | 11. $2222\frac{2}{5}$. | 14. $5614\frac{2}{7}$. | 17. $121\frac{7}{8}$ mi. |
| 3. $8\frac{2}{3}$. | 6. $78\frac{3}{4}$. | 9. 250. | 12. 4020. | 15. 9599. | 18. $1\frac{3}{8}\frac{5}{4}$ in. |
| | | | | | 19. $16\frac{5}{16}$ in. |

Exercise 114. Page 209

- | | | | | |
|---------------------|----------------------|-----------------------|-----------------------|----------------------------|
| 1. $\frac{5}{48}$. | 5. $\frac{3}{32}$. | 9. $\frac{1}{72}$. | 13. $\frac{1}{128}$. | 17. $17\frac{27}{10000}$. |
| 2. $\frac{3}{32}$. | 6. $\frac{5}{96}$. | 10. $\frac{1}{44}$. | 14. $\frac{1}{320}$. | 18. $9\frac{1}{100}$. |
| 3. $\frac{3}{64}$. | 7. $\frac{3}{80}$. | 11. $\frac{1}{160}$. | 15. $\frac{1}{44}$. | 19. $7\frac{3}{1000}$. |
| 4. $\frac{1}{32}$. | 8. $\frac{3}{128}$. | 12. $\frac{1}{160}$. | 16. $\frac{1}{168}$. | 20. $1\frac{3}{10000}$. |
| | | | | 21. $1\frac{11}{100000}$. |

Exercise 115. Page 210

- | | | | | | |
|---------------------|----------------------|-------------------------|-----------------------|-------------------------|------------------------|
| 1. $\frac{5}{12}$. | 5. $2\frac{1}{8}$. | 9. $3\frac{5}{8}$. | 13. $28\frac{1}{2}$. | 17. $17\frac{19}{24}$. | 21. $\$1\frac{1}{8}$. |
| 2. $\frac{2}{35}$. | 6. $\frac{1}{8}$. | 10. $6\frac{1}{16}$. | 14. $16\frac{2}{3}$. | 18. $13\frac{1}{16}$. | 22. $\$4\frac{1}{2}$. |
| 3. $\frac{1}{18}$. | 7. $2\frac{3}{24}$. | 11. $4\frac{97}{112}$. | 15. $6\frac{2}{3}$. | 19. $\frac{3}{4}$. | |
| 4. $1\frac{3}{8}$. | 8. $4\frac{3}{8}$. | 12. $8\frac{1}{16}$. | 16. $22\frac{7}{8}$. | 20. $\$1\frac{1}{4}$. | |

Exercise 116. Page 211

- | | | | | |
|---------|----------|--------|-----------------------|----------|
| 1. 56. | 4. 280. | 7. 12. | 10. $21\frac{7}{8}$. | 13. 42. |
| 2. 111. | 5. 651. | 8. 28. | 11. 5. | 14. 81. |
| 3. 196. | 6. 6. | 9. 60. | 12. 20. | 15. 828. |
| | 16. 18. | | 18. 100. | 20. 284. |
| | 17. 500. | | 19. 30. | 21. 213. |

Exercise 117. Page 212

1. $4\frac{1}{2}$.	8. $10\frac{4}{5}$.	15. 60.	22. $1\frac{1}{8}$.	29. $7\frac{9}{4}$.	36. $13\frac{4}{5}$.
2. $6\frac{2}{3}$.	9. 20.	16. 80.	23. $\frac{3}{5}$.	30. $\frac{15}{2}$.	37. 11.
3. $8\frac{3}{4}$.	10. 21.	17. 64.	24. $1\frac{2}{5}$.	31. $3\frac{9}{8}$.	38. $8\frac{3}{4}$.
4. 15.	11. $22\frac{1}{2}$.	18. $22\frac{6}{7}$.	25. $\frac{1}{2}$.	32. $4\frac{8}{9}$.	39. $12\frac{5}{2}$.
5. $21\frac{1}{3}$.	12. 20.	19. $38\frac{2}{5}$.	26. $\frac{6}{7}$.	33. $10\frac{1}{4}$.	40. $18\frac{9}{4}$.
6. 8.	13. 20.	20. 135.	27. $\frac{5}{9}$.	34. $7\frac{1}{6}$.	
7. $9\frac{1}{7}$.	14. 35.	21. $\frac{8}{9}$.	28. $\frac{7}{8}$.	35. $20\frac{1}{3}$.	

Exercise 118. Page 213

1. $\frac{12}{25}$.	6. $\frac{8}{11}$.	11. $4\frac{8}{25}$.	16. $\frac{9}{80}$.	21. $1\frac{3}{60}$.	26. $\frac{1}{2}$.
2. $1\frac{1}{19}$.	7. $2\frac{2}{9}$.	12. $\frac{39}{50}$.	17. $\frac{3}{10}$.	22. 2.	27. 6.
3. $1\frac{5}{13}$.	8. $7\frac{1}{2}$.	13. $\frac{18}{25}$.	18. $\frac{1}{14}$.	23. $3\frac{3}{4}$.	28. $\$11\frac{1}{2}$.
4. $1\frac{15}{41}$.	9. $9\frac{3}{10}$.	14. $1\frac{1}{3}$.	19. $\frac{5}{2}$.	24. $1\frac{7}{8}$.	29. 18.
5. $1\frac{1}{7}$.	10. $3\frac{2}{25}$.	15. $\frac{3}{10}$.	20. $\frac{9}{320}$.	25. $1\frac{1}{3}$.	30. $\$33$; $\$2\frac{3}{4}$.
					31. $\$69$; $\$5\frac{3}{4}$.

Exercise 119. Page 214

1. $\frac{1}{3}$.	11. $\frac{1}{16}$.	21. 2.	31. $\frac{1}{3}$.	41. $\frac{39}{85}$; $\frac{46}{85}$.
2. $\frac{1}{3}$.	12. $\frac{1}{32}$.	22. $\frac{1}{2}$.	32. $\frac{2}{3}$.	42. $\frac{15}{16}$.
3. $\frac{2}{7}$.	13. $1\frac{1}{9}$.	23. $\frac{1}{3}$.	33. $\frac{1}{16}$.	43. $\frac{3}{4}$.
4. $\frac{1}{3}$.	14. $1\frac{1}{5}$.	24. $\frac{2}{3}$.	34. $\frac{3}{8}$.	44. $1\frac{4}{1050}$.
5. $\frac{1}{5}$.	15. $2\frac{4}{5}$.	25. $\frac{1}{10}$.	35. $\frac{5}{8}$.	45. $\frac{9}{64}$.
6. $\frac{1}{3}$.	16. $2\frac{2}{9}$.	26. $\frac{1}{5}$.	36. $\frac{7}{8}$.	46. $1\frac{104}{165}$.
7. 2.	17. $2\frac{4}{5}$.	27. $\frac{1}{2}$.	37. $\frac{1}{5}$.	47. $\frac{7}{10}$.
8. 4.	18. $2\frac{7}{10}$.	28. $\frac{1}{4}$.	38. $\frac{4}{5}$.	48. $\frac{2}{3}$.
9. $\frac{1}{5}$.	19. $\frac{2}{3}$.	29. $\frac{3}{4}$.	39. $\frac{1}{2}$.	49. $\frac{2}{5}$.
10. $\frac{1}{8}$.	20. $\frac{34}{43}$.	30. $\frac{1}{8}$.	40. $\$41\frac{1}{4}$; $\$57\frac{3}{4}$.	50. $\frac{3}{5}$; 1.

Exercise 120. Page 216

1. \$28.	13. 160 ft.	25. 320.	37. $1\frac{5}{16}$.
2. \$72.	14. \$984.	26. 150.	38. $\frac{1}{10}$.
3. 108.	15. \$1144.	27. 208.	39. $\frac{1}{53}$.
4. $21\frac{1}{2}$.	16. 96.	28. 160.	40. $\frac{1}{56}$; $1\frac{1}{2}$; $1\frac{3}{2}$;
5. $304\frac{1}{4}$.	17. 64.	29. 189.	$\frac{1}{32}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$.
6. \$56.	18. 64.	30. 270.	41. $\frac{15}{58}$.
7. \$96.	19. 81.	31. 259.	42. $\frac{1}{36}$.
8. 45.	20. 108.	32. 423.	43. $\frac{1}{3}$.
9. $42\frac{3}{4}$.	21. 110.	33. 370.	44. $\frac{1}{9}$.
10. \$60.	22. 48.	34. 1400.	45. $\frac{27}{200}$.
11. \$100.	23. 204.	35. 1560.	46. $\frac{1}{6}$; 6.
12. 112 ft.	24. 136.	36. 2750.	

Exercise 121. Page 219

- | | | | | |
|--------------------------|--------------------------|------------------------|-----------------------------|-----------------------------|
| 1. $\frac{7}{8}$. | 37. $113\frac{11}{20}$. | 73. $21\frac{23}{8}$. | 109. $49\frac{1}{2}$. | 145. $\frac{4}{5}$. |
| 2. $\frac{7}{16}$. | 38. $69\frac{49}{60}$. | 74. $66\frac{5}{32}$. | 110. $59\frac{1}{2}$. | 146. $7\frac{7}{7}$. |
| 3. $\frac{9}{16}$. | 39. $16\frac{3}{5}$. | 75. $1\frac{1}{3}$. | 111. $59\frac{17}{20}$. | 147. $\frac{1}{4}$. |
| 4. $1\frac{3}{16}$. | 40. $16\frac{5}{16}$. | 76. $1\frac{9}{10}$. | 112. $83\frac{3}{8}$. | 148. $\frac{6}{25}$ cu. ft. |
| 5. $1\frac{9}{16}$. | 41. $12\frac{29}{30}$. | 77. $\frac{23}{24}$. | 113. $31,603$. | 149. 20 sq. ft. |
| 6. $\frac{5}{16}$. | 42. $14\frac{7}{32}$. | 78. $\frac{27}{40}$. | 114. $495\frac{1}{32}$. | 150. 56 sq. in. |
| 7. $1\frac{13}{16}$. | 43. $12\frac{13}{24}$. | 79. $\frac{5}{16}$. | 115. $118\frac{3}{4}$. | 151. $\frac{2}{5}$ cu. ft. |
| 8. $1\frac{5}{16}$. | 44. 14 . | 80. $\frac{1}{8}$. | 116. $270\frac{1}{16}$. | 152. $\frac{1}{3}$ cu. in. |
| 9. $1\frac{9}{16}$. | 45. $15\frac{21}{64}$. | 81. $\frac{1}{8}$. | 117. $264\frac{17}{56}$. | 153. 432 sq. ft. |
| 10. $1\frac{11}{16}$. | 46. $38\frac{1}{12}$. | 82. $2\frac{1}{3}$. | 118. $247\frac{1}{2}$. | 154. 78 sq. in. |
| 11. $2\frac{3}{32}$. | 47. $\frac{5}{8}$. | 83. $5\frac{5}{6}$. | 119. $17,787$. | 155. $2\frac{7}{16}$. |
| 12. $2\frac{1}{32}$. | 48. $\frac{7}{8}$. | 84. $13\frac{4}{5}$. | 120. $27,054$. | 156. $6\frac{10}{17}$. |
| 13. $1\frac{1}{32}$. | 49. $\frac{7}{16}$. | 85. $19\frac{1}{8}$. | 121. $2345\frac{1}{8}$. | 157. $1\frac{23}{42}$. |
| 14. $1\frac{3}{32}$. | 50. $\frac{9}{16}$. | 86. 20 . | 122. $8028\frac{3}{8}$. | 158. $1\frac{3}{8}$. |
| 15. $1\frac{7}{32}$. | 51. $1\frac{1}{32}$. | 87. $42\frac{1}{2}$. | 123. $8035\frac{1}{2}$. | 159. $1\frac{21}{21}$. |
| 16. $1\frac{1}{24}$. | 52. $2\frac{3}{32}$. | 88. 84 . | 124. $14,708\frac{3}{5}$. | 160. $\frac{1}{9}$. |
| 17. $1\frac{11}{24}$. | 53. $2\frac{7}{64}$. | 89. $56\frac{1}{4}$. | 125. $17,917\frac{9}{10}$. | 161. $\frac{1}{9}$. |
| 18. $\frac{5}{6}$. | 54. $27\frac{1}{8}$. | 90. $75\frac{5}{8}$. | 126. $1\frac{1}{5}$. | 162. $\frac{1}{7}$. |
| 19. $1\frac{5}{6}$. | 55. $57\frac{1}{4}$. | 91. $97\frac{1}{2}$. | 127. $1\frac{5}{9}$. | 163. $\frac{1}{2}$. |
| 20. $2\frac{1}{64}$. | 56. $63\frac{1}{10}$. | 92. 225 . | 128. $1\frac{1}{7}$. | 164. $\frac{856}{1675}$. |
| 21. $21\frac{103}{20}$. | 57. $44\frac{25}{32}$. | 93. $11\frac{1}{4}$. | 129. $2\frac{5}{7}$. | 165. $\frac{1}{2}$. |
| 22. $3\frac{1}{2}$. | 58. $61\frac{13}{16}$. | 94. 14 . | 130. $21\frac{9}{10}$. | 166. $\frac{1}{2}$. |
| 23. $14\frac{1}{12}$. | 59. $64\frac{19}{24}$. | 95. 45 . | 131. $87\frac{3}{5}$. | 167. $\frac{1}{8}$. |
| 24. $15\frac{5}{12}$. | 60. $70\frac{3}{8}$. | 96. 60 . | 132. $1\frac{53}{64}$. | 168. $\frac{3}{8}$. |
| 25. $9\frac{11}{24}$. | 61. $2\frac{1}{16}$. | 97. 28 . | 133. $\frac{5}{6}$. | 169. $\frac{1}{3}$. |
| 26. $11\frac{7}{24}$. | 62. $5\frac{3}{16}$. | 98. 40 . | 134. $1\frac{3}{32}$. | 170. $\frac{1}{6}$. |
| 27. $18\frac{7}{24}$. | 63. $2\frac{1}{16}$. | 99. 45 . | 135. $\frac{7}{8}$. | 171. $\frac{8}{9}$. |
| 28. $16\frac{13}{24}$. | 64. $4\frac{1}{16}$. | 100. $\frac{9}{16}$. | 136. $\frac{1}{7}$. | 172. $\frac{1}{9}$. |
| 29. $9\frac{1}{6}$. | 65. $3\frac{7}{32}$. | 101. $\frac{3}{8}$. | 137. $3\frac{35}{35}$. | 173. $\frac{1}{2}$. |
| 30. $10\frac{11}{16}$. | 66. $3\frac{17}{32}$. | 102. $\frac{7}{18}$. | 138. $1\frac{7}{10}$. | 174. $\frac{1}{4}$. |
| 31. $92\frac{13}{30}$. | 67. $14\frac{15}{16}$. | 103. $\frac{7}{100}$. | 139. 4 . | 175. $\frac{1}{6}$. |
| 32. $138\frac{13}{16}$. | 68. $1\frac{7}{32}$. | 104. $\frac{5}{96}$. | 140. $\frac{9}{14}$. | 176. $\frac{9}{71}$. |
| 33. $168\frac{1}{10}$. | 69. $34\frac{5}{12}$. | 105. $\frac{3}{64}$. | 141. $\frac{32}{35}$. | 177. $\frac{20}{27}$. |
| 34. $142\frac{2}{15}$. | 70. $37\frac{3}{10}$. | 106. $\frac{1}{24}$. | 142. $\frac{3}{22}$. | 178. $\frac{1}{7}$. |
| 35. $128\frac{2}{3}$. | 71. $42\frac{23}{30}$. | 107. $\frac{1}{20}$. | 143. $4\frac{4}{49}$. | 179. $1\frac{1}{11}$. |
| 36. $122\frac{49}{60}$. | 72. $55\frac{3}{4}$. | 108. $38\frac{1}{2}$. | 144. 5 . | |

Exercise 122. Page 222

- | | | |
|---|--|--|
| 1. 11,139 $\frac{3}{16}$. | 9. 435. | 17. 49 $\frac{1}{2}$. |
| 2. 8812 $\frac{1}{2}$. | 10. 2914 $\frac{3}{4}$ $\frac{2}{7}$. | 18. 159 $\frac{1}{2}$ cu. in. larger. |
| 3. 24,990. | 11. 16. | 19. 489 $\frac{1}{2}$ cu. in. larger. |
| 4. 213,511 $\frac{1}{9}$. | 12. 1 $\frac{7}{8}$. | 20. 9120. |
| 5. 9887 $\frac{5}{8}$. | 13. $\frac{9}{10}$. | 21. 1st; 370 $\frac{5}{32}$. |
| 6. 1824. | 14. 4. | 22. 1235 $\frac{1}{4}$. |
| 7. 262 $\frac{1}{2}$. | 15. 2. | 23. 1st; 50 $\frac{5}{8}$ $\frac{5}{4}$ cu. in. |
| 8. 5002 $1\frac{1}{8}$ $\frac{3}{10}$. | 16. 70 $\frac{5}{8}$. | 24. 869 $\frac{1}{2}$; 427 $\frac{3}{2}$ $\frac{1}{2}$;
1847 $1\frac{1}{6}$; 1059 $4\frac{5}{4}$. |

Exercise 123. Page 224

- | | | | | |
|--|--------------------------|-------------------------------------|---------------------------------------|--------------------------------------|
| 1. 140 $\frac{5}{6}$. | 23. 1741 $\frac{1}{2}$. | 45. 1 $\frac{3}{4}$ in. | 67. 1 $\frac{5}{16}$. | 89. 6 $\frac{1}{17}$. |
| 2. 97 $\frac{8}{15}$. | 24. 2754. | 46. 960 ft. | 68. $\frac{10}{24}$. | 90. $\frac{2}{7}$ $\frac{5}{7}$. |
| 3. 96 $1\frac{1}{8}$. | 25. 156 $\frac{1}{4}$. | 47. 54 in. | 69. $\frac{5}{14}$. | 91. 2. |
| 4. 68 $1\frac{3}{20}$. | 26. 138 $\frac{3}{5}$. | 48. 69. | 70. $\frac{1}{4}$. | 92. 6 $\frac{9}{10}$. |
| 5. 155 $\frac{5}{12}$. | 27. 6684. | 49. 200. | 71. 1 $\frac{1}{15}$. | 93. $1\frac{9}{12}$. |
| 6. 103 $\frac{7}{15}$. | 28. 2960. | 50. 259. | 72. 1 $\frac{1}{24}$. | 94. $1\frac{4}{11}$. |
| 7. 112 $1\frac{3}{14}$. | 29. 2074. | 51. \$250. | 73. 1 $\frac{1}{16}$. | 95. 1 $\frac{3}{3}$ $\frac{4}{3}$. |
| 8. 150 $\frac{3}{10}$. | 30. 4008. | 52. 57. | 74. 1 $\frac{1}{9}$. | 96. $\frac{3}{5}$ $\frac{9}{10}$. |
| 9. 76 $1\frac{1}{2}$ $\frac{9}{10}$. | 31. 35 $1\frac{4}{5}$. | 53. 50. | 75. $\frac{9}{10}$. | 97. 1 $\frac{1}{2}$. |
| 10. 107 $\frac{1}{2}$. | 32. 1128. | 54. 72. | 76. 1 $\frac{1}{5}$. | 98. $\frac{2}{5}$ $\frac{1}{5}$. |
| 11. 117 $\frac{3}{10}$. | 33. 4629. | 55. 160. | 77. $\frac{7}{8}$. | 99. 21 $\frac{0}{77}$. |
| 12. 55 $\frac{9}{14}$. | 34. 4984 $\frac{4}{5}$. | 56. \$3.60. | 78. $\frac{1}{2}$ $\frac{6}{5}$. | 100. 4 $\frac{1}{2}$ $\frac{8}{3}$. |
| 13. 57 $\frac{3}{10}$. | 35. 3872. | 57. $\frac{1}{5}$. | 79. $\frac{9}{10}$. | 101. 5 $\frac{9}{11}$. |
| 14. 54 $1\frac{7}{20}$. | 36. 4560. | 58. $\frac{8}{21}$. | 80. 1 $\frac{1}{8}$. | 102. 9 $\frac{1}{36}$. |
| 15. 16 $1\frac{1}{4}$ $\frac{1}{10}$. | 37. 22 $\frac{1}{10}$. | 59. 1 $\frac{1}{2}$. | 81. 4 $\frac{2}{3}$. | 103. \$18.38. |
| 16. 50 $\frac{5}{8}$. | 38. 1304. | 60. 1. | 82. $\frac{5}{16}$. | 104. \$19. |
| 17. 10 $1\frac{1}{2}$ $\frac{9}{4}$. | 39. 1. | 61. 3 $\frac{7}{8}$ $\frac{7}{1}$. | 83. $1\frac{2}{7}$. | 105. \$34. |
| 18. 24 $1\frac{3}{4}$. | 40. $\frac{1}{7}$. | 62. $\frac{3}{5}$. | 84. $\frac{4}{15}$ $\frac{2}{5}$. | 106. \$220. |
| 19. 24 $\frac{7}{8}$. | 41. $\frac{3}{10}$ ft. | 63. 1 $\frac{1}{8}$ $\frac{1}{4}$. | 85. 4 $\frac{1}{18}$. | 107. \$165.67. |
| 20. 13 $\frac{3}{8}$. | 42. 30 ft. | 64. 2. | 86. $\frac{5}{11}$. | 108. \$126. |
| 21. 25 $\frac{1}{3}$. | 43. 54 in. | 65. $\frac{1}{2}$. | 87. 1 $\frac{7}{3}$ $\frac{1}{35}$. | |
| 22. 396. | 44. $\frac{3}{4}$ ft. | 66. 1 $\frac{1}{7}$. | 88. 5 $1\frac{1}{8}$ $\frac{0}{10}$. | |

Exercise 124. Page 226

- | | | | |
|------------|---------------------------|------------------------|--------------|
| 1. \$4. | 6. \$19.06. | 11. 23. | 16. \$2.40. |
| 2. 80¢. | 7. \$1.12 $\frac{1}{2}$. | 12. 9 $\frac{1}{2}$ ¢. | 17. \$92.50. |
| 3. \$1.13. | 8. \$6.87 $\frac{1}{2}$. | 13. 7¢. | 18. 23¢. |
| 4. \$1.79. | 9. 7 $\frac{1}{5}$ mi. | 14. 10 hr. | 19. 28¢. |
| 5. \$1.09. | 10. \$1.20. | 15. \$2.69. | 20. \$40. |

- | | | | |
|--|---|-------------------------|--------------------------|
| 21. \$60. | 31. $133\frac{4}{7}\frac{9}{2}$ oz.; | 39. \$0.65. | 49. 12. |
| 22. \$1.12. | $33\frac{1}{2}\frac{2}{8}\frac{1}{8}$ oz. | 40. \$1.92. | 50. $\frac{3}{10}$. |
| 23. \$1.14. | 32. $449\frac{7}{3}\frac{2}{2}$ lb. | 41. \$4.40. | 51. 12. |
| 24. 13 gal.; \$1.95. | 33. $732\frac{2}{6}\frac{7}{4}$ lb. | 42. \$0.62. | 52. $\frac{1}{4}$. |
| 25. \$4.80. | 34. 231 lb. 9 oz. | 43. \$0.07. | 53. 3 cups; |
| 26. \$3.60. | 35. 57.5 lb.; | 44. \$1.70. | $1\frac{1}{2}$ cups. |
| 27. \$4.50; $\frac{1}{3}$. | $586\frac{4}{4}\frac{7}{8}$ lb. | 45. \$1.57. | 54. \$0.61. |
| 28. \$339; $\frac{1}{2}\frac{1}{4}\frac{3}{5}$. | 36. 38,994 $\frac{9}{6}\frac{4}{4}$ lb. | 46. \$7.29. | 55. 312. |
| 29. \$2.43. | 37. \$7.94. | 47. 196. | 56. $7\frac{1}{7}\phi$. |
| 30. $\frac{1}{2}\frac{2}{16}\frac{5}{6}$. | 38. $8\frac{1}{2}$ mi. | 48. $\frac{3}{8}$; 16. | 57. 1728. |

Exercise 125. Page 232

- | | | | | |
|---------------------|-----------------------|----------------------|---------------------|------------------|
| 1. $\frac{1}{10}$. | 6. $\frac{1}{25}$. | 11. 1. | 16. $\frac{1}{3}$. | 21. 7; 7; 70. |
| 2. $\frac{1}{9}$. | 7. $\frac{1}{20}$. | 12. 2. | 17. $\frac{3}{8}$. | 22. 10; 10; 100. |
| 3. $\frac{3}{10}$. | 8. $\frac{3}{50}$. | 13. 3. | 18. $\frac{2}{3}$. | 23. 33; 33; 330. |
| 4. $\frac{2}{5}$. | 9. $\frac{2}{25}$. | 14. $1\frac{1}{2}$. | 19. $\frac{5}{8}$. | 24. 36; 36; 360. |
| 5. $\frac{1}{2}$. | 10. $\frac{1}{100}$. | 15. $2\frac{3}{4}$. | 20. $\frac{1}{6}$. | 25. 72; 72; 720. |

Exercise 126. Page 232

- | | | | | |
|----------|------------------------|-----------|------------|-------------------|
| 1. 0.25. | 6. 0.125. | 11. 1.25. | 16. 0.375. | 21. 0.5; 0.005. |
| 2. 0.3. | 7. $0.33\frac{1}{3}$. | 12. 1.5. | 17. 0.165. | 22. 0.25; 0.0025. |
| 3. 0.5. | 8. 0.375. | 13. 1.75. | 18. 0.348. | |
| 4. 0.75. | 9. $0.66\frac{2}{3}$. | 14. 2.5. | 19. 0.625. | |
| 5. 0.8. | 10. 0.875. | 15. 3.75. | 20. 0.783. | |

Exercise 127. Page 233

- | | | | | | |
|---------|---------|----------|-----------|-------------------------|------------------------------|
| 1. 25%. | 3. 50%. | 5. 150%. | 7. 37.5%. | 9. $33\frac{1}{3}\%$. | 11. 50%; $12\frac{1}{2}\%$. |
| 2. 75%. | 4. 80%. | 6. 250%. | 8. 62.5%. | 10. $66\frac{2}{3}\%$. | 12. 10%; 1000%. |

Exercise 128. Page 233

- | | | | | | |
|---------|---------|------------------------|-------------------------|--------------------------|---|
| 1. 50%. | 3. 75%. | 5. 60%. | 7. $37\frac{1}{2}\%$. | 9. 375%. | 11. $33\frac{1}{3}\%$; $16\frac{2}{3}\%$. |
| 2. 25%. | 4. 20%. | 6. $12\frac{1}{2}\%$. | 8. $262\frac{1}{2}\%$. | 10. $687\frac{1}{2}\%$. | 12. $6\frac{1}{4}\%$; $3\frac{1}{8}\%$. |

Exercise 129. Page 234

- | | | | |
|-------------|---------------------|---|-------------------------|
| 1. \$137. | 10. \$59.25. | 19. \$2700; \$2700. | 27. $\frac{1}{2}$; 15. |
| 2. \$93. | 11. \$147.03. | 20. \$5047; \$5047. | 28. 0.52; 234; 216. |
| 3. \$64. | 12. \$244.50. | 21. $\frac{1}{12}$; $8\frac{1}{3}\%$. | 29. 0.225; 4140. |
| 4. \$211. | 13. \$440.49. | 22. $\frac{1}{8}$; $12\frac{1}{2}\%$. | 30. 630 men; |
| 5. \$6.66. | 14. 81; 99. | 23. $\frac{1}{20}$; 5%. | 840 women; |
| 6. \$7.45. | 15. 132; 121. | 24. $\frac{1}{3}$. | 210 boys. |
| 7. \$30.17. | 16. 84; 112. | 25. 20%. | 31. \$3400. |
| 8. \$23.10. | 17. \$330; \$330. | 26. 25%; 50%; | |
| 9. \$53.38. | 18. \$1260; \$1260. | 75%. | |

Exercise 130. Page 236

- | | | | | |
|------------|-------------|----------|--------------------------|------------------------------------|
| 1. 8. | 7. \$2. | 13. 18. | 19. $57\frac{3}{5}$. | 25. 20. |
| 2. 12. | 8. \$4. | 14. 105. | 20. $8\frac{2}{5}$. | 26. $1480\frac{5}{8}$ lb. |
| 3. 72. | 9. \$2.70. | 15. 60. | 21. $235\frac{1}{5}$ lb. | 27. 4.77 ft. |
| 4. 9. | 10. \$3.60. | 16. 33. | 22. 60,000 lb. | 28. 39.37. |
| 5. 16. | 11. 22. | 17. 12. | 23. 225. | 29. 6082.56. |
| 6. \$4.50. | 12. 66. | 18. 49. | 24. 25.52. | 30. 14; 10 in.; $163\frac{1}{3}$. |

Exercise 131. Page 238

- | | | | | |
|------------------------|-------------------------|-------------------------------|-------------------------|---|
| 1. 60%. | 7. $16\frac{2}{3}\%$. | 13. 5%. | 19. 11%. | 25. $37\frac{1}{2}\%$. |
| 2. 50%. | 8. $14\frac{2}{7}\%$. | 14. 3%. | 20. 5%; 10%. | 26. 8%; 92%. |
| 3. $33\frac{1}{3}\%$. | 9. $33\frac{1}{3}\%$. | 15. 5%. | 21. $37\frac{1}{2}\%$. | 27. $53\frac{1}{3}\%$; $46\frac{2}{3}\%$. |
| 4. 20%. | 10. $20\frac{1}{2}\%$. | 16. \$25; $16\frac{2}{3}\%$. | 22. $6\frac{1}{4}\%$. | 28. $12\frac{1}{2}\%$. |
| 5. 20%. | 11. 5%. | 17. \$650; 10%. | 23. 16%. | 29. $12\frac{1}{2}\%$. |
| 6. 6%. | 12. 6%. | 18. 10%. | 24. 8%. | 30. 25%; 20%;
25%. |

Exercise 132. Page 240

- | | | | | |
|----------|------------|-------------|---------------------|---------------------------|
| 1. 6000. | 7. \$1400. | 13. \$250. | 19. 23,800. | 24. 20; $17\frac{1}{2}$. |
| 2. 2000. | 8. \$1500. | 14. \$400. | 20. 32,500; | 25. February. |
| 3. 3000. | 9. \$2200. | 15. \$500. | 37,700. | 26. \$3240. |
| 4. 700. | 10. \$60. | 16. \$1200. | 21. \$2400. | 27. 40. |
| 5. 480. | 11. \$360. | 17. \$7250. | 22. 280. | 28. \$7500. |
| 6. 3840. | 12. \$400. | 18. \$375. | 23. \$9.50; \$8.55. | 29. 840. |

Exercise 134. Page 244

- | | | | |
|---------------|------------------|---------------|---------------|
| 1. \$33. | 16. \$867.30. | 31. \$165. | 46. \$75. |
| 2. \$50.40. | 17. \$560. | 32. \$623. | 47. \$82. |
| 3. \$86.25. | 18. \$720. | 33. \$810. | 48. \$175. |
| 4. \$28. | 19. \$810. | 34. \$675. | 49. \$825. |
| 5. \$41.04. | 20. \$194.40. | 35. \$735. | 50. \$840. |
| 6. \$74.75. | 21. \$435.40. | 36. \$530. | 51. \$575. |
| 7. \$84.96. | 22. \$53,521.25. | 37. \$780. | 52. \$450. |
| 8. \$106.25. | 23. \$33,519 50. | 38. \$720. | 53. \$969.60. |
| 9. \$270. | 24. \$94,062.50. | 39. \$654. | 54. \$842.16. |
| 10. \$431.25. | 25. \$51. | 40. \$169.05. | 55. \$296. |
| 11. \$783. | 26. \$69.70. | 41. \$21,800. | 56. \$320. |
| 12. \$189.88. | 27. \$56.16. | 42. \$18,480. | 57. \$460. |
| 13. \$207.58. | 28. \$67.50. | 43. \$75. | 58. \$720. |
| 14. \$280. | 29. \$57.20. | 44. \$82. | 59. \$260. |
| 15. \$661.25. | 30. \$81.60. | 45. \$68. | 60. \$14,725. |

61. \$15,000.	77. \$1402.01.	93. 20%; 75¢.	108. \$223.20.
62. \$16,500.	78. \$623.70.	94. $38\frac{8}{9}\%$.	109. \$150.
63. \$16,800.	79. \$16.50.	95. \$1.	110. \$400.
64. \$41.25.	80. \$13.80.	96. \$1.20; \$1.50;	111. 50%.
65. \$45.63.	81. \$37.04.	\$0.30; 25%.	112. \$800, loss.
66. \$661.22.	82. \$14.50.	97. 25%.	113. \$450.
67. \$634.40.	83. \$34.20.	98. $45\frac{5}{11}\%$.	114. 10%.
68. \$1603.14.	84. \$87.87.	99. C C X.	115. 25%.
69. \$41.31.	85. \$378.02.	100. A C H X; H R Y.	116. 40%.
70. \$55.44.	86. \$315.90.	101. \$262.50.	117. Neither.
71. \$88.70.	87. \$625.53.	102. \$360; 50%.	118. \$112.50, loss.
72. \$138.12.	88. \$461.50.	103. \$10; $33\frac{1}{3}\%$.	119. \$50, loss.
73. \$130.	89. 40%, gain.	104. \$337.50.	120. \$3180.
74. \$342.04.	90. \$1006.25.	105. \$3.15.	121. The same.
75. \$352.37.	91. \$6000.	106. \$15.40.	122. $26\frac{9}{16}\%$.
76. \$524.12.	92. \$4647.50.	107. \$2.40, loss.	

Exercise 135. Page 250

1. $3\frac{1}{3}\%$.	14. $66\frac{2}{3}\%$; 50%;	26. 145.	39. 3800.
2. 25%.	58 $\frac{1}{3}\%$.	27. Yes.	40. $9\frac{5}{7}\%$.
3. $13\frac{1}{3}\%$.	15. \$125.	28. \$90.	41. 390.
4. 5%.	16. \$300.	29. \$6250.	42. 52 ft.
5. $11\frac{1}{9}\%$.	17. \$150.	30. 120%; \$50.	43. 65.
6. 20%.	18. 250 ft.	31. 140; 259.	44. \$1500.
7. 60%.	19. \$350.	32. \$451.	45. 19.18 mi.
8. $33\frac{1}{3}\%$.	20. 625.	33. \$1250.	46. 54.
9. $16\frac{2}{3}\%$.	21. 30.	34. 150.	47. \$450; \$900.
10. $12\frac{1}{2}\%$.	22. 119.	35. 55 lb.	48. 84 words.
11. 58%.	23. \$337.	36. 61,000.	49. 76 words.
12. 48%; 52%.	24. \$2812.50.	37. 7134.	50. 0.826%.
13. 75%.	25. \$2110.	38. \$76.50.	51. \$1512.
			52. \$280.80.

Exercise 136. Page 255

1. \$25.	7. \$25.	13. \$437.	19. \$43.75.
2. \$75.	8. \$112.	14. \$492.80.	20. \$736.
3. \$67.50.	9. \$270.	15. \$658.75.	21. \$780.20.
4. \$23.	10. \$200.	16. \$400.	22. \$37.50.
5. \$40.50.	11. \$157.50.	17. \$630.	23. \$3.
6. \$24.75.	12. \$324.	18. \$540.	24. 10%.

25. \$90.	33. \$520.	42. 25%.	50. \$617.63.
26. \$74.40.	34. \$16.40.	43. \$385.40.	51. \$1750.
27. \$150.	35. \$39.60.	44. 6%.	52. 8%.
28. \$2115.	36. \$385.	45. \$16.68.	53. The first ;
29. 8%.	37. \$1911.	46. \$238.70.	\$0.42.
30. \$1350 ;	38. \$41.82.	47. \$66.64.	54. The second ;
\$1255.50.	39. \$0.12.	48. \$770 ; \$825 ;	\$0.38.
31. 9%.	40. \$47.25.	\$660 ; \$858.	55. The second ;
32. 25%.	41. \$0.03.	49. \$1509.75.	\$0.14.

Exercise 137. Page 259

1. \$127.93.	5. \$16.12.	9. \$280.87.	13. \$167.96.
2. \$372.97.	6. \$154.79.	10. \$396.55.	14. \$481.01.
3. \$160.06.	7. \$13.58.	11. \$403.36.	15. \$226.41.
4. \$75.02.	8. \$984.50.	12. \$222.86.	16. \$563.77.
			17. \$463.24.

Exercise 138. Page 262

1. \$169.20.	7. \$307.80.	13. \$194.40.	18. \$528.
2. \$262.20.	8. \$325.92.	14. \$51.84.	19. \$119.88.
3. \$407.40.	9. \$393.66.	15. \$51.84.	20. \$144.
4. \$285.60.	10. \$170.10.	16. 20% and 10%	21. \$480.
5. \$378.	11. \$1324.80.	by 15¢.	22. \$594.81.
6. \$162.	12. \$2358.75.	17. \$40.50.	

Exercise 139. Page 264

1. \$126.	5. \$846.45.	9. \$405.	13. \$463.73.
2. \$907.20.	6. \$24.37.	10. \$121.87.	14. \$194.09.
3. \$164.45.	7. \$1486.75.	11. \$310.50.	15. \$3317.28.
4. \$53.82.	8. \$65.87.	12. \$168.47.	16. \$3037.73.
			17. \$455.43.

Exercise 140. Page 266

1. 17.2%.	6. 37%.	11. 49.6%.	16. $40\frac{1}{8}\%$.
2. 20.1%.	7. $34\frac{3}{8}\%$.	12. 43%.	17. They are equal.
3. $20.4\frac{2}{3}\%$.	8. 27.1%.	13. 46%.	18. 24%.
4. 21.74%.	9. 42.625%.	14. $32\frac{1}{2}\%$.	19. They are equal.
5. 28%.	10. 46%.	15. 37%.	

Exercise 141. Page 267

- | | | |
|--------------|-------------------|----------------------------------|
| 1. \$733.59. | 5. 46.199%. | 9. The single discount; \$4.17. |
| 2. \$559.55. | 6. \$650. | 10. The trade discounts; \$0.30. |
| 3. \$361.42. | 7. \$357.20; 24%. | 11. The same. |
| 4. \$10.25. | 8. 58.96%. | |

Exercise 143. Page 270

- | | | | |
|---------------|-------------------------|------------------------|-------------------------|
| 1. \$5.50. | 29. \$576. | 57. \$117. | 82. $11\frac{1}{5}\%$. |
| 2. \$10.35. | 30. \$807.50. | 58. \$906.95. | 83. 5%. |
| 3. \$27. | 31. \$1193.75. | 59. \$906.95. | 84. 227,500 ; |
| 4. \$36.25. | 32. \$2737. | 60. 3%. | \$23,798.25. |
| 5. \$52.50. | 33. \$5481. | 61. \$935; 3%. | 85. 3%. |
| 6. \$82.50. | 34. \$6176. | 62. \$473.60. | 86. $3\frac{1}{2}\%$. |
| 7. \$217.50. | 35. \$1477.50. | 63. 5%. | 87. \$4.75. |
| 8. \$373. | 36. \$2172.50. | 64. \$21.48. | 88. \$308.14. |
| 9. \$41.25. | 37. \$3537. | 65. \$168. | 89. 5%. |
| 10. \$81.88. | 38. \$4620. | 66. $5\frac{1}{3}\%$. | 90. \$12,500, |
| 11. \$96.60. | 39. \$264.48. | 67. \$4.88. | 91. \$597.71. |
| 12. \$187.88. | 40. \$563.50. | 68. \$66.70. | 92. \$1680. |
| 13. \$212.50. | 41. \$678.21. | 69. \$41.11. | 93. \$3162 ; \$15.81 |
| 14. \$327. | 42. \$5593. | 70. \$4750. | 94. 350,000 ; |
| 15. \$519. | 43. \$4189.50. | 71. 8%. | \$42,385. |
| 16. \$622. | 44. \$3188. | 72. 3%. | 95. The amounts |
| 17. \$10. | 45. \$6360. | 73. \$25.53. | are equal. |
| 18. \$18.75. | 46. \$1140. | 74. \$93.75. | 96. \$11.44. |
| 19. \$31. | 47. \$3104. | 75. \$18.75. | 97. The amounts |
| 20. \$13.20. | 48. \$4200. | 76. \$824.50. | are equal. |
| 21. \$54. | 49. \$3000. | 77. \$42.50. | 98. \$17.38. |
| 22. \$100. | 50. \$247. | 78. 255,000 ; | 99. \$784. |
| 23. \$30. | 51. 15%. | \$18.75. | 100. Equal ; \$300. |
| 24. \$120. | 52. \$44.45. | 79. \$8787.50 ; | 101. \$109.20 ; |
| 25. \$4794. | 53. $2\frac{1}{2}\%$. | 425,000 ; | \$153.40. |
| 26. \$6384. | 54. $2\frac{1}{2}\%$. | \$31.25. | 102. \$253.20. |
| 27. \$3493. | 55. $12\frac{1}{2}\%$. | 80. \$786.60. | |
| 28. \$392. | 56. \$186.30. | 81. \$0.21. | |

Exercise 145. Page 278

- | | | | | |
|---------------------------------------|---------------|-----------------|--------------------------------------|---------------|
| 1. \$12. | 15. \$63.75. | 29. \$107.25. | 40. \$24; \$48; | 47. \$60. |
| 2. \$12. | 16. \$152.40. | 30. \$97.88. | \$12; \$36; | 48. \$50. |
| 3. \$12. | 17. \$105. | 31. \$58.45. | \$60. | 49. \$52.50. |
| 4. \$36. | 18. \$231.25. | 32. \$63.83. | 41. \$12; \$1; | 50. \$52.50. |
| 5. \$36. | 19. \$15. | 33. \$78.91. | \$2; \$7. | 51. \$85. |
| 6. \$36. | 20. \$12.50. | 34. \$47.31. | 42. \$36; \$3; \$9; | 52. \$164. |
| 7. \$30. | 21. \$39. | 35. \$367.50. | \$30; \$24. | 53. \$101.25. |
| 8. \$45. | 22. \$22.50. | 36. \$491.56. | 43. \$60; | 54. \$114. |
| 9. \$195. | 23. \$56.25. | 37. \$15. | \$72.50; | 55. \$95. |
| 10. \$27.50. | 24. \$87.75. | 38. \$30. | \$72.50; | 56. \$156. |
| 11. \$42. | 25. \$79.20. | 39. \$24; \$48; | \$77.50. | 57. \$195. |
| 12. \$55.20. | 26. \$121.50. | \$12; | 44. \$29. | 58. \$247.50. |
| 13. \$78.75. | 27. \$88. | \$36; | 45. \$29. | 59. \$206.25. |
| 14. \$21.20. | 28. \$78. | \$60. | 46. \$54. | 60. \$396. |
| 61. \$342. | | | 66. \$23.46; \$46.92; \$164.22. | |
| 62. \$285. | | | 67. \$3.04; \$15.20; \$139.84. | |
| 63. \$637.50. | | | 68. \$21.60; \$1.80; \$0.90; \$4.50; | |
| 64. \$26.16; \$13.08; \$8.72; | | | \$9.60. | |
| \$17.44; \$100.28. | | | 69. \$7.02; \$3.51; \$2.34; \$1.76; | |
| 65. \$18; \$9; \$15; \$1.50; \$49.50. | | | \$0.59; \$1.56. | |

Exercise 146. Page 280

- | | | | |
|-------------|--------------|--------------|---------------|
| 1. \$15. | 5. \$55.83. | 9. \$152.78. | 13. \$174.74. |
| 2. \$15.17. | 6. \$19.71. | 10. \$76.88. | 14. \$240. |
| 3. \$20.25. | 7. \$52.34. | 11. \$93.68. | 15. \$207.81. |
| 4. \$33.89. | 8. \$103.95. | 12. \$74.53. | 16. \$442.36. |

Exercise 147. Page 281

- | | | | |
|-----------------------|-----------------------|--------------|---|
| 1. 2 yr. 1 mo. 13 da. | 4. 3 yr. 1 mo. 25 da. | 7. \$92.08. | 10. ~~\$52.13~~ ^{\$57.13} |
| 2. 5 yr. 3 mo. 20 da. | 5. 4 yr. 4 mo. 16 da. | 8. \$139.22. | 11. \$48.33. |
| 3. 5 yr. 9 mo. 27 da. | 6. \$67.50. | 9. \$39.48. | 12. \$745.88. |

Exercise 148. Page 283

- | | | | |
|-----------------------------|---------------|---------------|----------------|
| 1. Sept. 5, 1913; \$303. | 8. \$10. | 15. \$825. | 22. \$261.25. |
| 2. July 4, 1913; \$512.50. | 9. \$11.25. | 16. \$78.75. | 23. \$372.75. |
| 3. Feb. 8, 1914; \$418. | 10. \$201.67. | 17. \$293.75. | 24. \$546.75. |
| 4. June 7, 1913; \$762.77. | 11. \$1218. | 18. \$133.75. | 25. \$6564.92. |
| 5. July 14, 1914; \$533.13. | 12. \$394.28. | 19. \$282.08. | |
| 6. June 15, 1914; \$256.57. | 13. \$371. | 20. \$369.69. | |
| 7. \$12. | 14. \$386.25. | 21. \$311.25. | |

Exercise 149. Page 286

- | | | | |
|-------------|-----------------|-----------------|-------------------------|
| 1. \$125. | 6. \$240. | 11. 2 yr. 3 mo. | 16. 6%. |
| 2. \$560. | 7. \$402. | 12. 2 yr. 4 mo. | 17. $16\frac{2}{3}$ yr. |
| 3. \$68.50. | 8. \$1042. | 13. 2 yr. 6 mo. | 18. 20 yr. |
| 4. \$426. | 9. \$1080. | 14. 5%. | 19. \$500. |
| 5. \$130. | 10. 3 yr. 6 mo. | 15. 4%. | 20. \$500. |

Exercise 150. Page 288

- | | | | |
|-----------|------------|------------|---------------|
| 1. \$500. | 9. 4%. | 17. \$700. | 25. \$400. |
| 2. 4%. | 10. 4 yr. | 18. 5%. | 26. 4%. |
| 3. 2 yr. | 11. \$105. | 19. \$84. | 27. 3 yr. |
| 4. 5%. | 12. \$200. | 20. 3 yr. | 28. \$324.50. |
| 5. \$400. | 13. 6%. | 21. \$200. | 29. 2 yr. |
| 6. 2 yr. | 14. 3 yr. | 22. 5%. | 30. 5%. |
| 7. \$65. | 15. \$64. | 23. \$120. | 31. 6%. |
| 8. \$200. | 16. \$60. | 24. 4 yr. | 32. \$316.25. |

Exercise 151. Page 289

- | | | |
|-------------------------------|-----------------------|------------------------|
| 1. \$4.51; \$9.03. | 22. \$63.56. | 43. 1 yr. 8 mo. |
| 2. \$85.80; \$87.97. | 23. 5%. | 44. 1 yr. 6 mo. |
| 3. \$171; \$172.58. | 24. 4%. | 45. 2 yr. |
| 4. \$188.13; \$190.46. | 25. 4%. | 46. 4 mo. |
| 5. \$114.58; \$115.45. | 26. 6%. | 47. 25 yr. |
| 6. \$389.58; \$392.26. | 27. 4%. | 48. 2 yr. 6 mo. |
| 7. \$490.07; \$490.77. | 28. 3%. | 49. 2 yr. 6 mo. |
| 8. \$304.03; \$305.35. | 29. 3%. | 50. 6 mo.; 2 yr. |
| 9. \$180.25; \$182; \$187.25. | 30. 6%. | 1 mo. 28 da.; |
| 10. \$243.11; \$247.61. | 31. 5%. | 2 yr. 7 mo. 28 da. |
| 11. \$635.68; \$698.18. | 32. 4%. | 51. 11 yr. |
| 12. \$1267.01; \$1392.01. | 33. 4%. | 52. 14 yr. |
| 13. \$2384.14; \$2759.51. | 34. $3\frac{1}{2}$ %. | 53. The first; \$3.13. |
| 14. \$48.13. | 35. $5\frac{1}{2}$ %. | 54. 5%. |
| 15. \$593.75. | 36. $3\frac{1}{2}$ %. | 55. \$30. |
| 16. \$88.73. | 37. 2 yr. 6 mo. | 56. \$55. |
| 17. \$38.96. | 38. 2 yr. 7 mo. | 57. \$405. |
| 18. \$21.95. | 39. 1 yr. 9 mo. | 58. \$1035. |
| 19. \$25.08. | 40. 1 yr. 5 mo. | 59. \$3327.50. |
| 20. \$105.08. | 3 da. | 60. \$5302.50. |
| 21. \$268.54; \$219.71; | 41. 1 yr. 2 da. | 61. $19\frac{9}{5}$ %. |
| \$292.95. | 42. 2 yr. 6 mo. | |

Exercise 153. Page 293

1. \$5.04.	5. \$11.48.	9. \$14.03.	13. \$35.55.
2. \$5.58.	6. \$14.04.	10. \$51.73.	14. \$143.49.
3. \$11.63.	7. \$5.63.	11. \$96.75.	15. \$255.75.
4. \$7.60.	8. \$7.80.	12. \$43.75.	16. \$331.25.

Exercise 155. Page 296

1. \$10.40.	6. \$41.25.	11. \$16.63.	16. \$38.18.
2. \$8.60.	7. \$1.67.	12. \$20.	17. \$1.13.
3. \$6.27.	8. \$2.35.	13. \$2.52.	18. \$12.19.
4. \$10.50.	9. \$4.76.	14. \$10.81.	19. \$18.56.
5. \$19.38.	10. \$16.67.	15. \$21.66.	20. \$28.91.

Exercise 156. Page 297

1. 92.	5. 174.	9. 243.	13. 162.
2. 153.	6. 188.	10. 186.	14. 141.
3. 227.	7. 104.	11. 122.	15. 191.
4. 220.	8. 48.	12. 141.	16. 235.

Exercise 157. Page 298

1. \$8.92.	8. \$7.13.	15. \$2.74.	22. \$19.88.
2. \$10.13.	9. \$9.60.	16. \$7.92.	23. \$19.67.
3. \$8.80.	10. \$25.20.	17. \$12.31.	24. \$12.59.
4. \$18.75.	11. \$3.85.	18. \$7.80.	25. \$7.14.
5. \$7.75.	12. \$7.13.	19. \$11.07.	26. \$10.79.
6. \$11.25.	13. \$15.87.	20. \$6.03.	27. \$9.15.
7. \$14.79.	14. \$18.	21. \$8.37.	28. \$14.88.

Exercise 158. Page 299

1. \$40.49.	4. \$39.48.	7. \$76.97.	10. \$35.27.
2. \$49.14.	5. \$51.46.	8. \$58.51.	11. \$56.21.
3. \$63.75.	6. \$62.10.	9. \$236.25.	12. \$65.88.

Exercise 159. Page 300

- | | | | |
|------------|-------------|--------------|---------------|
| 1. \$3.08. | 5. \$14.67. | 9. \$4.80. | 13. \$76.23. |
| 2. \$3.20. | 6. \$11.34. | 10. \$20.10. | 14. \$236.50. |
| 3. \$4.01. | 7. \$17.67. | 11. \$19.20. | 15. \$110.25. |
| 4. \$5.99. | 8. \$4.50. | 12. \$48. | |

Exercise 160. Page 302

- | | | | | |
|--------------|--------------|---------------|--------------|------------|
| 1. \$71.97. | 3. \$487.25. | 5. \$1538.21. | 7. \$534. | 9. \$1.80. |
| 2. \$182.70. | 4. \$347.43. | 6. \$378.44. | 8. \$406.98. | |

Exercise 161. Page 304

- | | | |
|--------------|--------------|--------------|
| 1. \$147.33. | 3. \$231.48. | 5. \$26.05. |
| 2. \$142.43. | 4. \$161.04. | 6. \$254.40. |

Exercise 162. Page 305

- | | | |
|--|----------------|------------------------|
| 1. \$18.83. | 9. \$261. | 19. \$750. |
| 2. \$2.40. | 10. \$432.39. | 20. \$840. |
| 3. \$28.08. | 11. \$10.90. | 21. 4 yr. 11 mo. 6 da. |
| 4. \$75.50; \$37.75;
\$113.25;
\$117.03. | 12. \$29.90 | 22. 3 yr. 4 mo. |
| 5. \$23.56. | 13. \$277. | 23. 1 yr. 7 mo. 6 da. |
| 6. \$26.18. | 14. \$1151.16. | 24. 7 yr. 9 mo. 15 da. |
| 7. \$1484.96. | 15. \$775. | 25. 5 yr. 7 mo. 9 da. |
| 8. \$45.44. | 16. \$240. | 26. 3%. |
| | 17. \$350. | 27. $4\frac{1}{2}\%$. |
| | 18. \$450. | 28. 6%. |

Exercise 163. Page 308

- | | | |
|--------------|--------------|----------------|
| 1. \$622. | 5. \$774.40. | 9. \$799.40. |
| 2. \$725.12. | 6. \$682.67. | 10. \$324.45. |
| 3. \$936.25. | 7. \$626.90. | 11. \$321.25. |
| 4. \$1180. | 8. \$547.52. | 12. \$631.27. |
| | | 13. \$1802.84. |

Exercise 164. Page 310

- | | | | | |
|--------------|--------------|--------------|--------------|--------------|
| 3. \$158.42. | 4. \$308.71. | 5. \$706.49. | 6. \$813.96. | 7. \$449.89. |
|--------------|--------------|--------------|--------------|--------------|

Exercise 165. Page 312

- | | | |
|-------------------------|-------------------------|----------------|
| 1. \$1.25 ; \$248.75. | 6. \$0.73 ; \$174.27. | 11. \$1237.50. |
| 2. \$2.88 ; \$572.12. | 7. \$11.69 ; \$1263.31. | 12. \$714.12. |
| 3. \$6.25 ; \$618.75. | 8. \$21.54 ; \$2328.46. | 13. \$7.08. |
| 4. \$10.42 ; \$1239.58. | 9. \$4.13 ; \$271.37. | |
| 5. \$18.75 ; \$2231.25. | 10. \$7.91 ; \$519.59. | |

Exercise 166. Page 313

- | | | |
|-------------------------|------------------------|-------------------------|
| 1. \$7.13 ; \$249.12. | 7. \$4.92 ; \$378.09. | 13. \$6.25 ; \$2493.75. |
| 2. \$5.82 ; \$373.87. | 8. \$17.55 ; \$838.65. | 14. \$1653.92. |
| 3. \$8.89 ; \$424.61. | 9. \$12.25. | 15. \$1660.79. |
| 4. \$7.46 ; \$551.71. | 10. \$12.35. | 16. \$1649.63. |
| 5. \$16.03 ; \$679.22. | 11. \$10.31. | 17. \$1625.25. |
| 6. \$14.95 ; \$1266.30. | 12. \$6.19. | 18. \$6.75. |
19. On the 4-months' note ; \$0.30.

Exercise 167. Page 316

- | | | | |
|---------------|---------------|------------------------|---------------|
| 1. \$971.10. | 5. \$1656.42. | 9. \$2059.20. | 14. \$249.75. |
| 2. \$348.42. | 6. \$2140.75. | 10. $\frac{4}{15}\%$. | 15. \$99.90. |
| 3. \$718.35. | 7. \$615.13. | 11. \$74.85. | 16. \$0.75. |
| 4. \$1214.72. | 8. \$1480.92. | 13. $\frac{1}{20}\%$. | |

Exercise 168. Page 318

- | | | |
|--------------|---------------|---------------|
| 1. \$502.53. | 3. \$502.51. | 5. \$2062.31. |
| 2. \$1800. | 4. \$1772.73. | 6. \$2836.14. |

Exercise 169. Page 319

- | | | |
|---------------|---------------|----------------|
| 1. \$281.22. | 7. \$1591.20. | 13. \$710.74. |
| 2. \$438.69. | 8. \$3040.51. | 14. \$1137.32. |
| 3. \$478.07. | 9. \$3042.50. | 15. \$1454.32. |
| 4. \$790.83. | 10. \$112.65. | 16. \$2774.60. |
| 5. \$1107.16. | 11. \$273.36. | 17. \$1886.59. |
| 6. \$1379.04. | 12. \$610.29. | 18. \$1943.75. |

Exercise 170. Page 322

- | | | | |
|---------------|------------------------|------------------------|------------------------|
| 1. \$97.07. | 5. \$888.80. | 9. \$2098.86. | 13. 5%. |
| 2. \$369.11. | 6. 7%. | 10. 4%. | 14. $4\frac{1}{2}\%$. |
| 3. \$1497.27. | 7. $10\frac{1}{2}$ yr. | 11. \$2000. | 15. $2\frac{1}{2}\%$. |
| 4. \$1482.29. | 8. 13 yr. | 12. $9\frac{1}{2}$ yr. | |

Exercise 171. Page 324

- | | | |
|--------------|---------------|---------------|
| 1. \$970.75. | 3. \$518.18. | 5. \$1572.38. |
| 2. \$873.53. | 4. \$1087.20. | 6. \$59.60. |

Exercise 172. Page 325

- | | | | | |
|---|---------------|---------------|---------------|--------------|
| 1. \$609.49. | 2. \$1917.86. | 3. \$1505.19. | 4. \$2343.20. | 5. \$146.25. |
| 6. Proceeds of the \$750 note; \$13.25. | | | | |

Exercise 174. Page 328

- | | | | | |
|---------------------|---------------------|---------|---------------------------|-----------------|
| 1. $\frac{2}{3}$. | 7. 4. | 13. 3. | 19. 12; 16. | 25. \$18; \$60. |
| 2. $\frac{3}{4}$. | 8. $\frac{1}{4}$. | 14. 20. | 20. 20; 25. | 26. 165; 176. |
| 3. $\frac{2}{5}$. | 9. $\frac{2}{5}$. | 15. 6. | 21. 100; 160. | 27. 132; 360. |
| 4. $\frac{8}{3}$. | 10. $\frac{3}{4}$. | 16. 35. | 22. $10\frac{1}{2}$; 21. | 28. \$14; \$16. |
| 5. $\frac{2}{15}$. | 11. 8. | 17. 18. | 23. \$8; \$64. | 29. \$8980. |
| 6. $\frac{4}{7}$. | 12. $\frac{3}{4}$. | 18. 14. | 24. \$36; \$60. | |

Exercise 175. Page 329

- | | | | |
|-----------------------|-------------|-----------------------|-------------------|
| 1. $211\frac{1}{2}$. | 3. \$1125. | 5. \$1650; \$2100. | 7. 16. |
| 2. 129.6. | 4. 150; 65. | 6. $187\frac{1}{2}$. | 8. \$896; \$1344. |
| | | | 9. \$28; \$56. |

Exercise 176. Page 331

- | | | | |
|---------------------|---------|------------|----------------------|
| 1. 18. | 9. 18. | 17. \$11. | 25. 60. |
| 2. 9. | 10. 35. | 18. \$5. | 26. 12. |
| 3. 80. | 11. 27. | 19. 85. | 27. 15. |
| 4. 6. | 12. 14. | 20. 35. | 28. 10. |
| 5. 12. | 13. 63. | 21. 11 ft. | 29. 1250. |
| 6. 25. | 14. 63. | 22. 280. | 30. 5.02656. |
| 7. $6\frac{2}{3}$. | 15. 63. | 23. 40. | 33. $9\frac{3}{8}$. |
| 8. 30. | 16. 45. | 24. 60. | 34. 144. |
35. $401:311 = 6817:5287$; $5287:6817 = 311:401$;
 $401:6817 = 311:5287$.

Exercise 177. Page 333

- | | | | |
|--------------|-------------|-------------------------|-------------|
| 1. \$249.60. | 4. \$7.75. | 7. $266\frac{1}{4}$ yd. | 10. 39 da. |
| 2. \$15.12. | 5. \$196. | 8. \$20.70. | 11. 15. |
| 3. \$16.50. | 6. \$68.75. | 9. $112\frac{1}{2}$. | 12. 216 mi. |

Exercise 178. Page 334

- | | | | | | |
|---------|---------|-----------|------------|-----------|-----------|
| 1. 625. | 2. 1.7. | 3. 40 ft. | 4. 180 ft. | 5. 30 ft. | 6. 64 ft. |
|---------|---------|-----------|------------|-----------|-----------|

Exercise 179. Page 335

- | | | | |
|-----------------------|------------------------|---------------------------|---------------------------|
| 1. $4\frac{1}{4}$ oz. | 5. $16\frac{2}{3}$ lb. | 9. 50 lb. | 13. 750 lb. |
| 2. $8\frac{1}{3}$ oz. | 6. 60 lb. | 10. $181\frac{9}{11}$ lb. | 14. 225 lb. |
| 3. $5\frac{1}{7}$ lb. | 7. $37\frac{1}{2}$ lb. | 11. 60 lb. | 15. $1031\frac{1}{4}$ lb. |
| 4. $7\frac{1}{2}$ lb. | 8. 150 lb. | 12. 360 lb. | 16. $1462\frac{1}{2}$ lb. |

Exercise 180. Page 336

- | | | | |
|------------------------|------------------------|------------------------|-------------------------|
| 1. \$45; \$70. | 8. \$30; \$54. | 15. 20 ft. | 22. \$121.50. |
| 2. \$2100. | 9. 4 lb.; 3 lb.; 3 lb. | 16. 90%. | 23. \$40. |
| 3. 49 ft. | 10. \$154.70. | 17. No. | 24. $13\frac{5}{12}$ T. |
| 4. $12\frac{4}{7}$ ft. | 11. \$50. | 18. 1431 lb. | 25. 3625. |
| 5. $62\frac{1}{2}$. | 12. 750. | 19. \$67.50. | 26. 14 da. |
| 6. 14. | 13. 4680. | 20. $10\frac{4}{15}$. | 27. $38\frac{1}{4}$. |
| 7. 99. | 14. 315 gal. | 21. 28.405 in. | 28. $35\frac{3}{4}$. |
| | | | 29. 46,100 cu. ft. |

Exercise 181. Page 339

- | | |
|------------------------------------|-----------------------------|
| 1. 77; 99; 121. | 8. \$224; \$288; \$336. |
| 2. 46; 138; 69. | 9. \$160; \$200; \$180. |
| 3. \$1176; \$1470. | 10. \$21.60; \$21.60. |
| 4. \$936; \$1248; \$1560. | 11. \$1235; \$1558. |
| 5. \$1350; \$1650; \$1750; \$2000. | 12. \$9; \$6; \$7; \$10.50. |
| 6. \$67.50; \$57; \$100.50. | 13. \$600; \$420. |
| 7. \$11.64; \$14.07; \$13.05. | 14. \$12; \$12. |

Exercise 182. Page 341

- | | | | | |
|-----------------|-----------|------------|----------|-----------------------|
| 1. 38 ft. 6 in. | 2. 37 ft. | 3. \$5.50. | 4. 4 ft. | 5. $8\frac{1}{3}$ lb. |
|-----------------|-----------|------------|----------|-----------------------|

Exercise 184. Page 344

- | | | | |
|---|---------|--------------|-------------------------------|
| 58. 2.89 sq. ft. | 70. 28. | 82. 35. | 94. 96 in. |
| 59. 3.61 sq. ft. | 71. 66. | 83. 33. | 95. 288 in. |
| 60. 5.29 sq. ft. | 72. 64. | 84. 36. | 96. 19.2 in. |
| 61. 8.41 sq. in. | 73. 60. | 85. 1.2 in. | 97. 25.6 in. |
| 62. 13.69 sq. in. | 74. 63. | 86. 1.1 in. | 98. 32.4 ft. |
| 63. 18.49 sq. in. | 75. 55. | 87. 2.4 in. | 99. 42 ft. |
| 64. 0.3844 sq. yd. | 76. 44. | 88. 50 ft. | 100. 44 rd. |
| 65. 1.6129 sq. yd. | 77. 45. | 89. 60 yd. | 101. 80 rd. |
| 66. 24.3049 sq. yd. | 78. 54. | 90. 80 rd. | 102. 52. |
| 67. 0.64 sq. mi. | 79. 30. | 91. 0.3 mi. | 103. $141\frac{1}{4}$ sq. in. |
| 68. $\frac{49}{64}$ sq. in. | 80. 27. | 92. 0.9 ft. | 104. 1.7 ft. |
| 69. $1\frac{8}{10}\frac{1}{24}$ sq. ft. | 81. 32. | 93. 0.12 ft. | 105. 2.2 in.; 8.8 in. |

Exercise 185. Page 347

1. 29.	5. 41.	9. 43.	13. 83.
2. 31.	6. 39.	10. 91.	14. 73.
3. 23.	7. 57.	11. 99.	15. 22 in.
4. 37.	8. 61.	12. 79.	16. 87.

Exercise 186. Page 349

1. 437.	7. 441.	13. 0.251.	19. 0.427.
2. 862.	8. 416.	14. 1.23.	20. 0.234.
3. 634.	9. 418.	15. 1.62.	21. 111 ft.
4. 871.	10. 714.	16. 1.98.	22. 89.7 in.
5. 343.	11. 32.2.	17. 21.5.	23. 176.4 in.
6. 897.	12. 9.07.	18. 0.063.	24. 7.7 in. ; 456.533 cu. in.

Exercise 187. Page 352

1. $\frac{7}{12}$.	13. 3.32.	25. 0.94.	37. 8 ft.
2. $\frac{11}{13}$.	14. 3.87.	26. 0.79.	38. 8 in.
3. $\frac{15}{18}$.	15. 5.48.	27. 0.76.	39. 24 in.
4. $\frac{21}{25}$.	16. 7.07.	28. 0.77.	40. 13.748 ft.
5. $\frac{23}{27}$.	17. 11.18.	29. 0.65.	41. 13.416 ft.
6. $\frac{31}{33}$.	18. 25.50.	30. 0.96.	42. 19.621 in.
7. $\frac{37}{41}$.	19. 0.75.	31. 65 ft.	43. 28.284 in.
8. $\frac{43}{45}$.	20. 0.76.	32. 75 ft.	44. 45.255 in.
9. $\frac{47}{51}$.	21. 0.58.	33. 85 ft.	45. 63.640 in.
10. $\frac{53}{55}$.	22. 0.63.	34. 89.157 ft.	46. 98.995 in.
11. 1.41.	23. 0.82.	35. 38.601 ft.	47. 106.066 in.
12. 2.65.	24. 0.61.	36. 50.774 ft.	

Exercise 188. Page 353

1. 17.32 sq. in.	3. 5.33 sq. in.	5. 74.83 sq. in.	7. $68\frac{3}{4}$ sq. ft.
2. 23.53 sq. in.	4. 42.43 sq. in.	6. 176.56 sq. in.	8. \$4000.

Exercise 189. Page 354

1. 907.9224 sq. in.	8. 660.5214 sq. in.	15. 1.95 ft.
2. 2642.0856 sq. in.	9. 44.65124664 sq. in.	16. 11.28 in.
3. 4300.8504 sq. in.	10. 74.20333536 sq. ft.	17. 42.22 ft.
4. 87.25134264 sq. in.	11. 10 in.	18. 22.36 ft. ; 6283.2 sq. ft.
5. 115.75193784 sq. ft.	12. 6 in.	
6. 1134.1176 sq. in.	13. 8.92 in.	19. 23.1 sq. in. ; 2.71 in.
7. 1661.9064 sq. in.	14. 1.60 in.	

Exercise 190. Page 355

- | | | | | |
|-----------|--------------|--------------|------------|------------|
| 1. 346. | 4. 5.477. | 7. 21.21 ft. | 10. 8 in. | 13. 14 ft. |
| 2. 413. | 5. 1074.8. | 8. 6.08 ft. | 11. 7 ft. | 14. 35 ft. |
| 3. 2.449. | 6. 10.61 ft. | 9. 126. | 12. 35 ft. | |

Exercise 192. Page 357

- | | | | |
|----------------------------|----------------------|----------------------|-----------------------------------|
| 1. 1358. | 4. $18\frac{1}{3}$. | 7. $84\frac{2}{3}$. | 10. $13\frac{1}{2}\frac{9}{7}$. |
| 2. $633\frac{1}{3}$. | 5. $78\frac{5}{9}$. | 8. 404. | 11. $10\frac{5}{9}$. |
| 3. 480 ; $53\frac{1}{3}$. | 6. $67\frac{1}{9}$. | 9. $8\frac{1}{6}$. | 12. $48\frac{3}{8}\frac{2}{1}$. |
| | | | 13. $100\frac{1}{8}\frac{3}{8}$. |

Exercise 193. Page 359

- | | | | |
|--------|-------------|-------------------------------|--------------|
| 1. 14. | 4. 8. | 7. \$3.48 ; \$3.72 ; \$4.19 ; | 9. \$11.70. |
| 2. 14. | 5. \$4.90. | \$4.32 ; \$3.36 ; \$3.58. | 10. \$22.40. |
| 3. 8. | 6. \$10.50. | 8. \$8.40. | 11. \$1.95. |

Exercise 194. Page 361

- | | | | | |
|---------------------|--------------------------------|------------------------|-----------|------------------------|
| 1. $5\frac{1}{3}$. | 6. $13\frac{1}{3}$. | 11. 420. | 16. 400. | 21. $666\frac{2}{3}$. |
| 2. 7. | 7. $14\frac{1}{4}$. | 12. $533\frac{1}{3}$. | 17. 280. | 22. $746\frac{2}{3}$. |
| 3. 8. | 8. $8\frac{2}{3}$. | 13. 210. | 18. 1600. | 23. 3450. |
| 4. 12. | 9. $9\frac{1}{2}\frac{1}{2}$. | 14. 288. | 19. 960. | 24. \$2.30. |
| 5. 12. | 10. 160. | 15. 600. | 20. 240. | 25. \$11.52. |
| | | | | 26. \$19.44. |

Exercise 195. Page 362

- | | | | | | |
|---------|-----------------------|---------|------------------------|------------------------|--------------|
| 1. 336. | 4. $373\frac{1}{3}$. | 7. 512. | 10. 320. | 13. $453\frac{1}{3}$. | 16. \$17.55. |
| 2. 224. | 5. $426\frac{2}{3}$. | 8. 480. | 11. $277\frac{1}{3}$. | 14. 528. | 17. \$35.33. |
| 3. 256. | 6. $469\frac{1}{3}$. | 9. 576. | 12. 400. | 15. 640. | |

Exercise 196. Page 363

- | | | | | | |
|--------|--------|------------|-------------|-------------|-------------|
| 1. 12. | 2. 14. | 3. \$9.80. | 4. \$11.52. | 5. \$13.11. | 6. \$36.96. |
|--------|--------|------------|-------------|-------------|-------------|

Exercise 197. Page 364

- | | | | | |
|--------|--------|--------|---------------|---------------|
| 1. 54. | 4. 62. | 7. 62. | 10. \$64. | 13. \$158.68. |
| 2. 59. | 5. 63. | 8. 59. | 11. \$72.60. | |
| 3. 48. | 6. 61. | 9. 79. | 12. \$127.10. | |

Exercise 198. Page 365

- | | | | | |
|--------------|------------------------|-------------------------|------------------------------------|--------------|
| 1. \$56.25. | 5. \$83.20. | 9. $2847\frac{2}{9}$. | 13. $1396\frac{8}{7}$. | 17. \$936. |
| 2. \$108.90. | 6. \$160.42. | 10. $1726\frac{2}{3}$. | 14. $1951\frac{2}{7}\frac{3}{7}$. | 18. \$47.92. |
| 3. \$130.56. | 7. \$37.32. | 11. 1750. | 15. $1710\frac{5}{4}$. | |
| 4. \$99.75. | 8. $1477\frac{7}{9}$. | 12. $1013\frac{8}{9}$. | 16. \$33,250. | |

Exercise 199. Page 366

- | | | | |
|------------|---------------|-----------------------------------|------------------------|
| 1. \$7. | 8. \$41.92. | 15. \$276.75. | 22. $742\frac{1}{2}$. |
| 2. 4. | 9. \$801.36. | 16. \$433.65. | 23. 787.9872. |
| 3. 17. | 10. \$19.50. | 17. Lengthwise; $37\frac{1}{3}$. | 24. \$19.19. |
| 4. \$7.20. | 11. \$143.40. | 18. $852\frac{8}{9}$. | 25. \$148. |
| 5. 21. | 12. \$37.50. | 19. $2\frac{2}{3}$ yd. | 26. \$229.40. |
| 6. \$4.50. | 13. \$54.79. | 20. \$1.14 $\frac{2}{7}$. | 27. $93\frac{5}{9}$. |
| 7. 111—. | 14. \$17.10. | 21. 12; \$2.64. | |

Exercise 200. Page 369

- | | | | |
|-----------------------------|---------------------------|-----------------------------|-----------------|
| 1. $14\frac{7}{24}$ sq. ft. | 6. $6\frac{2}{3}$ ft. | 11. $6\frac{2}{3}$ sq. ft. | 16. 136 ft. |
| 2. $8\frac{8}{9}$ sq. yd. | 7. 3 ft. | 12. $12\frac{1}{6}$ sq. ft. | 17. 12 sq. in.; |
| 3. 750 sq. ft. | 8. $16\frac{8}{9}$ in. | 13. $13\frac{7}{9}$ in. | 6 sq. in.; |
| 4. 640 sq. in. | 9. $3\frac{2}{3}$ sq. ft. | 14. $32\frac{4}{33}$ ft. | 3 sq. in. |
| 5. 7 ft. | 10. 273 sq. in. | 15. $141\frac{1}{3}$ in. | |

Exercise 201. Page 371

- | | | |
|--------------------|--------------------|----------------------|
| 1. 78.54 sq. in. | 5. 1385.4456. | 9. 1017.8784 sq. ft. |
| 2. 38.4846 sq. in. | 6. 11.0447 sq. in. | 10. 0.0123 sq. in. |
| 3. 3.1416 sq. in. | 7. 12.5664 sq. in. | 11. 9.62115 sq. in. |
| 4. 1385.4456. | 8. 19.635 sq. in. | 12. 28.2744 sq. ft. |
| | | 13. 8.7267 sq. ft. |

Exercise 202. Page 372

- | | | |
|-----------------------------|----------------------|----------------------|
| 1. 63 sq. in. | 12. 2068 sq. in. | 23. 4461.875 sq. ft. |
| 2. 80 sq. in. | 13. 60 ft. | 24. 1623.75 sq. ft. |
| 3. 90 sq. in. | 14. 100 ft. | 25. 1082.5 sq. ft. |
| 4. $94\frac{1}{2}$ sq. in. | 15. 44,376 sq. ft. | 26. 2706.25 sq. ft. |
| 5. 76 sq. in. | 16. 60 sq. in. | 27. 4330 sq. ft. |
| 6. $66\frac{1}{2}$ sq. in. | 17. 125 sq. in. | 28. 2706.25 sq. ft. |
| 7. 286 sq. in. | 18. 60 sq. in. | 29. 2165 sq. ft. |
| 8. 287 sq. in. | 19. 90 sq. in. | 30. 88 sq. in. |
| 9. $560\frac{1}{2}$ sq. in. | 20. 4056.25 sq. ft. | 31. 54 sq. in. |
| 10. 1025 sq. in. | 21. 6084.375 sq. ft. | 32. 63 sq. in. |
| 11. 1218 sq. in. | 22. 9731.25 sq. ft. | |

Exercise 203. Page 374

- | | | |
|-----------------------------|---------------------------------------|-------------------|
| 1. 72 sq. in. | 4. $39\frac{2}{3}\frac{1}{2}$ sq. in. | 7. 189 sq. in. |
| 2. 96 sq. in.; 112 sq. in. | 5. 77 sq. in. | 8. 48 sq. ft. |
| 3. $365\frac{5}{8}$ sq. in. | 6. 84 sq. in. | 9. 1085.6 sq. in. |

- | | | |
|------------------------------|-----------------------------|-----------------------------|
| 10. $104\frac{5}{8}$ sq. in. | 17. $26\frac{1}{4}$ sq. ft. | 24. $16\frac{1}{3}$ sq. ft. |
| 11. 142. | 18. $73\frac{1}{2}$ sq. ft. | 25. $19\frac{1}{4}$ sq. ft. |
| 12. $7\frac{7}{8}$. | 19. $36\frac{1}{2}$ sq. ft. | 26. $37\frac{1}{8}$ sq. ft. |
| 13. 1350 sq. in. | 20. 46 sq. ft. | 27. $4\frac{7}{12}$ sq. in. |
| 14. 4374 sq. in. | 21. $31\frac{1}{2}$ sq. ft. | 28. 35 sq. in. |
| 15. $73\frac{1}{2}$ sq. in. | 22. 60 sq. ft. | 29. $38\frac{3}{4}$ sq. in. |
| 16. $135\frac{3}{8}$ sq. in. | 23. $46\frac{7}{8}$ sq. ft. | 30. $48\frac{3}{4}$ sq. in. |

Exercise 204. Page 376

- | | | | |
|-------------------------------------|---|------------------------|-----------------|
| 1. 250 ft. | 9. $4\frac{3}{8}$ A. | 17. 24 sq. in. | 22. 1120. |
| 2. 8 in.; 9 in.; $4\frac{3}{4}$ in. | 10. $4\frac{2}{5}$ A. | 18. $\frac{1}{6}$. | 23. 160 sq. ft. |
| 3. 180 ft.; 320 ft. | 14. $3\frac{3}{8}$ in.; $4\frac{1}{16}$ in. | 19. $9\frac{3}{4}$ in. | 24. 600 sq. ft. |
| 4. 22 ft.; 35 ft. | 15. $\frac{1}{4}$. | 20. 295. | 25. 600. |
| 5. $11\frac{1}{4}$ in. | 16. $\frac{1}{80}$. | 21. 100. | 26. 10,000. |

Exercise 205. Page 379

- | | | |
|------------------------------|-------------------------------|-------------------------|
| 1. $91\frac{7}{8}$ cu. in. | 22. 18,000 cu. ft. | 43. 380.92 cu. in. |
| 2. 120 cu. in. | 23. 23,600 cu. ft. | 44. 768.6875 cu. in. |
| 3. $218\frac{2}{3}$. | 24. 18,000 cu. ft. | 45. 866.264 cu. in. |
| 4. 216 cu. in. | 25. 56,700 cu. ft. | 46. 1118.43 cu. in. |
| 5. 1.728 cu. in. | 26. 810. | 47. 1275.844 cu. in. |
| 6. 68.921 cu. in. | 27. 74.088 cu. in. | 48. 2114.541 cu. in. |
| 7. 132.651 cu. in. | 28. 185.193 cu. in. | 49. 500.5 cu. in. |
| 8. 74.088 cu. in. | 29. 328.509 cu. in. | 50. 602.07 cu. in. |
| 9. 35.937 cu. in. | 30. $52\frac{7}{8}$ cu. ft. | 51. 1717.98 cu. in. |
| 10. $18\frac{3}{4}$. | 31. $98\frac{4}{5}$ cu. in. | 52. 863.28 cu. in. |
| 11. $4\frac{3}{8}$. | 32. $259\frac{4}{5}$ cu. in. | 53. 451.98 cu. in. |
| 12. $3\frac{1}{3}$. | 33. $83\frac{7}{8}$ cu. in. | 54. 803.58 cu. in. |
| 13. 255.2 cu. ft. | 34. $401\frac{6}{11}$ cu. in. | 55. 2620.56 cu. in. |
| 14. $153\frac{1}{2}$ cu. in. | 35. $202\frac{9}{11}$ cu. in. | 56. 2093.34 cu. in. |
| 15. $39\frac{3}{4}$ cu. in. | 36. $11\frac{5}{8}$ cu. ft. | 57. 963.33 cu. in. |
| 16. 4200 cu. ft. | 37. $31\frac{6}{7}$ cu. ft. | 58. 1326.78 cu. in. |
| 17. 42 cu. ft. | 38. $51\frac{3}{8}$ cu. ft. | 59. $41\frac{7}{8}$ ft. |
| 18. 5700 cu. ft. | 39. 216 cu. in. | 60. $205\frac{1}{3}$. |
| 19. 2200 cu. ft. | 40. 247 cu. in. | 61. 320. |
| 20. 2900 cu. ft. | 41. 354 cu. in. | 62. 1904; 65,280 lb. |
| 21. 6000 cu. ft. | 42. 58.32 cu. in. | |

Exercice 206. Page 382

- | | | |
|--|---------------------------------|------------------------|
| 1. 96 cu. in. | 15. $9262\frac{1}{3}$ cu. ft. | 29. 21 sq. in. |
| 2. 208 cu. in. | 16. $6639\frac{1}{3}$ cu. ft. | 30. 27 sq. in. |
| 3. 320 cu. in. | 17. $2343\frac{1}{3}$ cu. in. | 31. 21 sq. ft. |
| 4. 704 cu. in. | 18. 457.95 cu. in. | 32. 24 sq. ft. |
| 5. 1144 cu. in. | 19. $195.41\frac{1}{3}$ cu. in. | 33. 27 sq. yd. |
| 6. 3825 cu. in. | 20. 160.332 cu. in. | 34. 21 sq. in. |
| 7. $5333\frac{1}{3}$ cu. in. | 21. 600.9404 cu. ft. | 35. 18 sq. in. |
| 8. 5896 cu. ft. | 22. 20.33 cu. ft. | 36. 33 sq. ft. |
| 9. $6\frac{7}{8}$ cu. in. | 23. $2496\frac{2}{3}$ cu. ft. | 37. 63 sq. ft. |
| 10. $11\frac{2}{3}\frac{1}{4}$ cu. in. | 24. $1482\frac{1}{4}$ cu. ft. | 38. 36 sq. yd. |
| 11. 8.99 cu. in. | 25. $2846\frac{1}{2}$ cu. in. | 39. 72,576 lb. |
| 12. 19.27 cu. in. | 26. 1720 cu. in. | 40. 93,537,284 cu. ft. |
| 13. 42.525 cu. in. | 27. 3933.384 cu. in. | 41. 163,328. |
| 14. $37.5\frac{2}{3}$ cu. ft. | 28. 667.038 cu. ft. | 42. 87,210 cu. ft. |

Exercise 207. Page 384

- | | | |
|--------------------------------|------------------------------------|--------------------------------|
| 2. 48 sq. in. ;
81 sq. in. | 20. $26\frac{1}{4}$. | 39. 128.1537 cu. ft. |
| 3. 5. | 21. $1244\frac{1}{4}$. | 40. 374.3656 cu. ft. |
| 4. $890\frac{1}{2}$ sq. in. | 22. 851 cu. in. | 41. 1397.0381 cu. ft. |
| 5. $96\frac{3}{4}$. | 23. 1242 cu. in. | 42. $1160\frac{5}{32}$. |
| 6. $3\frac{1}{4}$. | 24. 8400 cu. in. | 43. $2268\frac{837}{896}$. |
| 7. 133. | 25. $522\frac{1}{2}$ cu. in. | 44. $9898\frac{71}{448}$. |
| 8. $7\frac{7}{11}$ ft. | 26. $433\frac{1}{2}$ cu. in. | 45. 103,810.245. |
| 9. $183\frac{3}{11}$ sq. ft. | 27. $42\frac{5}{8}$ cu. ft. | 46. 820,095.1746. |
| 10. $5498\frac{2}{11}$ cu. ft. | 28. 6.4 in. | 47. 701,733.992. |
| 11. 59.5 cu. in. | 29. 4.2 in. | 48. $230\frac{85}{448}$. |
| 12. 290.08 cu. in. | 30. 7.2 in. | 49. $466\frac{739}{896}$. |
| 13. 354.0625 cu. in. | 31. 7.25 in. | 50. 660. |
| 14. 649.6875 cu. in. | 32. 32.4 sq. ft. | 51. $425\frac{5}{32}$. |
| 15. 2007.5 sq. in. | 33. $14.74\frac{2}{7}$ sq. ft. | 52. $677\frac{329}{2048}$. |
| 16. 3037.5 sq. in. | 34. $118.76\frac{84}{331}$ sq. ft. | 53. $957\frac{23}{448}$. |
| 17. $2308\frac{1}{3}$ sq. in. | 35. 222.2 sq. ft. | 54. $3069\frac{1}{56}$. |
| 18. 5379.5 sq. in. | 36. 6605.1009 cu. in. | 55. $5387\frac{4343}{14336}$. |
| 19. 4500. | 37. 2847.7693 cu. in. | 56. $659\frac{3973}{16128}$. |
| | 38. 3813.2741 cu. in. | |

Exercise 208. Page 387

- | | | |
|---|-------------------------------|--------------------------------|
| 1. 2; 3; 10; 5. | 6. $4537\frac{1}{2}$. | 12. 68,777.89 $\frac{2}{7}$. |
| 2. $7\frac{1}{2}$; 15; 75; 750;
75,000. | 7. $524\frac{4}{5}$. | 13. 217,464.34 $\frac{2}{7}$. |
| 3. 147.6. | 8. 1235. | 14. 597,365.32232. |
| 4. 210.6. | 9. $2944.73\frac{1}{7}$. | 15. $340,371\frac{3}{7}$. |
| 5. $1687\frac{1}{5}$. | 10. $16,602.24\frac{3}{8}$. | 16. 169,400. |
| | 11. $29,438.003\frac{4}{7}$. | 17. 2.6374 ft. |

Exercise 209. Page 388

- | | | | |
|---------------------------------------|----------------------------------|------------|----------------------------|
| 1. 8 cu. in. | 9. 63.56 cu. in. | 17. 7 in. | 25. 12 sq. ft. |
| 2. 35 cu. in. | 10. 146.575 cu. in. | 18. 6 in. | 26. 15 sq. ft. |
| 3. 60 cu. in. | 11. 75.3984 cu. in. | 19. 6 in. | 27. 11 sq. ft. |
| 4. 170 cu. in. | 12. 9.4248. | 20. 7 ft. | 28. 21 sq. in. |
| 5. 476 cu. in. | 13. $208\frac{1}{3}$. | 21. 10 ft. | 29. 3.3 sq. in. |
| 6. $5\frac{5}{8}$ cu. in. | 14. $15\frac{1}{2}\frac{7}{4}$. | 22. 11 yd. | 30. 33 sq. yd. |
| 7. $12\frac{1}{3}\frac{1}{2}$ cu. in. | 15. 9 in. | 23. 9 yd. | 31. $113,016\frac{2}{3}$. |
| 8. $28\frac{7}{32}$ cu. in. | 16. 4 in. | 24. 30. | |

Exercise 210. Page 390

- | | | | |
|------------------|-----------------------------|-----------------------------|------------------------|
| 1. 40 sq. ft. | 4. 48 sq. in. | 7. 10 sq. ft. | 10. $183\frac{1}{3}$. |
| 2. 13.65 sq. in. | 5. 100 sq. in. | 8. $294\frac{1}{2}$ sq. in. | |
| 3. 25 sq. in. | 6. $40\frac{7}{32}$ sq. in. | 9. $115\frac{1}{2}$ sq. ft. | |

Exercise 211. Page 392

- | | | |
|---------------------------------------|-----------------------|---|
| 1. 616 sq. in. | 10. 157,696 sq. in. | 19. 15.205344 sq. in. |
| 2. 3850 sq. in. | 11. 432,586 sq. in. | 20. 38.4846 sq. in. |
| 3. 5544 sq. in. | 12. 591,976 sq. in. | 21. 12.5664 sq. in. |
| 4. 13.86 sq. in. | 13. 153.9384 sq. in. | 22. 314.16 sq. in. |
| 5. 24.64 sq. in. | 14. 113.0976 sq. in. | 23. 3.1416 sq. in. |
| 6. 75.46 sq. in. | 15. 254.4696 sq. in. | 24. 113.0976 sq. ft. |
| 7. $2\frac{1}{3}\frac{3}{2}$ sq. in. | 16. 50.2656 sq. in. | 25. 615.7536 sq. in. |
| 8. 1.54 sq. in. | 17. 176.715 sq. in. | 26. 1017.8784 sq. in. |
| 9. 0.9856 sq. in. | 18. 122.71875 sq. in. | 27. 78.54 sq. in. |
| 28. 172.034016 sq. in. | | 31. 2,359,727,071,428 $\frac{1}{7}$ sq. mi. |
| 29. 201,142,857 $\frac{1}{7}$ sq. mi. | | 32. 6084 $\frac{4}{7}$. |
| 30. $28\frac{2}{7}$. | | 33. 7 in. |

Exercise 212. Page 393

- | | | |
|-------------------------------------|---------------------------------------|-----------------------|
| 1. 33.5104 cu. in. | 5. 50,965.1296 cu. in. | 9. 179.5948 cu. ft. |
| 2. 523.6 cu. in. | 6. 310,339.8144 cu. in. | 10. 65.45 cu. in. |
| 3. 1436.7584 cu. in. | 7. 57.9059712 cu. ft. | 11. 220.89375 cu. in. |
| 4. 14,137.2 cu. in. | 8. 288.6962848 cu. ft. | 12. 425.7062 cu. in. |
| 13. 27.5132 oz. | 21. 113.0976 cu. in. ; | |
| 14. 47.71305 cu. in. | 904.7808 cu. in. | |
| 15. 340,647,604,331,650,000 cu. mi. | 22. 243.9168. | |
| 16. 7.0144 cu. in. | 23. 56.5488. | |
| 17. 88.3575 lb. | 24. 0.5236 cu. ft. ; 0.7854 cu. ft. ; | |
| 18. 18.77236 lb. | 0.2618 cu. ft. ; twice ; three ; | |
| 19. 5,276,681,625.6 cu. mi. | two thirds. | |
| 20. 14,137.2. | 25. 38.7 lb. | |

Exercise 214. Page 396

- | | | |
|-------------------------------|--------------------------------|-----------------------|
| 1. 68.25 cu. in. | 24. $\frac{529}{2304}$ sq. in. | 48. 1.9 in. |
| 2. 10 sq. in. | 25. 8.2369 sq. ft. | 49. 17 in. |
| 3. $22\frac{3}{4}$ cu. in. | 26. $\frac{169}{256}$ sq. in. | 50. 64 in. |
| 4. 36 sq. in. | 27. 15.3664 sq. ft. | 51. 7.8 in. |
| 5. 2513.28 sq. ft. | 28. $\frac{289}{1296}$ sq. in. | 52. 0.44 in. |
| 6. 2.1 in. | 29. 306.25 sq. rd. | 53. 6.4 ft. |
| 7. 21 sq. in. | 30. $\frac{225}{4096}$ sq. in. | 54. 8.4 in. |
| 8. 12.8975 cu. in. | 31. 0.1369 sq. in. | 55. 11 ft. 10 in. |
| 9. 29.791 cu. in. | 32. 31.28 sq. ft. | 56. 17.89 rd. |
| 10. \$37.50. | 33. $8\frac{1}{2}$ sq. ft. | 57. 12.37 in. |
| 11. $227\frac{5}{4}$. | 34. 4.7724 sq. in. | 58. 2.74 in. |
| 12. $31\frac{3}{2}$ lb. | 35. 46.72 sq. in. | 59. 13.08 ft. |
| 13. 86.394 lb. | 36. $3.86\frac{1}{2}$ sq. ft. | 60. 18.76 rd. |
| 14. 6702 $\frac{6}{11}$. | 37. 38 sq. ft. | 61. 2.46 yd. |
| 15. 19.3962. | 38. 8.93 sq. ft. | 62. 19.62 ft. |
| 16. 2415.541 lb. | 39. 0.0777 sq. in. | 63. 15.84 yd. |
| 17. 1,457,777 $\frac{7}{8}$. | 40. $25\frac{5}{16}$ sq. ft. | 64. 2.92 in. |
| 18. 0.074 in. | 41. $2\frac{1}{2}$ sq. yd. | 65. 616 sq. in. |
| 19. 50.66694 lb. | 42. 15.68 sq. in. | 66. 13,034.56 sq. ft. |
| 20. 76.94 lb. | 43. $3.37\frac{1}{6}$ sq. ft. | 67. 48,246.66 sq. rd. |
| 21. 15.59 sq. ft. | 44. 17 in. | 68. 745.36 sq. in. |
| 22. Twice. | 45. 3.4 in. | 69. 16,022.16 sq. ft. |
| 23. 268,083,200,000 cu. mi. ; | 46. 11 in. | 70. 85,046.5 sq. rd. |
| 201,062,000 sq. mi. | 47. 2.1 in. | 71. 5003.46 sq. in. |

- | | | |
|------------------------|--|---|
| 72. 26,832.96 sq. yd. | 81. 28.86 ft. | 83. 24,848,438 $\frac{1}{2}$ cu. in. ; |
| 73. 174,896.26 sq. yd. | 82. 6.71 yd. | 411,852 $\frac{4}{7}$ sq. in. |
| 74. 9.90 in. | 84. 134,754.953 cu. in. ; 12,712.73 sq. in. | |
| 75. 23.22 ft. | 85. 21,323,020 $\frac{4}{21}$ cu. in. ; 371,913 $\frac{1}{7}$ sq. in. | |
| 76. 3.57 yd. | 86. 88,102.985 cu. in. ; 9576.41 sq. in. | |
| 77. 15.65 in. | 87. 1,023,065,476 $\frac{4}{21}$ cu. in. ; 4,910,714 $\frac{2}{7}$ sq. in. | |
| 78. 24.25 ft. | 88. 167,626.121 cu. in. ; 14,704.05 sq. in. | |
| 79. 3.77 yd. | 89. 21.6 cu. ft. | 91. 93.48 cu. in. |
| 80. 21 in. | 90. 71.34 cu. in. | 92. 44.3 $\frac{1}{8}$ cu. ft. |
| 93. 25.432 cu. ft. | 97. 109.185 cu. ft. | 101. 23.534 cu. ft. |
| 94. 225.255 cu. in. | 98. 62.832 cu. in. | 102. 427.039 cu. in. |
| 95. 439.899 cu. in. | 99. 28.189 cu. in. | 103. 498.968 cu. ft. |
| 96. 2928.081 cu. ft. | 100. 36,011 $\frac{1}{2}$ cu. in. | 104. 70 $\frac{5}{6}$ $\frac{0}{7}$ cu. ft. |

Exercise 216. Page 403

- | | | | | |
|---------|----------|----------|----------------|--------------|
| 1. 30. | 20. 30. | 39. 490. | 58. 3548. | 77. 877. |
| 2. 30. | 21. 37. | 40. 530. | 59. 4621. | 78. 738. |
| 3. 30. | 22. 32. | 41. 510. | 60. 5628. | 79. 935. |
| 4. 36. | 23. 25. | 42. 366. | 61. \$422.49. | 80. 726. |
| 5. 24. | 24. 30. | 43. 409. | 62. \$235.49. | 81. 718. |
| 6. 34. | 25. 43. | 44. 527. | 63. \$389.85. | 82. 675. |
| 7. 38. | 26. 31. | 45. 595. | 64. \$819.82. | 83. 951. |
| 8. 40. | 27. 30. | 46. 560. | 65. \$3589.76. | 84. 831. |
| 9. 27. | 28. 320. | 47. 486. | 66. \$4501. | 85. 782. |
| 10. 30. | 29. 318. | 48. 607. | 67. 672. | 86. 878. |
| 11. 30. | 30. 432. | 49. 419. | 68. 736. | 87. 41.96. |
| 12. 39. | 31. 419. | 50. 510. | 69. 653. | 88. 550. |
| 13. 25. | 32. 482. | 51. 560. | 70. 700. | 89. 602. |
| 14. 37. | 33. 505. | 52. 543. | 71. 771. | 90. \$36.50. |
| 15. 29. | 34. 499. | 53. 555. | 72. 803. | 91. \$30. |
| 16. 23. | 35. 430. | 54. 535. | 73. 850. | 92. \$36.25. |
| 17. 37. | 36. 500. | 55. 433. | 74. 858. | 93. \$39.45. |
| 18. 30. | 37. 365. | 56. 488. | 75. 777. | |
| 19. 30. | 38. 540. | 57. 518. | 76. 710. | |

Exercise 217. Page 407

- | | | | | |
|----------|-------------|---------------|---------------|---------------|
| 1. 4237. | 6. 920. | 11. 43,428. | 16. \$271.50. | 21. \$518.25. |
| 2. 1704. | 7. 23,508. | 12. 52,400. | 17. \$669.80. | 22. \$344.75. |
| 3. 3703. | 8. 28,571. | 13. \$24.38. | 18. \$594. | 23. \$337.75. |
| 4. 5224. | 9. 22,208. | 14. \$171.36. | 19. \$174.35. | 24. \$144.25. |
| 5. 4674. | 10. 49,922. | 15. \$224.45. | 20. \$267.50. | |

Exercise 218. Page 410

1. 13,800.	22. \$355.	43. 12.4.	64. \$89.
2. 68,700.	23. \$610.	44. 1.24.	65. \$310.
3. 70,200.	24. \$581.	45. 1860.	66. \$34.
4. \$111.	25. \$518.	46. 18.6.	67. \$27.
5. \$240.	26. \$46.	47. 1.86.	68. \$126.30.
6. 1,846,200.	27. \$112.	48. 0.08.	69. \$42.48.
7. \$336.	28. \$99.	49. \$12,000.	70. 1; 50; 104;
8. \$624.	29. \$248.	50. \$10,800.	133.
9. 1827.	30. \$328.	51. \$9300.	71. 1; 10; 100;
10. 178.7.	31. \$93.	52. \$21,000.	91.
11. 134.	32. \$112.	53. \$184.	72. 0.05; 1; 6.05.
12. 1.34.	33. 847.	54. \$245.	73. 1800 lb.
13. 0.134.	34. 484,400.	55. \$213.	74. \$400; \$200.
14. \$176.	35. 562,400.	56. \$423.	75. \$1600; \$1733 $\frac{1}{3}$.
15. \$43.	36. 56,240.	57. \$159.	76. \$600; \$800;
16. \$97.	37. 5624.	58. \$1220.	\$866 $\frac{2}{3}$.
17. 36.	38. 3216.	59. \$372.	77. 2160; 2160;
18. 15,900.	39. 669.	60. \$270.	2160.
19. 66,600.	40. 3976.	61. \$48.	78. 5760; 5760;
20. \$366.	41. 147.6.	62. \$170.	5760.
21. \$246.	42. 12.4.	63. \$1.50.	

Exercise 219. Page 412

1. \$6521.04.	4. \$5432.02.	7. \$32,092.06.	10. \$62,734.28.
2. \$7883.37.	5. \$12,743.28.	8. \$45,284.82.	11. \$27,657.63.
3. \$7512.67.	6. \$42,218.55.	9. \$43,706.25.	12. \$325,399.50.

Exercise 220. Page 413

1. 80,146.	10. 660,660.	19. 222,200.	28. 18,315.
2. 102,091.	11. 778,470.	20. 666,666.	29. 21,780.
3. 41,239.	12. 1,099,989.	21. 733,326.	30. 4884.
4. 448,008.	13. 2706.	22. 1,042,954.	31. 11,110.
5. 667,799.	14. 9592.	23. 1,211,210.	32. 20,592.
6. 549,978.	15. 11,462.	24. 1,599,994.	33. 17,094.
7. 299,706.	16. 13,640.	25. 3366.	34. 9768.
8. 429,077.	17. 26,576.	26. 7029.	35. 10,989.
9. 297,770.	18. 464,464.	27. 13,332.	36. 19,998.

Exercise 221. Page 415

- | | | | |
|--------------|--------------|--------------|---------------|
| 1. \$41.74. | 5. \$51.28. | 9. \$81.25. | 13. 7500. |
| 2. \$118.79. | 6. \$83.03. | 10. \$8. | 14. \$50.89. |
| 3. \$390.83. | 7. \$402.19. | 11. \$76.54. | 15. \$20.30. |
| 4. \$454.09. | 8. \$46.67. | 12. \$6. | 16. \$187.78. |
| | | | 17. \$202.50. |

Exercise 222. Page 416

- | | | |
|--------------|------------------------|-------------------------|
| 1. \$964.20. | 19. 9999.99. | 37. 654.96. |
| 2. \$27.96. | 20. 810.81. | 38. 266.76. |
| 3. \$5.275. | 21. 75. | 39. 319.104. |
| 4. \$684.02. | 22. 7.25 hr. | 40. 537.12; \$1505.68; |
| 5. \$540.02. | 23. $6\frac{2}{3}$ hr. | \$5424.84; \$17,107.72. |
| 6. 24,246.8. | 24. 1448. | 41. 137.6; \$546.40; |
| 7. 4435. | 25. 97.26. | \$2211.20; \$2540. |
| 8. 6835. | 26. 176.22. | 42. 2091.6; \$988.80; |
| 9. 85,489. | 27. 738.96. | \$1462.50; \$8289.60. |
| 10. 1885. | 28. 206.67. | 43. 149. |
| 11. 923. | 29. 39.488. | 44. 294. |
| 12. 2355. | 30. 109.2. | 45. 142. |
| 13. 3106. | 31. 289.86. | 46. 149,707.88. |
| 14. 501.2. | 32. 591.68. | 47. 549. |
| 15. 3014. | 33. 148.77. | 48. 36. |
| 16. 24,381. | 34. 539.2. | 49. 72. |
| 17. 12,528. | 35. 139.14. | 50. 48 mi. |
| 18. 6666.66. | 36. 548.04. | 51. 10 mi. |
| | | 52. 3 mi. |

Exercise 223. Page 418

- | | | | |
|-----------------------|---------------|---------------|-------------|
| 1. \$6. | 11. \$46,875. | 21. 63. | 31. \$9375. |
| 2. 28.8. | 12. \$1375. | 22. 87. | 32. \$3000. |
| 3. 13¢. | 13. \$742.50. | 23. \$800. | 33. \$1750. |
| 4. \$5; \$10. | 14. \$108. | 24. \$13,000. | 34. 230. |
| 5. 16%. | 15. \$156.25. | 25. \$2500. | 35. 340. |
| 6. $6\frac{6}{9}$ yd. | 16. 1760. | 26. \$1200. | 36. 330. |
| 7. \$1.80. | 17. \$2250. | 27. \$150. | 37. 32. |
| 8. \$54. | 18. 2700. | 28. \$3000. | 38. 72. |
| 9. \$560. | 19. 4000. | 29. \$2250. | 39. 213. |
| 10. \$1400. | 20. 4000. | 30. \$6750. | |

Exercise 224. Page 421

- | | | | |
|-----------------|-----------------|------------------------|------------------------|
| 1. \$125. | 15. \$640. | 31. 6%. | 47. \$13.02. |
| 2. \$80. | 16. \$720. | 32. 4%. | 48. \$10. |
| 3. \$1200. | 17. \$360. | 33. 5%. | 49. \$5.21. |
| 4. 4 yr.; | 18. \$480. | 34. 6%. | 50. \$7.33. |
| 5 yr. ; 8 yr. ; | 19. \$500. | 35. $3\frac{1}{3}\%$. | 51. \$16.40. |
| 3 yr. 4 mo. | 20. \$500. | 36. 4%. | 52. \$13.88. |
| 5. \$37.50. | 21. \$750. | 37. 4%. | 53. \$0.21. |
| 6. \$28.80. | 22. \$375. | 38. 4%. | 54. \$0.60. |
| 7. \$85. | 23. \$1200. | 39. 3%. | 55. No difference. |
| 8. \$88. | 24. \$1500. | 40. 6%. | 56. No difference. |
| 9. \$63.25. | 25. 2 yr. 6 mo. | 41. \$1.75. | 57. No difference. |
| 10. \$200. | 26. 2 yr. 4 mo. | 42. \$2.75. | 58. \$27.13. |
| 11. \$368.50. | 27. 3 yr. | 43. \$1.75. | 59. \$13.13 ; \$13.13. |
| 12. \$306.25. | 28. 2 yr. 8 mo. | 44. \$2.10. | 60. \$97.60. |
| 13. \$433.13. | 29. 2 yr. 9 mo. | 45. \$9.60. | |
| 14. \$11.27. | 30. 2 yr. 4 mo. | 46. \$10.80. | |

Exercise 225. Page 423

- | | | | | |
|--------------|--------------|--------------|---------------|----------------|
| 1. \$328.50. | 3. \$253.64. | 5. \$325.21. | 7. \$282.24. | 9. \$545.27. |
| 2. \$254. | 4. \$94.40. | 6. \$237.97. | 8. \$1780.03. | 10. \$1351.84. |

Exercise 226. Page 425

- | | | | |
|--------------|--------------|--------------|--------------|
| 1. \$304.84. | 2. \$260.08. | 3. \$843.21. | 4. \$520.05. |
|--------------|--------------|--------------|--------------|

Exercise 227. Page 427

- | | | | | |
|---------------|---------------|---------------|---------------|----------------|
| 1. \$179.35. | 3. \$132.35. | 5. \$1193.45. | 7. \$998.25. | 9. \$3102.25. |
| 2. \$1254.05. | 4. \$1508.22. | 6. \$4315.10. | 8. \$2676.75. | 10. \$8417.40. |

Exercise 228. Page 428

- | | | |
|-------------|--------------|--------------|
| 1. \$64.86. | 2. \$256.27. | 3. \$130.24. |
|-------------|--------------|--------------|

Exercise 229. Page 432

- | | | | |
|-----------------|-----------------|------------------|------------------|
| 1. \$10,650. | 7. \$6030. | 13. \$4837.50. | 19. \$35,687.50. |
| 2. \$14,093.75. | 8. \$6410.63. | 14. \$11,805. | 20. \$27,465.63. |
| 3. \$16,187.50. | 9. \$12,406.25. | 15. \$31,218.75. | 21. \$143.75. |
| 4. \$35,028.13. | 10. \$3190.63. | 16. \$13,734.38. | 22. \$393.75. |
| 5. \$50,006.25. | 11. \$6140.63. | 17. \$16,856.25. | 23. \$31.26. |
| 6. \$4659.38. | 12. \$1975. | 18. \$21,196.88. | 24. 40 ; \$15. |

Exercise 230. Page 433

- | | | |
|---------------------|---------------------|-----------------------|
| 1. \$2812.50. | 18. \$115.62, gain. | 35. 200. |
| 2. \$17,500. | 19. \$34.37, gain. | 36. 401; \$20.12. |
| 3. \$7565.63. | 20. \$40.63, loss. | 37. 200; \$50. |
| 4. \$2681.25. | 21. \$21.87, loss. | 38. 101; \$53.62. |
| 5. \$5225. | 22. \$96.87, gain. | 39. 50; \$43.75. |
| 6. \$26,162.50. | 23. \$253.12, gain. | 40. 100; \$25. |
| 7. \$48,802.50. | 24. \$156.25, loss. | 41. 402; \$5.25. |
| 8. \$5100. | 25. \$593.75, gain. | 42. 35; \$71.25. |
| 9. \$1056.25. | 26. \$1350, loss. | 43. 78; \$88.50. |
| 10. \$3780. | 27. \$30, gain. | 44. 60; \$100. |
| 11. \$9056.25. | 28. \$262.49, gain. | 45. 50; \$12.50. |
| 12. \$21.87, gain. | 29. 100. | 46. 400; \$50. |
| 13. \$34.37, gain. | 30. 200. | 47. 50; \$18.75. |
| 14. \$221.88, gain. | 31. 200. | 48. 70; \$28.75. |
| 15. \$143.75, loss. | 32. 200. | 49. 150; \$75. |
| 16. \$106.25, gain. | 33. 200. | 50. 50; \$43.75; 100; |
| 17. \$25, gain. | 34. 200. | \$50. |

Exercise 231. Page 436

- | | | | |
|-----------------------|------------------|-----------------------|-----------------------|
| 1. $2\frac{1}{2}\%$. | 3. \$90,000. | 5. \$300. | 7. $4\frac{1}{2}\%$. |
| 2. \$2625. | 4. \$75,500,000. | 6. $2\frac{1}{2}\%$. | 8. 6%. |

Exercise 232. Page 437

- | | | | |
|--------|-------------------------|---------------------------|---|
| 1. 4%. | 7. 4%. | 13. $4\frac{6}{11}\%$. | 19. The same. |
| 2. 4%. | 8. $5\frac{1}{10}\%$. | 14. $4.04\frac{1}{2}\%$. | 20. The 5% note; $0.34\frac{1}{2}\%$. |
| 3. 4%. | 9. 8%. | 15. $4.98\frac{1}{2}\%$. | 21. The 8% stock; $\frac{1}{3}\%$. |
| 4. 5%. | 10. 6%. | 16. $5.37\frac{1}{2}\%$. | 22. The 6% stock; $0.12\frac{1}{2}\%$. |
| 5. 6%. | 11. $4\frac{4}{9}\%$. | 17. $5.26\frac{1}{2}\%$. | 23. \$120; $119\frac{7}{8}$. |
| 6. 8%. | 12. $5\frac{5}{11}\%$. | 18. 5%. | 24. \$150; $149\frac{7}{8}$. |

Exercise 233. Page 438

- | | | | |
|--------------|---------------------------|----------------|------------------|
| 1. \$360. | 9. \$1105. | 17. 10; \$200. | 25. 8; \$320. |
| 2. \$350. | 10. $4.90\frac{1}{2}\%$. | 18. 20; \$50. | 26. 9; \$112.50. |
| 3. \$360. | 11. $6.37\frac{1}{2}\%$. | 19. 2; \$30. | 27. 9; \$25. |
| 4. \$255. | 12. $3\frac{1}{3}\%$. | 20. 3; \$100. | 28. 10; \$37.50. |
| 5. \$260. | 13. 2. | 21. 4. | 29. 14; \$140. |
| 6. \$472.50. | 14. 3. | 22. 5; \$6.25. | 30. 80; \$50. |
| 7. \$765. | 15. 4. | 23. 6; \$10. | 31. 10; \$25. |
| 8. \$660. | 16. 7; \$150. | 24. 7; \$290. | |

Exercise 234. Page 440

- | | | |
|-----------------------|-------------------------|------------------------|
| 1. \$4711.50. | 8. \$19.50 ; | 14. Promissory note. |
| 2. \$12,180. | \$15,034.50. | 15. 7% stock. |
| 3. $3\frac{2}{5}\%$. | 9. $11\frac{8}{11}\%$. | 16. 5% bond ; |
| 4. 42. | 10. 20 ; \$100. | 0.0085%. |
| 5. $4\frac{1}{2}\%$. | 11. \$100.47, loss. | 17. The same. |
| 6. \$4.75. | 12. 5% stock. | 18. The same. |
| 7. \$23,647.50. | 13. The same. | 19. 5% bond ; 0.0045%. |
| | | 20. The same. |

Exercise 236. Page 443

- | | | | | |
|-------------|---------------|------------------------|------------------------|-------------------------------|
| 1. \$18.75. | 10. \$146.25. | 19. \$6500. | 28. $1\frac{3}{4}\%$. | 37. \$41.25. |
| 2. \$22.50. | 11. \$187.50. | 20. \$2400. | 29. $1\frac{1}{2}\%$. | 38. \$71.50. |
| 3. \$30.63. | 12. \$125. | 21. \$3800. | 30. $1\frac{1}{2}\%$. | 39. \$52.50. |
| 4. \$50.63. | 13. \$270. | 22. \$3200. | 31. \$34.38. | 40. \$5500. |
| 5. \$52.50. | 14. \$187.50. | 23. \$1975. | 32. \$48.75. | 41. $1\frac{1}{2}\%$. |
| 6. \$71.50. | 15. \$223.13. | 24. \$1650. | 33. \$84. | 42. \$43.20. |
| 7. \$95. | 16. \$515.63. | 25. 1% . | 34. \$126. | 43. $\frac{4}{5}\%$. |
| 8. \$78.75. | 17. \$319.20. | 26. $2\frac{1}{2}\%$. | 35. \$375. | 44. \$3000. |
| 9. \$75. | 18. \$198. | 27. 2% . | 36. \$756.25. | 45. \$4666.67 ;
\$3333.33. |

Exercise 237. Page 445

- | | | | |
|--------------|--------------|--------------|---------------|
| 1. \$108.75. | 4. \$118.62. | 7. \$86.28. | 10. \$456.63. |
| 2. \$169.88. | 5. \$194.25. | 8. \$307.44. | 11. \$3954. |
| 3. \$99.58. | 6. \$100.58. | 9. \$369.32. | 12. \$131.23. |

Exercise 238. Page 446

- | | | | | |
|-------------|--------------|-----------------------|---------------|---------------|
| 1. \$2800. | 3. \$16.50. | 5. \$4600. | 7. \$1072.50. | 9. \$825. |
| 2. \$20.10. | 4. \$14,000. | 6. $1\frac{1}{4}\%$. | 8. \$7500. | 10. \$12,500. |
| | | | | 11. \$112.50. |

Exercise 239. Page 449

- | | | | |
|-------------|-------------|---------------|--------------|
| 1. \$1.39. | 5. \$20.28. | 9. \$66.66. | 13. \$3.18. |
| 2. \$4.73. | 6. \$25.28. | 10. \$63.88. | 14. \$5. |
| 3. \$5. | 7. \$41.95. | 11. \$62.50. | 15. \$6.36. |
| 4. \$22.22. | 8. \$50. | 12. \$220.82. | 16. \$32.72. |

17. \$45.45.	22. \$72.72.	27. \$443.14.	32. 12 mills.
18. \$32.27.	23. \$118.17.	28. \$748.56.	33. 12 mills.
19. \$43.18.	24. \$318.15.	29. \$49.50.	34. \$3760.
20. \$34.09.	25. \$113.63.	30. \$162.50.	35. \$42.50.
21. \$50.45.	26. \$240.89.	31. \$7500.	36. \$135.

Exercise 240. Page 451

1. \$142.50.	9. \$1460.	17. \$343.75.	25. \$3375.
2. \$218.75.	10. \$3787.50.	18. \$7500.	26. \$2750.
3. \$300.	11. \$2490.	19. \$415.80.	27. \$6600.
4. \$375.75.	12. \$12,712.50.	20. 15%.	28. \$168.50.
5. \$585.	13. \$10,276.70.	21. 40%.	29. \$271.80.
6. \$375.	14. \$16,137.50.	22. \$8275.	30. 1750.
7. \$495.	15. \$45,177.	23. 35%.	31. \$750.
8. \$945.	16. \$24.48.	24. \$331.	

Exercise 241. Page 453

1. $21\frac{3}{4}\%$.	19. \$105.12.	34. $4\frac{1}{2}\%$; $5\frac{1}{2}\%$.
2. \$49,280.	20. Sand, 108 ; soda, 21 ; lime, 18 ; alumina, 3.	35. $1\frac{28}{33}$.
3. A, \$72 ; B, \$81.	21. Copper, 962.5 ; tin, 262.5 ; zinc, 25.	36. $66\frac{2}{7}\%$.
4. 224.64 lb.	22. 484.	37. $29.79\frac{5}{7}$ in.
5. 60 in.	23. 29.5305903 da.	38. 10 lb. ; 50 lb.
6. 908.855 lb.	24. $34\frac{1}{8}$.	39. 201.68.
7. 77.98 lb.	25. 468 lb.	40. 60 lb.
8. 2126.4705 sq. in.	26. 11.2.	41. \$720.17.
9. 5 hr.	27. \$145.80.	42. 28.97 ft.
10. \$4500 ; \$1200	28. \$3750.	43. 5.67 bbl. ; 3.54 cu. yd.
11. $1,974,857\frac{1}{7}$.	29. 105.	44. 1350 ; 3375.
12. 14.14 ft.	30. \$13.95.	45. 2.
13. 0.7 in.	31. $4\frac{1}{2}$ hr.	46. 4.
14. $3764\frac{4}{9}$.	32. \$46.80.	47. $4\frac{4}{9}$.
15. \$237.50.	33. 38.1831.	48. $4\frac{3}{7}$.
16. 1960.		49. 96.
17. 1320.		50. $1\frac{2}{3}$.
18. 71.00016 in. ; 71.02857 $\frac{1}{7}$ in. ; 71 in.		51. 7.35 mi.
		52. 96.

- | | | |
|------------------------|-----------------------------|-------------------------|
| 53. 7 hr. | 62. $13\frac{889}{760}$ mi. | 73. 700 gal. |
| 54. 1st, \$1800; | 63. \$420. | 74. 27 min. |
| 2d, \$2100; | 64. 52.95 ft. | 75. 300. |
| 3d, \$900. | 65. Neither. | 76. 360 ft. |
| 55. $157\frac{1}{2}$. | 66. $32\frac{1}{6}$ min. | 77. 48,906. |
| 57. \$40. | 67. 17,000. | 78. 60,451,437.5. |
| 58. 256 lb. | 68. $42\frac{1}{2}$ mi. | 79. \$262.50; \$437.50; |
| 59. $7\frac{1}{3}$. | 69. 51.2. | \$210; \$227.50; |
| 60. 36 sq. in.; | 70. 4. | $28\frac{2}{3}\%$. |
| 72 sq. in. | 71. 8316. | 80. 21. |
| 61. 113.0976 sq. in. | 72. 2 hr. 9 min. | |

Exercise 242. Page 461

- | | |
|---|---|
| 1. Sugar, $7\frac{1}{2}$ lb.; water, $3\frac{3}{4}$ pt. | 5. 50%; $33\frac{1}{3}\%$; $66\frac{2}{3}\%$. |
| 2. $5\frac{1}{3}$. | 6. 50%; $33\frac{1}{3}\%$; $66\frac{2}{3}\%$. |
| 3. $2\frac{1}{2}$ oz. | 7. 200 lb. |
| 4. Water, $14\frac{2}{7}\%$; sugar, $28\frac{4}{7}\%$; fruit, $57\frac{1}{7}\%$. | |

Exercise 243. Page 463

- | | | |
|--------------|-------------------------------|-----------------------|
| 1. \$0.21. | 3. \$0.90; $5\frac{5}{9}\%$. | 5. 5¢; \$1.23; 13¢. |
| 2. \$1; 20%. | 4. In buying honey. | 6. $7\frac{1}{9}\%$. |

Exercise 244. Page 464

- | | | | | |
|-------------|--------------|--------|--------|--------|
| 1. \$70.20. | 2. \$122.50. | 3. 24. | 4. 21. | 5. 18. |
|-------------|--------------|--------|--------|--------|

Exercise 245. Page 465

- | | | |
|------------------------------------|------------|-------------|
| 1. \$0.85; \$0.82; \$0.17; \$0.85. | 3. \$2.15. | 5. \$1.53. |
| 2. \$1; \$3.20; \$12.15. | 4. \$3.51. | 6. \$25.70. |

Exercise 246. Page 466

- | | | | | | |
|-------|-------|---------------------|-------------|---------------|---------------|
| 1. 8. | 3. 6. | 5. $6\frac{2}{3}$. | 7. 2552 ft. | 9. \$470.82. | 11. \$631.20. |
| 2. 3. | 4. 8. | 6. 735 ft. | 8. 7847 ft. | 10. \$225.88. | |

Exercise 247. Page 468

- | | |
|--|--|
| 1. 16%. | 3. Ensilage, 56%; clover hay, 24%; bran, 13%; corn meal, 7%. |
| 2. \$9. | 4. Resin, $37\frac{1}{2}\%$; beeswax, $37\frac{1}{2}\%$; tallow, $7\frac{7}{24}\%$. |
| 5. Use of land, $33\frac{1}{3}\frac{4}{3}\%$; labor, $45\frac{1}{3}\frac{9}{5}\%$; fertilizers, $18\frac{7}{3}\frac{8}{9}\%$; seed, $2\frac{2}{3}\frac{4}{9}\%$. | |
| 6. 198.8514. | 7. \$14. |
| | 8. 14,520; $544\frac{1}{2}$ T. |

Exercise 248. Page 469

- | | | | |
|-------------------------|----------------|-----------------|---------------|
| 1. 500. | 3. 1.032. | 5. 3400; \$102. | 7. 3302.4. |
| 2. $714\frac{2}{7}$ lb. | 4. 251.748 lb. | 6. 1724 sq. ft. | 8. \$1684.80. |

Exercise 249. Page 470

- | | | |
|-------------------------|-------------------------|--|
| 1. 138.6. | 4. 384. | 7. 625; $20' \times 5' \times 6\frac{1}{4}'$; $10' \times 10' \times 6\frac{1}{4}'$. |
| 2. $4259\frac{1}{27}$. | 5. \$190.08. | 8. $1131\frac{3}{7}$. |
| 3. 90. | 6. $127\frac{6}{7}$ ft. | 9. $125\frac{5}{7}$. |

Exercise 250. Page 471

- | | | | |
|--------------|-----------------|-----------------------|--------|
| 1. 1102.5. | 3. 16,800. | 5. \$19.45. | 7. 36. |
| 2. \$346.08. | 4. 2d cow; 96¢. | 6. $185\frac{5}{8}$. | |

Exercise 251. Page 472

- | | | |
|---|-------------|----------------------|
| 1. 2.28 lb. | 2. 0.18 lb. | 3. 0.4149%. |
| 4. Copper sulphate, $3\frac{1}{8}$ lb.; unslacked lime, $3\frac{1}{8}$ lb.; water, $243\frac{3}{4}$ lb. | | |
| 5. Acid phosphate, 875 lb.; meal, 950 lb.; kainite, 175 lb. | | |
| 6. Acid phosphate, 1075 lb.; fish scrap, 800 lb.; muriate of potash, 125 lb. | | |
| 7. Timothy, 16 lb.; redtop, 16 lb.; red clover, 8 lb. | | |
| 8. Blue grass, 8 lb.; white clover, 4 lb.; perennial rye, 9 lb.; red fescue, 3 lb.; redtop, 8 lb. | | |
| 9. \$6.75. | 11. 150. | 13. 1000. |
| 10. 3680. | 12. 81¢. | 14. $1\frac{1}{2}$. |
| | | 15. \$2. |
| | | 17. 1:6. |
| | | 16. 4. |
| | | 18. \$18.82. |

Exercise 252. Page 474

- | | | | |
|-----------------------|--|-------------------|--------------|
| 1. \$303.14. | 6. The second; $3\frac{1}{2}\frac{3}{9}\%$. | 11. \$4837.50. | 16. \$76.34. |
| 2. \$38.34. | 7. \$929.19. | 12. 922 lb. 4 oz. | 17. \$82.02. |
| 3. $4\frac{3}{4}$ in. | 8. $5\frac{8}{3}\frac{1}{3}$ in. | 13. 27.713 in. | 18. \$73.37. |
| 4. \$67,031.25. | 9. 520 lb. | 14. 7875 lb. | 19. \$72.06. |
| 5. \$4. | 10. $62,304\frac{1}{6}$ lb. | 15. \$89.23. | 20. \$65.61. |

Exercise 253. Page 477

- | | | | |
|------------|-----------|--------------|----------------------|
| 1. \$1850. | 3. 90%. | 5. \$535.50. | 7. \$1650. |
| 2. \$1850. | 4. \$800. | 6. \$28.30. | 8. 20%. |
| | | | 9. The former; \$40. |

1. 23.562 ft. 3. 1884.96 ft. 5. 14.927 in. 7. 68,706.792 lb. 9. 17.
2. 5236 ft. 4. 1200. 6. 36.319 in. 8. 1847.2608 sq.ft. 10. 750.

1. 15. 3. 39. 5. 147.96. 7. $3300\frac{1}{6}$. 9. 40,474.4375.
2. $17\frac{1}{2}$. 4. 69. 6. 415.54. 8. $7205\frac{5}{8}$. 10. 99,373.1875.

1. 24.48.	4. 335.5625.	7. 756.	10. 407.925.
2. 41.83.	5. 525.	8. 56.525.	11. lw ; 7750.
3. 3500.35.	6. 342.	9. 404.79.	12. $a = \frac{1}{2}fh$.

1. 15. 3. 57. 5. 126. 7. 456. 9. 6.225. 11. $37\frac{1}{2}$ sq. in.
2. 49. 4. 70. 6. 109.8. 8. 162.54. 10. 10. 12. $\frac{1}{2}(x+y)z$; 672.

1. 44.	6. 24.2.	11. 57.2.	16. 78.54.	21. 10.79925 in.
2. 66.	7. 44.	12. 66.	17. 188.496.	22. 75.3984.
3. 88.	8. 22.	13. 31.416.	18. 251.328.	23. 43.9824 in.
4. 11.	9. 35.2.	14. 62.832.	19. 15.708.	
5. 15.4.	10. 39.6.	15. 157.08.	20. 47.124.	

1. 78.54.	6. 7854.	11. 24.64.	16. 314.16.
2. 314.16.	7. 154.	12. 3850.	17. 1256.64.
3. 1256.64.	8. $38\frac{1}{2}$.	13. 75.46.	18. 7854.
4. 19.635.	9. 616.	14. 186.34.	19. 12.5664.
5. 1963.5.	10. 13.86.	15. 78.54.	20. 706.86 ;
			707 $\frac{1}{2}$; 0.28 $\frac{1}{2}$.

$$\begin{array}{lll} 1. l = ch. & 3. l = 2\pi rh. & 5. v = \frac{1}{3}bh. \\ 2. l = \pi dh. & 4. v = bh. & 6. v = \frac{1}{3}\pi r^2h. \end{array}$$

7. The lateral surface of a cone equals half the product of the distance from the vertex to the circumference of the base by the circumference of the base.

8. The lateral surface of a cone equals 3.1416 times the distance from the vertex to the circumference of the base multiplied by the radius of the base.

9. The surface of a sphere equals 4 times 3.1416 multiplied by the square of the radius.

10. The volume of a sphere equals $\frac{4}{3}$ times 3.1416 multiplied by the cube of the radius.

11. 5. 12. 40. 13. 45. 14. 110; 60. 15. 70; 14.

Exercise 261. Page 486

- | | | |
|----------|-----------------|-------------------------------|
| 1. \$60. | 4. \$450. | 7. 7 yr.; 14 yr.; 9 yr. 4 mo. |
| 2. 2 yr. | 5. \$750. | 8. $p = \frac{a}{1 + tr}$. |
| 3. 5%. | 6. 4 yr.; 8 yr. | |

Exercise 262. Page 489

- | | | |
|------------------|--|-------------------------|
| 1. 1 hr. 1 min. | 9. 8 hr. 28 min. 1 sec. | 17. 2 hr. 49 min. |
| 2. 1 min. 1 sec. | 10. 11 hr. 12 min. 2 sec. | 51 sec. |
| 3. 2 hr. 1 min. | 11. 20 min. 41 sec. | 18. 1 P.M. |
| 4. 3 hr. 2 min. | 12. 29 min. 42 sec. | 19. 1.01.01 P.M. |
| 5. 4 hr. 2 sec. | 13. 1 hr. 54 min. 50 sec. | 20. 5 A.M. |
| 6. 1 hr. 41 min. | 14. 4 hr. 9 min. 51 sec. | 21. 6 A.M. |
| 7. 2 hr. 10 min. | 15. 8 hr. 32 min. $24\frac{1}{2}$ sec. | 22. 2.30.50 P.M. |
| 8. 3 hr. 15 min. | 16. 2 hr. 10 min. 42 sec. | 23. 1 hr. 20 min. slow. |

Exercise 263. Page 490

- | | | | | |
|-----------------------|-----------------------|------------------------|-------------------------|--------------------------|
| 1. $30^{\circ} 45'$. | 4. $150^{\circ} 1'$. | 7. $81^{\circ} 15'$. | 10. $179^{\circ} 42'$. | 13. $60^{\circ} W$. |
| 2. $61^{\circ} 45'$. | 5. $165^{\circ} 2'$. | 8. $106^{\circ} 32'$. | 11. $30^{\circ} E$. | 14. $26^{\circ} 15' W$. |
| 3. $92^{\circ} 15'$. | 6. 65° . | 9. $143^{\circ} 55'$. | 12. $45^{\circ} W$. | |

Exercise 264. Page 492

- 11 A.M.; 10 A.M.; 1 P.M.; 12 M.
- 10 A.M.; 11 A.M.; 12 M.; 5 P.M.
- 2 P.M.; 2 P.M.; 12 M.; 3 P.M.; 8 P.M.
- 2.30 P.M.; 3.30 P.M.; 4.30 P.M.
- 11.45 A.M.; 11.45 A.M.; 11.45 A.M.; 11.45 A.M.; 12.45 P.M.
- 1.45 A.M. the next day; 1.45 A.M. the next day; 2.45 A.M. the next day; 2.45 A.M. the next day; 7.45 A.M. the next day.
- 2 P.M. 9. 2 P.M.; 1 P.M.; 8 A.M.; 5 A.M.
- 5 P.M. on Dec. 31. 10. 9 min. 42.72 sec.; standard.
11. 4 min. 6.3 sec.; local.

Exercise 265. Page 493

- | | |
|---------------------------------|---------------------------------|
| 1. 6 hr. 31 min. 19 sec. | 11. 135° E. |
| 2. 27 min. 8 sec. | 12. 150° E. |
| 3. 8 hr. 13 min. 32 sec. | 13. $37^{\circ} 30'$ W. |
| 4. 54 min. $31\frac{1}{5}$ sec. | 14. 3 hr. 30 min. 3 sec. fast. |
| 5. $80^{\circ} 14' 51''$ E. | 15. $12.09.38\frac{2}{15}$ P.M. |
| 6. $70^{\circ} 15' 18''$ W. | 16. 7.30 A.M. |
| 7. $86^{\circ} 48'$ W. | 17. $46^{\circ} 53' 45''$ W. |
| 8. $75^{\circ} 31' 15''$ W. | 18. $110^{\circ} 11' 12.2''$ W. |
| 9. 2 min. 22.2 sec. | 19. $17^{\circ} 26' 15''$ E. |
| 10. $41^{\circ} 22' 30''$ E. | |

Exercise 266. Page 497

- | | | | |
|--------------|---------------|----------------|----------------|
| 1. 25,000 m. | 12. 487.5 m. | 23. 1 m. | 34. 3.1 mi. |
| 2. 3700 m. | 13. 29.375 m. | 24. 9 m. | 35. 46.5 mi. |
| 3. 700 m. | 14. 96.4 m. | 25. 246.06 ft. | 36. 1.55 mi. |
| 4. 62,500 m. | 15. 250 m. | 26. 159.12 ft. | 37. 2.945 mi. |
| 5. 57,750 m. | 16. 0.5 m. | 27. 9.02 ft. | 38. 168.64 mi. |
| 6. 23.7 m. | 17. 2 m. | 28. 11.48 ft. | 39. 30.318 mi. |
| 7. 4.75 m. | 18. 3 m. | 29. 2.46 ft. | 40. 524.19 km. |
| 8. 0.275 m. | 19. 7.62 m. | 30. 22.47 ft. | 41. 68.87 km. |
| 9. 7.25 m. | 20. 1.46 m. | 31. 147.64 ft. | 42. 1.21 km. |
| 10. 0.685 m. | 21. 0.18 m. | 32. 22.56 ft. | 43. 5.7885. |
| 11. 29.65 m. | 22. 0.99 m. | 33. 70.46 ft. | 44. 194.06. |
| | | | 45. 351.61. |

Exercise 267. Page 498

- | | | |
|-----------------------|-------------------|---------------|
| 1. 275,000,000 sq. m. | 6. 6.4275 sq. m. | 11. 4.5 ha. |
| 2. 3,450,000 sq. m. | 7. 0.12345 sq. m. | 12. 17.5 ha. |
| 3. 750,000 sq. m. | 8. 2.25 sq. m. | 13. 32.55 ha. |
| 4. 0.4875 sq. m. | 9. 37.375 sq. m. | 14. 10 ha. |
| 5. 0.9235 sq. m. | 10. 1 ha. | 15. 600 ha. |
| | | 16. 90 m. |

Exercise 268. Page 499

- | | | |
|-------------------|-----------------|------------------|
| 1. 725.25 cu. m. | 4. 9125 cu. m. | 7. 25.325 cu. m. |
| 2. 2750 cu. m. | 5. 8.125 cu. m. | 8. 37.25 cu. m. |
| 3. 4625.75 cu. m. | 6. 9.275 cu. m. | 9. 71.75. |

Exercise 269. Page 500

- | | | | |
|--------------|----------------|--------------|-------------------|
| 1. 75,000 l. | 8. 6.25 l. | 15. 2.6 hl. | 22. 1260. |
| 2. 27,500 l. | 9. 37.5 hl. | 16. 1.26 hl. | 23. 11,390.625. |
| 3. 9275 l. | 10. 48.75 hl. | 17. 3700 qt. | 24. 864 ; 86,400. |
| 4. 937.5 l. | 11. 9.785 hl. | 18. 950 qt. | 25. 612,500. |
| 5. 75 l. | 12. 0.2575 hl. | 19. 75 qt. | 26. 763,408.8. |
| 6. 9.75 l. | 13. 0.975 hl. | 20. 82.5 qt. | |
| 7. 7.5 l. | 14. 0.3875 hl. | 21. 1000. | |

Exercise 270. Page 501

- | | | | |
|------------|--------------|--------------------|----------------|
| 1. 1700 g. | 7. 7.5 g. | 13. 15,000,000 mg. | 19. 0.25 kg. |
| 2. 900 g. | 8. 8.125 g. | 14. 2,700,000 mg. | 20. 2000 kg. |
| 3. 250 g. | 9. 7000 mg. | 15. 750 mg. | 21. 6.6 lb. |
| 4. 3750 g. | 10. 8700 mg. | 16. 98 mg. | 22. 165 lb. |
| 5. 7 g. | 11. 270 mg. | 17. 27,000 kg. | 23. 0.66 lb. |
| 6. 9.5 g. | 12. 4250 mg. | 18. 500 kg. | 24. 0.0198 lb. |

Exercise 271. Page 502

1. 0.79. 2. 0.7. 3. 0.24. 4. 1.03. 5. Yes. 6. 19.3. 7. 15.3153 kg.

Exercise 272. Page 503

- | | | |
|------------------------|-------------------|-----------------|
| 1. 1.8769 sq. m. | 6. 12,350 sq. cm. | 11. 2835. |
| 2. 5.372 sq. m. | 7. 2 m. | 12. 113. |
| 3. 17,671.5 sq. cm. | 8. 6.28 m. | 13. 0.8125. |
| 4. 82,448.1504 sq. mm. | 9. 1262.24 kg. | 14. 0.915 kg. |
| 5. 4.5375 sq. m. | 10. 137.05536 kg. | 15. 52.7558 in. |

